Unified Electro-Gravity (UEG) Theory Applied to Stellar Gravitation, and the Mass-Luminosity Relation (MLR)

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The Unified Electro-Gravity (UEG) theory is applied to model gravitational effects of an individual star or a binary-star system, including that of the sun which is the only star of our solar system. The basic UEG theory was originally developed to model elementary particles, as a substitute for the standard model of particle physics. The UEG theory is extended in this paper (a) to model the gravitational force due to light radiation from an individual star, which determines its energy output due to nuclear fusion in the star, as well as (b) to model the gravitational force between two nearby stars, which determines the orbital dynamics in a binary-star system. The mass-luminosity relation (MLR) derived separately from each of the above two models are compared and studied together with the MLR currently available from measured orbital data for binary stars, as well as from an existing energy-source model for stellar nuclear fusion (Eddington's model). The current MLR data uses conventional Newtonian gravity, where the gravitational force is produced only due to the gravitational mass of the star, which is assumed to be equal to the inertial mass as per the principle of equivalence. This Newtonian gravitational model is modified by including the new UEG effect due to the light radiation of a star, in order to establish the actual MLR which can be significantly different from the currently available MLR. The new UEG theory is applied to an individual isolated star (for modeling the force for stellar nuclear fusion), which is spherically symmetric about its own center, in a fundamentally different manner from its application to a binary-star system (for modeling orbital motion of a binary-star), which is not a spherically-symmetric structure.

I. INTRODUCTION

A new Unified Electro-Gravity (UEG) theory was developed in [1, 2], as a substitute for the standard model of particle physics, which successfully modeled elementary particles. The UEG theory introduced a new definition for the energy density in an electromagnetic field, which effectively resulted in having a new gravitational force proportional to the conventional energy density, directed toward the center of gravity of a particle. The center of gravity of the particle is located at its physical center due to spherical symmetry of the structure, because the particle is assumed to be spherically symmetric with respect to itself and it is treated ideally in isolation from any surrounding object. This simplicity of spherical symmetry may not be valid in general situations, for example in modeling gravitation due to light radiation from individual stars in a binary-star system [3], which clearly is not spherically symmetric with respect to the total structure, even though its two individual partner stars may be spherically symmetric with respect to their own physical centers. The basic UEG theory of [1] developed for particle physics, needs to be modified for a general radiating structure with no simple symmetry, particularly to model gravitation in a binary-star system with approximately identical partner stars.

Computation of an equivalent gravitational force due to stellar radiation from a given star, as per the new UEG theory, requires definition of (a) a suitable gravitational center toward which the force is directed at, determined by the gravitational parameters of the particular star as well as of its surrounding objects, and (b) a suitable effective energy density defined at the point of observation of the force, determined by the intensity distribution due to the star's radiation in the vicinity of the observation point. A rigorous UEG model that would be applicable for any stellar or galactic structure, with general distribution of radiation, appear premature at this point. In this paper, we postulate a suitable model for a binary-star structure, applied specifically when its individual partner stars are approximately similar to each other and are positioned relatively close to each other (eclipsing binary), as useful special situations. This would allow us to revisit the mass-luminosity relation (MLR) of [3, 4], deduced from measured observation of mostly closely spaced, eclipsing binaries [5]. Considering the large range of star luminosities in [3], with upper limit as large as 10^5 solar luminosity, we expect the effective gravitational mass of an individual star in a binary system to be in general appreciably different from its inertial mass, due to additional UEG effect from the stellar radiation. In other words, the mass term in the MLR of [3], which should in principle be the gravitational mass of a star but is assumed to be equal to its inertial mass as per the principle of equivalence of general relativity [6], might not be really equal to the inertial mass of a star. The actual inertial mass would be deduced using the proposed model to provide a new stellar MLR ((actual inertial)mass-luminosity relation). This would be a modern advancement in the physics of stellar gravitation, as it relates to the orbital dynamics of binary stars, based on the new UEG theory.

