Shorter refutation of Gödel’s completeness theorem

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Abstract: The completeness theorem rendered as $(\forall x.R(x,x))\rightarrow(\forall x \exists y.R(x,y))$ is not tautologous. The application of Isabelle/HOL to prove the same also is not tautologous, to invalidate that tool. These demonstrations form a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Let $\sim$ Not, ¬; $+$ Or, $\lor$, $\cup$; $-$ Not Or; $\&$ And, $\land$, $\cap$; \ Not And; $>$ Imply, greater than, $\rightarrow$, $\Rightarrow$, $\supset$; $<$ Not Imply, less than, $\in$, $\prec$, $\subset$; @ Not Equivalent, $\neq$;

Remark 1.2: Not distributing the quantifiers as $(\#q&(r&(q&q)))>(\#q&%s)&(r&(q&s))$ produces the same truth value table.

Eq. 1.2 as rendered is not tautologous, refuting Gödel's completeness theorem.


From: en.wikipedia.org/wiki/Gödel%27s_completeness_theorem

Gödel's completeness theorem is a fundamental theorem in mathematical logic that establishes a correspondence between semantic truth and syntactic provability in first-order logic. … The formula

$$(\forall x.R(x,x))\rightarrow(\forall x \exists y.R(x,y))$$

holds in all structures. By Gödel's completeness result, it must hence have a natural deduction proof.

Let $q, r, s$: $x, R, y$.

$$(r&(\#q&\#q))>(r&(\#q&\%s))$$

Remark 1.2: Not distributing the quantifiers as $(\#q&(r&(q&q))>((\#q&\%s)&(r&(q&s)))$ produces the same truth value table.