Refutation of a weak set theory H that proves its own consistency

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Abstract: For theory H, the axioms of extensionality and separation are *not* tautologous. The theorem to prove any instances of the scheme of ε-induction is also *not* tautologous. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Pakhomov, F. (2019). A weak set theory that proves its own consistency. arxiv.org/pdf/1907.00877.pdf

1. Introduction

We define a theory $H_{<\omega}$ and show that it proves its own Hilbert-style consistency. Unlike Willard's theories, our theory isn't arithmetical but rather a system in the language of set theory with additional unary function V. The axioms of H are

1. x = y ↔ ∀z(z ∈ x ↔ z ∈ y) (Extensionality); (1.1.1)

LET p, q, r, s: x, y, z, ϕ.

(p=q)=((#r<p)=(#r<q)) ; `TFFT TNNT TFFT TNNT` (1.1.2)

2. ∃y∀z(z ∈ y ↔ z ∈ x ∧ ϕ(z)), where ϕ(z) ranges over first-order formulas without free occurrences of y (Separation); (1.2.1)

(#r<%q)=(#r<(p&(s&#r))) ; `TTTT TTCC TTTT TCCT` (1.2.2)

A. Theory H is non-Gödelian

Lemma 10. Theory H prove any instances of the scheme of ε-induction:

∀x((∀y ∈ x)ϕ(y) → ϕ(x)) → ∀x ϕ(x). (10.1)

(((#r<#q)&(p& r))>(p&#q))>(p&#q) ;
`FFF``N` **`F`**`N`**`F`**`N` **`FFF`**`N` **`F`**`N`**`F`**`N` (10.2)

For theory H, the axioms of extensionality and separation in Eqs. 1.1.2 and 1.2.2 as rendered and the theorem to prove any instances of the scheme of ε-induction in 10.2 are *not* tautologous.