The MLR data from orbital measurement of binary stars were also believed to be confirmed in [4, 7], with sound theoretical results from a stellar energy-source model based on balancing of gravitational pressure with thermal pressure from nuclear fusion (Eddington's Model). Therefore, in order to definitively validate the UEG theory, the energy-source model also needs to be modified based on the UEG theory, and the results for both the models of [3, 7] must be shown to be consistent with each other. Accordingly, the new MLR ((actual inertial) mass-luminosity relation) as derived from a stellar energy-source model should be the same or sufficiently similar to that deduced, as discussed earlier, from binary-star measurements, when the additional gravitation effects of the new UEG theory are included. In equivalent terms, for a given actual mass (inertial) and its associated light output of a star, the presence of the additional gravitational effects due to the new UEG theory in the two MLR models should lead to the same or comparable "equivalent-mass" as the mass-term in the MLR data of [3, 4, 7], which is derived using only conventional Newtonian gravity. The "equivalent-mass" parameter, which is a function of the inertial mass and star light, is defined here such that the expression of the star's luminosity derived using any particular model based on only Newtonian gravity, and that using a rigorous derivation including additional gravitation due to the star light as per the UEG theory, would appear in functionally identical forms. Except, the mass term in the simple Newtonian case is replaced by the equivalent-mass in the rigorous derivation using the UEG theory.

The gravitational forces which determine nuclear fusion in the energy-source model of a star act differently from those in a binary-star system which determine the orbital motion of the partner stars. In the former case, the gravitational forces of a given star act upon the star's own mass (ignoring any small opposing gravitation from nearby star(s)), whereas in the later the gravitational forces from one partner star acts upon the mass of the other of the binary system. The former is a spherically symmetric problem, whereas the later is not. The existing models in [3, 7] are based on conventional Newtonian gravity, where the the same gravitational mass is used when the star is either a source or target of gravitation, which is equal to the inertial mass of the star. Consequently, the same unique mass term, equal to the inertial mass of the star, is used in the two existing MLR models of [3, 7]. However, due to the basic differing natures of gravitation acting in the two MLR models, as explained above, one might be inclined to expect that the presence of any additional gravitational forces due to the new UEG theory would in general result in two different "equivalent-mass" terms for the two models, leading to apparent inconsistency in the UEG theory. However, the "equivalent-mass" terms in the two MLR models, based on the new UEG theory, would be shown in this paper to end up with approximately the same functional trend and magnitude, thus resolving the apparent inconsistency. This would be a significant result, definitively validating the UEG theory, as extended to stellar nuclear fusion as well as orbital dynamics of binary stars.

It may be noted, that the MLR of [3, 4, 7] for solar-mass stars is implicitly assumed to confirm with the measured luminosity of the sun and its effective gravitational mass as observed from planetary motions in our solar system. The sun is the only star in our solar system, with orbiting planets having a fraction of the solar mass [8]. Accordingly, the sun may be treated essentially as an isolated star, for modeling its gravitation using the UEG theory. Under this condition, the effective gravitational mass m_g of the sun would be equal to the sum of its inertial mass m and the UEG mass m_u due to its total luminosity L_0. That is, $m_g = m + m_u$. The UEG mass m_u may be calculated using the known value of solar luminosity L_0 [9] and the UEG constant γ deduced in [1, 2] from modeling of elementary particles. As mentioned, the total effective gravitational mass of the sun is also known [9], based on the observed orbital periods of planets. The inertial mass of the sun may then be estimated by subtracting the UEG mass from the total effective gravitational mass. Now, in order that the UEG theory confirms with the two models of MLR [3, 4, 7], specifically for the mass parameters of the sun, we need to show that the mass (actual inertial) term in the "new" MLR derived using the UEG theory is equal to the known inertial mass m of the sun, whereas the mass (equivalent gravitational) term in the "existing" MLR [3, 4, 7] is equal to the known gravitational mass m_g of the sun, when the luminosity is equal to the solar luminosity. Conversely, the expressions of the MLRs (new and existing) derived in functional forms, with the additional functional constraint $m_g = m + m_u$ required specifically for the sun, could allow for an estimate solution of the unknown UEG mass m_u of the sun, from which the UEG constant γ could be deduced. This would provide a useful estimation for the UEG constant γ, which may be verified with estimation from the particle physics model [1]. This would independently support the UEG theory as well as the value of the UEG constant, for the stellar models.

