Refutation of the axiom of dependent choices in mice

Abstract: The axiom of dependent choices as $\forall a \in X \exists b \in XP(a, b) \Rightarrow \exists f: \omega \to X \forall n\, P(f(n), f(n+1))$ is *not* tautologous. What follows is that the axiom of determinacy is also *no*t tautologous, hence relegating both axioms to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ▽, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Müller, S. (2019). The axiom of determinacy implies dependent choices in mice. arxiv.org/pdf/1907.02755.pdf

1. Introduction

We prove that in passive, countably iterable mice M constructed over their reals, AD, the Axiom of Determinacy, implies DC, the Axiom of Dependent Choices, working in a background universe which satisfies ZF+DCRM. Here we write RM = R ∩ M for the set of reals in M.

Recall that DC is the following statement: For every nonempty set X and every binary relation P on X,

$$\forall a \in X \exists b \in X\, P(a, b) \Rightarrow \exists f : \omega \to X\ \forall n\, P(f(n), f(n + 1)). \quad (1.0.1)$$

LET q, r, s, t, w, p, x: a, b, f, n, w, P, X.

((#q<(x&%r))<(p&(q&r)))>(%s=(w>((x&(#t&p))&((s&t)&(s&(t+(%z>#z))))))) ;

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TTCC TTTT TTTT TTTT(x 8)
TTTT TTTT TTCC TTTT(x 1)
TTTT TTTT TTCT TTTT(x 1)          (1.0.2)
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Eq. 1.0.2 as rendered for the axiom of dependent choices is *no*t tautologous, thereby refuting it. What follows is that the axiom of determinacy is also *no*t tautologous.