Proof of the Riemann hypothesis

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Abstract

I treat Riemann hypothesis as an infinite series and proved it.

key words

Riemann-Zeta function, non-trivial zero point, infinite series

1 introduction

\[
\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad s = c + ix
\] (1)

If \( c = 1/2 \), \( x \) is non-trivial zero value, then Eq. (1) becomes zero.

\[
\zeta(s) = 2^s \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1 - s) \zeta(1 - s)
\] (2)

\[
\xi(s) = \frac{1}{2} s(s - 1) \pi^{-s/2} \Gamma \left( \frac{1}{2} s \right) \zeta(s)
\] (3)

which satisfies:

\[
\xi(s) = \xi(1 - s)
\] (4)

The formula

\[
\cos \theta + i \sin \theta = e^{i\theta}
\]

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can be rewritten as shown below.

\[ \cos \theta - i \sin \theta = e^{-i\theta} \]

Riemann-Zeta Function is also defined as below. (put \( s = c + ix \).)

\[
(1 - \frac{2}{2^s}) \zeta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^s} - \sum_{n=1}^{\infty} \frac{1}{(2n)^s} 
\]

\[
= \sum_{n=1}^{\infty} \left[ \frac{1}{(2n - 1)^{c+ix}} - \frac{1}{(2n)^{c+ix}} \right] = \sum_{n=1}^{\infty} \left[ \frac{(2n - 1)^{-ix}}{(2n - 1)^c} - \frac{(2n)^{-ix}}{(2n)^c} \right] 
\]

Using the formula \( a^{b+ix} = a^b \left[ \cos(x \ln a) + i \sin(x \ln a) \right] \)
equal if \( b=0 \)

\( a^{-ix} = \cos(x \ln a) - i \sin(x \ln a) \)
equal \( (2n)^{-ix} = \cos(x \ln(2n)) - i \sin(x \ln(2n)) \)

\[
= \sum_{n=1}^{\infty} \left[ \frac{\cos(x \ln(2n - 1)) - i \sin(x \ln(2n - 1))}{(2n - 1)^c} - \frac{\cos(x \ln(2n)) - i \sin(x \ln(2n))}{(2n)^c} \right] 
\]

\[
\zeta(s) = \left[ 1/(1 - 2^{1-s}) \right] \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} 
\]

\[
= \left[ 1/(1 - 2^{1-s}) \right] \sum_{n=1}^{\infty} \left[ \frac{\cos(x \ln(2n - 1)) - i \sin(x \ln(2n - 1))}{(2n - 1)^c} - \frac{\cos(x \ln(2n)) - i \sin(x \ln(2n))}{(2n)^c} \right] = 0 
\]

\[
\left[ 1/(1 - 2^{1-s}) \right] \neq 0 
\]

\[
\sum_{n=1}^{\infty} \left[ \frac{\cos(x \ln(2n - 1))}{(2n - 1)^c} - \frac{\cos(x \ln(2n))}{(2n)^c} \right] = 0 
\]

\[
\sum_{n=1}^{\infty} \left[ \frac{\sin(x \ln(2n - 1))}{(2n - 1)^c} - \frac{\sin(x \ln(2n))}{(2n)^c} \right] = 0 
\]

Although \( x \) is treated as a real number, \( x \) is a non-trivial zero value.

From Eq.(9), it is estimated that \( \cos \) is a real value and \( \sin \) is an imaginary value. When this
real value and the imaginary value reach zero simultaneously, they become non-trivial zero value. Both \( \cos \theta \) and \( \sin \theta \) just rotate the circle with radius 1. Therefore, no matter how large the value of \( x \) or how small the value of \( x \) is, no failure can occur.

Eq.(1) is the definition of Riemann-Zeta function itself, and Eq.(11) and Eq.(12) are modified Eq.(1).

Eq.(11) and Eq.(12) hold when \( c = 1/2 \), \( x \) is non-trivial zero value.

And, from Eq.(1), \( \Re(s) = 1/2 \)

Proof complete.

References


Please raise the prize money to my little son and daughter who are still young.