The expressions of the MLR, currently existing based on the conventional Newtonian gravity, is introduced in section II. This is followed by presentations of the two new models including the UEG effects. The spherically symmetric problem of the stellar energy-source model would be presented first in section III. The relatively simple and definitive results from this study would then be used together with the required constraints for the sun, in order to analytically estimate the UEG constant γ. This would be followed by modeling of the gravitation in a binary-star system in section IV, based on the UEG theory. A general treatment applicable for any separation between the partner stars, and any level of luminosity and associated UEG mass of each star, would be presented. This would require numerical integration for evaluating the gravitational force between the two stars. In addition, a useful limiting situation when the effective gravitational force of each star is much larger than its inertial mass, will be presented. The gravitational force in the limiting case can be evaluated using simple analytical formulas, which may be compared with the results from the general derivation for validation. The results for new MLR deduced from the energy-source and the binary-star models would be compared with each other, as well cross-checked with the parameters of the sun in section V, followed by general discussions and conclusions from the study in section V.

II. STELLAR MASS LUMINOSITY RELATION (MLR) BASED ON NEWTONIAN GRAVITATION

The relationship between stellar mass, m, and luminosity, L, as it currently exists to date, is expressed as [4]:

$$
\begin{aligned}
\frac{L}{L_0} &= 0.23(\frac{m}{m_0})^{2.3},\ m < 0.43m_0, \\
&= (\frac{m}{m_0})^4,\ 0.43m_0 < m < 2m_0, \\
&= 1.5(\frac{m}{m_0})^{3.5},\ 2m_0 < m < 20m_0, \\
&= 3200(\frac{m}{m_0}),\ m > 20m_0. \qquad (1)
\end{aligned}
$$

where the parameters with a subscript 0 are associated with the sun, which is the only star in our solar system. This existing MLR was deduced from measurements of binary stars based on stellar dynamics using Newtonian gravitation, and was independently supported by a theoretical model of the stellar energy source based on balancing the pressure of Newtonian gravitation with the thermal pressure due to nuclear fusion. According to the equivalent modelings presented in the following sections, the mass m in (1) would be equal to the equivalent gravitational mass $m_g = m_{ge}$ in the orbital dynamics model of section IV, or the equivalent mass m_e in the energy source model of section III. Therefore, the solar mass m_0 in (1) refers to the equivalent gravitational mass $m_{g0}$of the sun, which is also equal to the equivalent solar mass m_{e0} from the energy source model. It may be noted, the $m_0 = m_{g0} = m_{e0}$ may be different from the actual solar mass (inertial) based on the UEG theory, as discussed in the section I.

The m_g and m_e would be functions of both the conventional inertial mass m and the luminosity L of a star. Each of these equivalent masses is expected to be equal to the inertial mass m, when the UEG effects are excluded in the modeling, keeping the existing MLR (1) unchanged in this case. On the other hand, when the UEG effects are included, the luminosity derived from the following new models would be equal to that from the existing MLR (1), if the mass m in the existing MLR is replaced by the equivalent mass, m_g or m_e, of the respective models. The MLR (1) is shown in Fig.6, indicated as the (m_g - L) or (m_e - L) relationship, according to the above equivalence. The actual mass m (inertial) is expected to be in general different from the m_g or m_e. Therefore, an actual MLR (mass (actual, inertial)-luminosity relation) is also expected to be different from (1), as derived and presented in section V.

