

Two Identities

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abstract

This note presents two identities for Pi

The number Pi is defined by the integral

$$\pi = 2 \int_{-1}^1 \sqrt{1-x^2} dx = 3.14159265... \quad (1)$$

This note presents two formulas for Pi.

Formulas

Entry 1. If $0 \leq x < 1$ then

$$\pi = 2 \sin^{-1}(\sqrt{1-x^2}) + 6 \sin^{-1}\left(\frac{x}{3} + \frac{4}{3}\left(\frac{x}{3} + \frac{4}{3}\left(\frac{x}{3} + \dots\right)^3\right)^3\right) \quad (2)$$

Entry 2. If $\frac{\sqrt{3}}{2} \leq x \leq 1$ then

$$\pi = \frac{2}{3} \sin^{-1}(\sqrt{1-x^2}) + 2 \sin^{-1}\left(\frac{1}{2} \sqrt[3]{2x + 3\sqrt{2x + 3\sqrt{2x + \dots}}}\right) \quad (3)$$

References

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