A Diagonal Map of the Real Numbers in the Closed Interval $[0, 1]$ to the Natural Numbers

**Abstract.** The terminating decimal fractions in the open interval $(0, 1)$ are put in 1-1 correspondence with the set of positive integers. Defining the reals in $[0, 1]$ as the limits of their partial decimal sums, sets up a diagonal map to the natural numbers.

The positive integers can be put in one to one correspondence with the terminating decimal fractions in the open interval $(0, 1)$. Each terminating decimal is the mirror image reflection through the decimal point of a positive integer. The mapping does not include any repeating decimal fractions. From this mapping the set of all rational numbers would appear to be uncountable. This shows that attempting to map the real numbers in the closed interval $[0, 1]$ to the natural numbers, by listing them as unending decimal fractions is futile. Cantor’s diagonal proof that the real numbers are an uncountable set can never even get started.

The problem is in defining the real numbers in $[0, 1]$ as the set of all unending decimal expansions. That’s vague. Most non-algebraic real numbers cannot be explicitly referenced. We need a new definition.

Each real number $S_{n}$ in the closed interval $[0, 1]$ is:

\[
\text{the limit } m \rightarrow \infty \ n = 1 \text{ to } m, 0 \leq a_n \leq 9 \sum_{n=1}^{m} a_n / 10^n = S_{n}.
\]

We move through the terminating decimals in $[0, 1]$ as the mirror image reflection through the decimal point of the natural numbers.

The subscript of $S_{n}$ is the negative of the natural number that is reflected through the decimal point. $0 \rightarrow 0, 1 \rightarrow 1, 2 \rightarrow 2, \ldots, 10 \rightarrow .01$.

We treat the unending decimals in $[0, 1]$ similarly. $1 \rightarrow .1\ldots, 2 \rightarrow .2\ldots, 3 \rightarrow .3\ldots, \ldots, 10 \rightarrow .01\ldots$

But, each of these figures can represent any one of an infinite number of unending decimals. To represent them we make an infinite grid leaving out any duplicate representations.

The first column is all the potential unending decimals represented by $.1\ldots$

The second column is all the potential unending decimals represented by $.2\ldots$

The third column is all the potential unending decimals represented by $.3\ldots$ and so forth.

We use ordered pairs of positive integers to represent the numbers in the grid. $T(a, b)$ represents the $b(th)$ potential unending decimal represented by $.a\ldots$

We make a diagonal mapping of $S_n$ to $T(a, b)$.

All the decimals in $[0, 1]$ can be mapped to the natural numbers. We map the terminating decimals to the odd numbers and the unending decimals to the even numbers.

Explore the detailed proofs and fascinating consequences of the real numbers as a denumerable set in [https://arxiv.org/abs/1002.4433](https://arxiv.org/abs/1002.4433)

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