III. ENERGY SOURCE MODEL FOR THE MASS LUMINOSITY RELATION, USING CONVENTIONAL GRAVITY AND UEG THEORY

Let us first consider a derivation based on a conventional Newtonian model, but keeping in mind distinct contributions that may need be modified when the UEG effects are included in a new model. Note that there are actually two mass parameters that as a product would contribute to the energy or light output in a star: one mass parameter is the source of gravity, and the other is the target mass on which gravity is acting upon. However, in a Newtonian gravity model, the two gravitational mass parameters happen to be equal to each other and are equal to the inertial mass. Let us take for granted that the existing Eddington's model [4, 7] (1) for the mass-luminosity relation (MLR), which expresses the luminosity L as a function $L(m)$ of the mass m, is in principle correct, assuming only the Newtonian gravity is applied without any UEG effect. Let us rewrite the function $L(m)$ in the form of its inverse function $m(L)$, the square of which is the mass-square function $m^2(L)$. When the UEG effects are added, let us define an equivalent-mass m_e, such that the new MLR would be equal to the Eddington's MLR when its mass term m is replaced by the m_e. Accordingly, the Eddington's mass-square function $m^2(L)$ is actuality the $m_e^2(L)$ function when the UEG effects are added. Clearly, the m_e^2 is equal to m^2 when only the Newtonian gravitation is included. Consider the mass-squared function is a product of two mass terms, as discussed above. One of the mass terms, corresponding to the target mass of gravitation would remain unchanged with or without the UEG effect. This is because the UEG theory only changes the gravitational acceleration, which needs to be multiplied with the same inertial mass to get the gravitational force, just as in the Newtonian case. The source-mass term would be sum of two parts, one of which represents the Newtonian gravitation which remains unchanged as m. Whereas, the second part of the source mass is contributed due to the UEG effect, and is expected to be proportional to the luminosity L or equivalently to its UEG mass m_u.

$$m_e^2(L) = m(L)(m(L) + \alpha m_u(L)),\ m_e = [m(m + \alpha m_u)]^{0.5}.$$
$$m_e^2 = m^2 + \alpha m m_u \simeq (m + \frac{\alpha}{2} m_u)^2,$$
$$m_e \simeq (m + \frac{\alpha}{2} m_u); m_u << m.$$
$$m_e^2 = m(m + \alpha m_u) = m(m + m_{ue}) = m m_{ge};$$
$$E_{gu}(r = d >> R) = \frac{G m_u}{d^2} = \gamma \frac{L}{4\pi d^2 c},\ m_u = \frac{\gamma L}{4\pi G c}. \quad (2)$$

The validity of the equivalent model proposed above may be verified by deriving the average pressure in a star by including the UEG effects in addition to the conventional Newtonian gravitation, and equating it with that by including only the conventional Newtonian gravitation [7], when mass m in the later result is substituted with the equivalent mass m_e, as per the above definition. This assumes that the luminosity is proportional to the average pressure, in consistency of the Eddington's model [7].

With this goal, let us first derive the average pressure due to only the UEG effect. For a typical star like the sun, the volume density ρ_{vL} of luminosity, resulting in the total luminosity L, may be assumed to be uniform. The energy density of radiation produced by a volume element of the energy source located at (r', θ, ϕ), and observed at $(r = z, \theta = 0)$, may be integrated over the entire spherical volume of the star of radius R, to obtain the total energy density $W_T(r)$. Due to spherical symmetry, the energy density would be independent of the θ and ϕ coordinates of the observation location, dependent only on its radial distance r.

$$W_T(r) = \int_{r'=0}^{R} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\rho_{vL} r'^2 \sin\theta d\phi d\theta dr'}{4\pi c(r^2 + r'^2 - 2rr'\cos\theta)}$$
$$= \frac{\rho_{vL}}{4c} \int_{r'=0}^{R} \int_{t=(r-r')^2}^{(r+r')^2} (\frac{r'}{r})(\frac{1}{t}) dt dr'; (t = r^2 + r'^2 - 2rr'\cos\theta)$$
$$= \frac{\rho_{vL}}{4c} \int_{r'=0}^{R} (\frac{r'}{r}) \ln\frac{(r'+r)^2}{(r'-r)^2} dr' = \frac{\rho_{vL}}{4c}[(\frac{R^2}{2r}) \ln\frac{(R+r)^2}{(R-r)^2}$$
$$+ \int_0^R \frac{2r'^2}{(r'^2 - r^2)} dr'] = \frac{\rho_{vL}}{2c}[R + (\frac{R^2 - r^2}{2r}) \ln\frac{R+r}{R-r}];$$
$$W_T(r = 0) = \frac{\rho_{vL} R}{c},\ W_T(r = R) = \frac{\rho_{vL} R}{2c}. \quad (3)$$

