Shorter refutation of Gödel’s completeness theorem

Abstract: The completeness theorem rendered as (∀*x*.*R*(*x*,*x*))→(∀*x*∃*y*.*R*(*x*,*y*)) is *not* tautologous. The application of Isabelle/HOL to prove the same also is *not* tautologous, to invalidate that tool. These demonstrations form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Gödel%27s_completeness_theorem

Gödel's completeness theorem is a fundamental theorem in mathematical logic that establishes a correspondence between semantic truth and syntactic provability in first-order logic. … The formula

(∀*x*.*R*(*x*,*x*))→(∀*x*∃*y*.*R*(*x*,*y*)) (1.1)

holds in all structures. By Gödel's completeness result, it must hence have a natural deduction proof.

LET q, r, s: *x*, *R*, *y*.

(r&(#q&#q))>(r&(#q&%s)) ; TTTT TTCC TTTT TTTT (1.2)

Remark 1.2: Not distributing the quantifiers as (#q&(r&(q&q)))>((#q&%s)&(r&(q&s))) produces the same truth value table.

Eq. 1.2 as rendered is *not* tautologous, refuting Gödel's completeness theorem.

Remark 1.3: The following paper in using an assistant to prove Gödel's completeness theorem further refutes the Isabelle/HOL tool itself: Margetson, J. (2014). Proving the completeness theorem within Isabelle/HOL. isa-afp.org/entries/Completeness-paper.pdf.