Proof of the Riemann hypothesis

Toshiro Takami

mmm82889@yahoo.co.jp

Abstract

In my previous paper “Consideration of the Riemann hypothesis” c=0.5 and x is non-trivial zero value, and it was described that it converges to almost 0, but a serious proof in mathematical expression could not be obtained. In this paper, we give a proof of mathematical expression. “the non-trivial zero values of all positive infinity and negative infinity lie on the real value 0.5” I am here mathematically proved.

introduction

\[ \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = 0 \quad s = c + ix \]  

(1)

c=0.5 \quad x \text{ is non-trivial zero value.}

\[ \zeta(1 - s) = \frac{2}{(2\pi)^s} \Gamma(s) \sin\left(\frac{\pi s}{2}\right) \zeta(s) \]  

(2)

Substituting c=0.5 and x is non-trivial zero value, for Eq.(2). It results in almost zero.

\[ if \quad \zeta(s) = 0 \quad then \quad \zeta(1 - s) = 0 \]  

(3)

Eq.(3) holds as a mirror image of the axis(c=0.5).

\[ \sum_{n=1}^{\infty} \frac{1}{n^s} = \sum_{n=1}^{\infty} \frac{1}{n^{a+ix}} = \sum_{n=1}^{\infty} \frac{n^{-x}}{n^a} = \sum_{n=1}^{\infty} \frac{\exp(-ix \ln(n))}{n^a} = \sum_{n=1}^{\infty} \frac{\cos(x \ln(n)) - i \sin(x \ln(n))}{n^a} \]  

(4)

\[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{2c}} = \sum_{n=1}^{\infty} \left[ \frac{1}{(2n-1)^{2c}} - \frac{1}{(2n)^{2c}} \right] = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2c}} - \sum_{n=1}^{\infty} \frac{1}{(2n)^{2c}} \]  

(5)
\begin{align}
0 & < c < 1 \\
\frac{1}{(2n-1)^{2c}} &= \frac{(2n-1)^ix}{(2n-1)^c} = \frac{\cos(x \ln(2n-1)) - i \sin(x \ln(2n-1))}{(2n-1)^c} \tag{7} \\
\frac{1}{(2n)^{2c}} &= \frac{(2n)^ix}{(2n)^c} = \frac{\cos(x \ln(2n)) - i \sin(x \ln(2n))}{(2n)^c} \tag{8} \\
\sum_{n=1}^{\infty} \left[ \frac{\cos(x \ln(2n-1)) - i \sin(x \ln(2n-1))}{(2n-1)^c} - \frac{\cos(x \ln(2n)) - i \sin(x \ln(2n))}{(2n)^c} \right] \tag{9}
\end{align}

Although \(x\) is treated as a real number, \(x\) is a non-trivial zero values.

From equation Eq.(9), it is estimated that \(\cos\) is a real value and \(\sin\) is an imaginary value. When this real value and the imaginary value reach zero simultaneously, they become non-trivial zero values.

c is complex number but treated as a real number.

\begin{align}
\sum_{n=1}^{\infty} \left[ \frac{\cos(x \ln(2n-1))}{(2n-1)^c} - \frac{\cos(x \ln(2n))}{(2n)^c} \right] \tag{10} \\
\sum_{n=1}^{\infty} \left[ \frac{\sin(x \ln(2n-1))}{(2n-1)^c} - \frac{\sin(x \ln(2n))}{(2n)^c} \right] \tag{11}
\end{align}

And, from [5] (it is my previous paper “Consideration of the Riemann hypothesis”)
In the paper, Eq.(10) is calculated as \(x\) = non-trivial zero value.

(Summary 1) is from [5] (my previous paper “Consideration of the Riemann hypothesis”).
(Summary 1)
If \(c\) shifts to 0.00001, it converge around -1.69.
If \(c\) shifts to 0.1, it converge around -1.04.
If \(c\) shifts to 0.2, it converge around -0.462.
If \(c\) shifts to 0.3, it converge around -0.745.
If \(c\) shifts to 0.4, it converge around -0.200.
If \(c\) shifts to 0.49, it converge around -0.02.
If \(c\) shifts to 0.499, it converge around -0.009.
If \(c\) shifts to 0.4999, it converge around -0.0002.
If \(c\) shifts to 0.49999, it converge around -0.00003.
If \(c\) shifts to 0.499999, it converge around -0.0000077.
If \(c\) shifts to 0.5000001, it converge around 0.0000089.
If \(c\) shifts to 0.500001, it converge around 0.000017.
If \(c\) shifts to 0.5001, it converge around 0.0002.
If \(c\) shifts to 0.501, it converge around 0.009.
If \(c\) shifts to 0.51, it converge around 0.02.
If \(c\) shifts to 0.6, it converge around 0.199.
If \(c\) shifts to 0.7, it converge around 0.349.
If \(c\) shifts to 0.8, it converge around 0.477.
If \(c\) shifts to 0.9, it converge around 0.583.
If \(c\) shifts to 0.99999, it converge around 0.67.

From the above, it can be seen that if \(c\) deviate from 0.5, the converging value deviate from zero.
However, it lacks rigor.

Discussion

If \(c=0.5\) and \(x\) is non-trivial zeros, Eq.(10)=0 and Eq.(11)=0 must hold.

Eq.(10) and Eq.(11) have almost the same value.

\[Eq.(10) \quad \text{and} \quad Eq.(11) \quad \text{converges when} \quad (0 < c < 1).\] (12)

In order to converge to 0, no divergence occurs and convergence is the minimum condition.
If $c$ is less than 0.5, It converges to a negative number[see (Summary 1)].

If $c$ is more than 0.5, It converges to a positive number[see (Summary 1)].

It can not be concluded that $c = 0.5$ as far as the above-mentioned (Summary 1). is examined to what value it converges. This is because the possibility of being near the pole of $c = 0.5$ can not be denied.

But, It must be symmetrical around 0.5 between real numbers 0 and 1 from Eq.(3).

According to (Summary 1), when $c$ is smaller than 0.5, it converges negatively, and when $c$ is larger than 0.5, it converges positively. And from Eq.(3),

$$c = \frac{1}{2}$$

proof complete.

References


key words
Riemann hypothesis, infinite series, negative and positive infinity
Please raise the prize money to my little son and daughter who are still young.