The UEG acceleration $E_{gu}(r)$, directed towards the center, can now be expressed by multiplying the energy density with the UEG constant γ [1], from which the pressure $P_u(r)$ and average pressure $< P_u >$ may be obtained as follows. For an approximate reference and simplicity of understanding, the energy density function $W_T(r)$ may be roughly approximated over the region $r < R$ by linear interpolation of the values at $r = 0$ and $r = R$, which can then be analytically integrated to obtain the pressure function and the average pressure. Numerical integration would be needed for accurate calculations, which may be verified with the analytical results from the approximate reference.

$$E_{gu}(r) = \gamma W_T(r),$$
$$E_{gu}(r = 0) = \gamma W_T(r = 0) = \frac{\gamma \rho_{vL} R}{c} = \frac{3 G m_u}{R^2},$$
$$E_{gu}(r = R) = \gamma W_T(r = R) = \frac{\gamma \rho_{vL} R}{2c} = \frac{1.5 G m_u}{R^2},$$
$$E_{gu}(r < R) \simeq \frac{G m_u}{R^2}(3 - 1.5(\frac{r}{R})), \quad (4)$$

$$P_u(r) = \int_r^R \rho_{vm} E_{gu}(r) dr$$
$$\simeq \frac{G m_u m}{(4/3\pi R^4)}[3(1 - (\frac{r}{R}) - 0.75(1 - (\frac{r}{R})^2)],$$
$$< P_u > = \frac{1}{R}\int_0^R P_u(r) dr = 1.20 \frac{G m_u m}{(4/3)\pi R^4} \sim \frac{G m_u m}{(4/3)\pi R^4};$$
$$\rho_{vm} = \frac{m}{(4/3)\pi R^3},\ \rho_{vL} = \frac{L}{(4/3)\pi R^3}. \quad (5)$$

Similar steps as above may be used for Newtonian gravitation to obtain the acceleration function $E_{gm}(r)$, pressure function $P_m(r)$, and average pressure $< P_m >$.

$$E_{gm}(r < R) = \frac{G m r}{R^3},$$
$$P_m(r) = \int_r^R \rho_{vm} E_{gm}(r) dr = \frac{G m^2}{(4/3)\pi R^4}[0.5(1 - (\frac{r}{R})^2)],$$
$$< P_m > = \frac{1}{R}\int_0^R P_u(r) dr = \frac{G m^2}{4\pi R^4}. \quad (6)$$

The average pressure $< P >$ in the presence of both the Newtonian and UEG forces would be the sum of the two terms $< P_u >$ and $< P_m >$. Whereas, the average pressure for conventional Newtonian gravity is $< P_m >$. As prescribed earlier for the equivalent modeling in (2), the equivalent mass m_e may now be expressed in terms of the UEG mass m_u and the inertial mass m. This may be compared with what we expected in (2), from which the equivalent UEG mass $m_{ue} = \alpha m_u$, and consequently the parameter α, may be deduced. The α is roughly estimated to be 3.0 using analytical integration based on the reference approximation of $E_{gu}(r)$ in (4), but is accurately calculated to be 3.60, using numerical integration based on the rigorous expressions of $E_{gu}(r)$ and $W_T(r)$ in (3,4).

$$< P_m >_{m=m_e} = < P > = < P_m > + < P_u >,$$
$$\frac{G {m_e}^2}{4\pi R^4} = \frac{G m^2}{4\pi R^4} + \frac{1.20 G m m_u}{(4/3)\pi R^4} \sim \frac{G m^2}{4\pi R^4} + \frac{G m m_u}{(4/3)\pi R^4},$$
$$m_e^2 = m(m + 3.60 m_u) = m(m + m_{ue}),\ \alpha = 3.60. \quad (7)$$

This calculation for the α is expected to be generally valid for most typical stars, similar to our sun. For very high luminosity stars, as compared to the sun, the source distribution of the luminosity may not be uniform as assumed in the above analysis, but could be more concentrated towards the center where the pressure can be significantly higher. This may lead to a higher value for the α in these cases, as per the above modeling.

