Derivation of the Fine Structure Constant from the electron wave function

Abstract

The fine structure constant α characterizes the strength of the electromagnetic interaction between elementary charged particles. Its value can be derived from the wave function for the electron. The electron wave function describes a field of Harmonic Oscillators which have the energy of the rest-mass of an electron. However, it takes time for such an Oscillator to complete a cycle of oscillation, so during this time the electromagnetic field that propagates into space from the Oscillator has expanded away from its source to cover the surface area of a three-dimensional shape. Therefore, the amplitude of the oscillations has been reduced before it can interact with its surrounding environment. The amount of this diminution is what the fine structure constant α represents. The actual value of α also depends on the fact that the source of the spreading energy is rotating Harmonic Oscillators. Classically, a point source will spread evenly in all directions and the intensity will diminish with Inverse-Square Law. But in the actual electron wave-function, the Harmonic Oscillators are rotating whilst expanding, causing the area that the energy spreads out over to be that of an expanding cone as well as that of a sphere. The area of this shape is greater than that of a simple sphere; this is what causes the fine structure constant α to have the value that it does.

The Analysis and derivation

The period of oscillation for a Harmonic Oscillator in the electron wave function [1] is equal to:

$\frac{2\pi\hbar}{m_e c^2}$ = $8.09329821808179682839791813090 99^{-21}$ seconds

Where m_e is the mass of an electron.

The distance travelled by light in this time is:

$2.42630976612576191624201607854 83^{-12}$ meters

So, this distance is the circumference of the circle described by the displacement of the medium (traveling at the speed of light) during this time.

Thus, the radius of this circle (the wave Amplitude) is:

$$\frac{2.42630976612576191624201607854 83^{-12}}{2\pi}$$

$$= \quad 3.86159192751055519248517570695 99^{-13} \text{ meters}$$

The distance that the expanding sphere has traveled, at the moment when the Harmonic Oscillator has completed one whole cycle around the circle is:

$$\frac{2.42630976612576191624201607854 83^{-12}}{2}$$

Thus, the radius of the expanding sphere of the Harmonic Oscillator's energy will be:

$$= \quad 1.21315488306288095812100803927 41^{-12} \text{ meters}$$

So, the surface area of this expanding sphere is:

$$4\pi(1.21315488306288095812100803927 41^{-12})^2$$

$$= \quad 1.84944902333260698897108401571 6^{-23} \; m^2$$

The squared amplitude of the wave energy over the area of this sphere as a fraction of the squared amplitude of the Harmonic Oscillator at its source (the radius of the circle described by the Oscillator) [2]:

$$\frac{1.84944902333260698897108401571 6^{-23}}{(3.86159192751055519248517570695 99^{-13})^2}$$

$$= \quad 124.025106721199280701905260268 4$$

This amount is equivalent to $4\pi^3$ [See Ref 2]

During the period of one oscillation of the electron's Harmonic Oscillator, the energy of the Oscillator spreads out in a sphere of radius r, such that the energy is then distributed over a spherical shell (of radius r). However, there is also another effect that is occurring at the same time as this expansion.

During this time of this expansion, every point in space (occupied by a portion of the electron's total energy as Harmonic Oscillators) will expand via Huygens wavelets, as a Harmonic Oscillator that sweeps out a path following its expansion and circular oscillation.

Thus, as the Oscillators all rotate in the same plane, parallel to each other, each point will expand in space covering an area described by cones along the electron spin axes (in both directions, with its apex at its starting location, and its circular base on the surface of the spherical shell of expansion over the oscillator's time period. See Figure 1).

As the energy of a Harmonic Oscillator refers to a complete cycle (one wavelength) the cone shape is swept out during the period of one complete cycle, thus there will be one wavelength along its slanting sides and one wavelength around the circumference of the circle on its base. Thus, the energy of one complete cycle of the Harmonic Oscillator is spread over both of these surfaces.

Each point in space contains only a fraction of the total energy of the electron and there are many such cones, but all cones are identical in geometry, so the total effect of this expansion and rotation can be simplified by treating the total electron energy as expanding over a single cone.

So, the total area over which the energy of the electron's Harmonic Oscillator will be diluted is that of the spherical shell plus that of the cone.

The side length of such a cone is the same as the radius of the spherical expansion:

$$1.21315488306288095812100803927 41^{-12} \text{ meters}$$

The circumference of the circle on the base of the cone will also be this same amount, so the radius of the base will be that of the Harmonic Oscillator's oscillation amplitude:

$$3.86159192751055519248517570695 99^{-13} \text{ meters}$$

So, the additional surface area that the energy of the Harmonic Oscillator spreads over is the area of the sides of the cone + the area of the cone's base.

$$\pi r x + \pi x^2 \qquad \text{(see Ref [2])}$$

Where x is the radius of the cone's base (the Harmonic Oscillator's amplitude).

Thus, the total additional area swept out by the expanding Harmonic Oscillators is:

$$\pi * 1.2131548830628809581210080392741^{-12} * 3.8615919275105551924851757069599^{-13} + \pi * (3.8615919275105551924851757069599^{-13})^2$$

$$= 1.9402156806248756383962538050746^{-24}\ m^2$$

The squared amplitude of the wave energy over the area of this cone as a fraction of the squared amplitude of the Harmonic Oscillator at its source (the radius of the circle described by the Oscillator) [2]:

$$\frac{1.9402156806248756383962538050746^{-24}}{(3.8615919275105551924851757069599^{-13})^2}$$

= 13.011197054679151857297134383156

The total value for the factor is that attributed to the sphere + that attributed to the cone. So, the calculated value of $\frac{1}{\alpha}$ is:

124.02510672119928070190526026 84 + 13.011197054679151857297134383156

= 137.03630377587843255920239465156

A good way to understand the two components of the area that the energy is spread over is by separating the components:

1. Imagine that the Harmonic Oscillators are frozen in time, but they expand over the sphere such that their amplitude is diminished by inverse square law.

