Refutation of analytic choice principles: axioms of choice and dependent choice

Abstract: The axiom of Γ choice and axiom of Σ^1_1-dependent choice are *not* tautologous. Therefore an open problem on the Weihrauch degree of parallelization of the Σ^1_1-choice principle on the integers is not solved by using those axioms. These axioms form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Anglès d'Auriac, P-E.; Kihara, T. (2019). A comparison of various analytic choice principles. arxiv.org/pdf/1907.02769.pdf

2. Equivalence results in the Weihrauch lattice

2.1. Σ^1_1-Choice Principles.

One of the main notions in this article is the Σ^1_1-choice principle. … In logic, the axiom of Γ choice, Γ-AC, is known to be the following statement [where ϕ is a Γ formula]:

$$\forall a \exists b\, \phi(a, b) \rightarrow \exists f \forall a\, \phi(a, f(a)) \quad (2.1.1.1)$$

LET p, q, r, s, t: a, b, f, ϕ, n.

(s&(#p&%q))>(s&(#p&(%r&#p))) ;

`TTTT TTTT TTTC TTTT` (2.1.1.2)

In logic, the axiom of Σ^1_1-dependent choice on X is the following statement [where ϕ is a Σ^1_1-formula, and a and b range over X]:

$$\forall a \exists b\, \phi(a, b) \longrightarrow \forall a \exists f\, [f(0) = a \,\&\, \forall n\, \phi(f(n), f(n + 1))] \quad (2.1.2.1)$$

(s&(#p&%q))>((r&(z@z))=(#p&(s&((%r&#t)&(%r&(#t+(%z>#z))))))) ;

`TTTT TTTT TTTT TTTT( 1)`
`TTTT TTTT TTTT TTTC( 1)` (2.1.2.2)

The axiom of Γ choice, Eq. 2.1.1.2 as rendered, and axiom of Σ^1_1-dependent choice, 2.1.2.2, are *not* tautologous. Therefore an open problem on the Weihrauch degree of parallelization of the Σ^1_1-choice principle on the integers is not solved by using those axioms.