A. Estimating the UEG Constant from Energy Source Model for a Solar-Mass Star

The sun, which is the only star in our solar system, was used as the reference in the MLR of (1). Note that the mass of the sun that has been historically deduced from measurement of orbital motions of all planets, in consistency with that of our own planet earth, is actually the equivalent gravitational mass of the sun. Based on the UEG theory, introduced in the following section IV, the sun may be considered an isolated body, for evaluation of its gravitational force acting on the planets in the solar system. This is because the sun is an isolated star, and all the planets in the solar system are considerably much lighter than the sun. Accordingly, the equivalent gravitational mass m_g of the sun would be the sum of its inertial mass m and its UEG mass m_u (as defined in 2). This $m_g = m + m_u$ for the sun needs to be equated to the equivalent mass m_e used here for energy source modeling, as well as to the m_g in the orbital modeling in section IV, for all solar-mass stars, in full consistency with the MLR of (1).

Applying the above solar condition $m_e = m + m_u$ in (7) leads to calculation of the m_u from the known value of the solar gravitational mass $m_g = m_e = 1.989 \times 10^{30} kg$ [9]. This calculated value of m_u can then be used in (2) to estimate the UEG constant γ, given the solar luminosity $L = L_0 = 3.828 \times 10^{26} W$ [9].

$$\begin{aligned}
(m + m_u)^2 &= m^2 + m_u^2 + 2mm_u = m(m + m_{ue}) \\
&= m^2 + 3.60mm_u, m_u = 1.60m, \\
m_e &= m + m_u = (1 + 1/1.60)m_u, \\
m_u &= (1.60/2.60)m_e = 0.615m_e, m = 0.385m_e, \\
\gamma &= m_u 4\pi Gc/L = 0.615m_e 4\pi Gc/L \\
&= 0.804 \times 10^3\ (m/s^2)/(J/m^3). \qquad (8)
\end{aligned}$$

This estimate compares reasonably close to an accurate calculation of $\gamma = 0.6 \times 10^3$ $(m/s^2)/(J/m^3)$ from a particle physics model [1], with a difference of about 33%. Considering that the present estimation is based on some simple assumptions and formulations of the UEG effects on star luminosity, the above result is a reasonable support for the value of the UEG constant as well as for the UEG theory. With the accurate value of $\gamma = 0.6 \times 10^3$ $(m/s^2)/(J/m^3)$ from the particle physics model, the different mass parameters of (8) may be back-calculated as $m_u = 0.46m_e$, $m = 0.54m_e$, $m_u = 0.85m$, $m_{ue} = 2.85m_u$, with $\alpha = m_{ue}/m_u = 2.85$, which is within a reasonable ($\sim 26\%$) difference from the estimate of $\alpha = 3.60$ in (7). Further, using the accurate values of $m_u = 0.46m_e$ and $m = 0.54m_e$, the value of the effective mass that would be estimated using the estimated value of $\alpha = 3.60$ in (7), is equal to $m_e(\text{estimate}) = \sqrt{0.54(0.54 + 3.60 \times 0.46)}m_e = 1.09m_e$, which is within a small ($\sim 10\%$) difference from the actual m_e. Any such reasonable difference between the estimated and actual magnitudes of the equivalent mass may be accommodated (b) by using a more realistic energy-source distribution that is different from the ideal uniform distribution assumed in the present model, or (b) by adjusting the magnitude of the MLR (1) deduced from the energy source model of [4, 7] (Eddington's model) to be reasonably different from that from binary-star measurements, both of which we simply presumed to be identical. However, it is more significant to note the functional form of the equivalent mass (7), which would be shown in the following section IV to compare remarkably with an alternate relationship (19) derived from a binary-star model, for large m_u/m, that would provide a strong validation for the UEG theory.