2. Imagine the expansion is frozen and the Harmonic Oscillators are now allowed to oscillate over one cycle - thus, over the whole expanded volume they rotate in a cone shape, sweeping out the additional area, diluting their effect over that area too.

The accepted CODATA [3] value of $\frac{1}{\alpha}$ is 137.035999083695801114154891488 26

(Relative standard uncertainty   1.5 x 10^{-10})

The value calculated above is, therefore, accurate to 99.9998% of the accepted value.

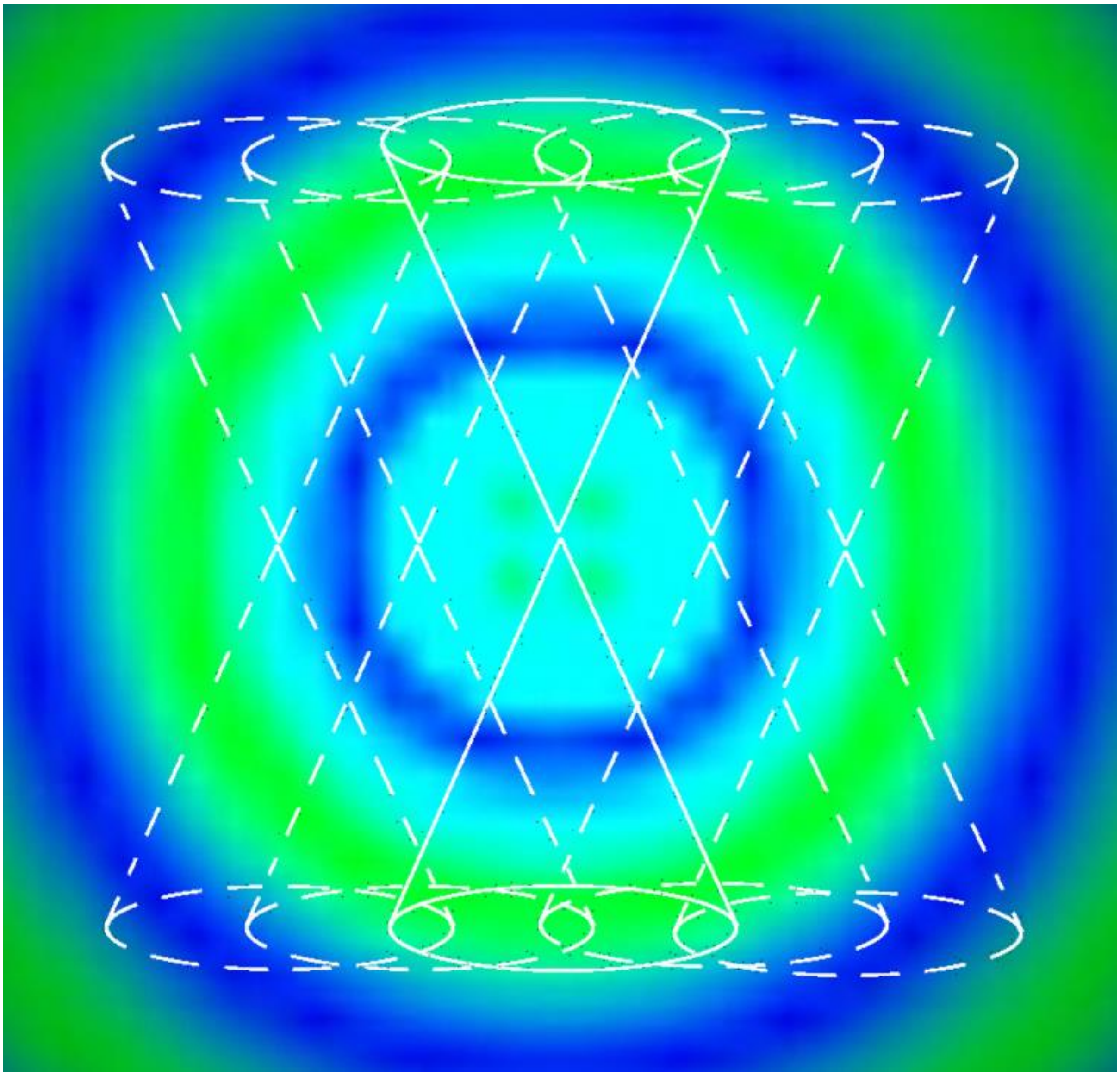

Figure 1.

References:

[1] Traill. D. A. “Wave functions for the electron and positron”, 2018,

https://www.researchgate.net/publication/326646134_Wave_functions_for_the_electron_and_positron Last accessed 6/7/2019

[2] Yee. J. “The Relationship of the Fine Structure Constant and Pi”, 2019,

https://www.researchgate.net/publication/333224718_The_Relationship_of_the_Fine_Structure_Constant_and_Pi Last accessed 6/7/2019

[3] “NIST Reference on Constants, Units and Uncertainty – fine-structure constant”, 2018 CODATA values.

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