Supported by the above estimations from the star-luminosity model, and to be supported even further by a UEG model of gravitation in a binary-star system in the following section IV, it is particularly significant to note the following consequence of the above results from the UEG theory. The inertial mass of the sun might not be what we have been believing [8, 9], estimated based on the Newton's Laws of gravitation and motion [10, 11], using observation of planetary motions, including orbital motion of the earth around the sun. We now find, as per the UEG theory, that the inertial mass m of the sun could actually be about half ($m = 0.54m_e$) of what is calculated from the planetary motions based on the Newton's Laws. The approximately other half ($m_u = 0.46m_e$) is a result of the new UEG force due to the sun's light. In other words, we are being pulled by the light of the sun about as much as by the actual mass of the sun! This would be a significant discovery, where the new UEG theory is shown to directly influence the gravitation in the solar system, and thus our common understanding of the basic nature of gravity that controls our own motion around the sun.

IV. UEG MODEL FOR EFFECTIVE GRAVITATIONAL MASS OF A STAR IN A BINARY SYSTEM

A. Estimation of Distance Between Partner Stars in an Observed Eclipsing Binary System

The stellar mass-luminosity relation (MLR) is based on measurement survey of mostly eclipsing binary stars, where the partner stars are closely spaced from each other. Accordingly, in order to evaluate the MLR in relation to the new UEG theory, it would be useful to analytically estimate a typical or median value for the surface-

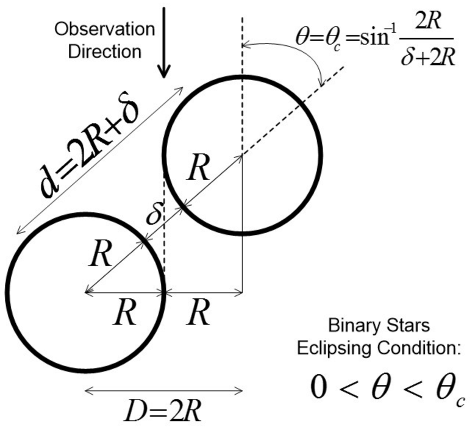


FIG. 1.

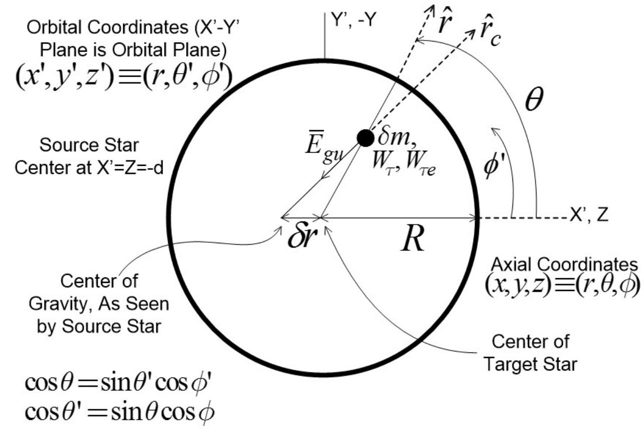


FIG. 2.

to-surface distance δ between the two partner stars. For simplicity the two stars may be assumed to be identical, each of radius R.

As shown in Fig.1, the critical angle $\theta = \theta_C$ between the orbital axis and the direction of observation, larger than which eclipsing would not be observed, may be expressed in terms of the surface-to surface distance δ and the star radius R. Assuming that orientation angle θ is equally likely between zero and $\pi/2$, stars with smaller θ_C (which is equivalent to a larger δ/R ratio), are less likely to be observed in a survey. A median probability $P > 0.5$ of observation would provide a useful estimate for the separation factor δ/R in an observed eclipsing binary system

$$P(0<\theta<\theta_C) = \frac{2\theta_C}{\pi},\ \sin\theta_C = \frac{2R}{2R+\delta} = \frac{D}{d} = \frac{1}{1+\frac{\delta}{2R}},$$
$$\frac{\delta}{2R} = (\csc\theta_C - 1);$$
$$\theta_C \geq \frac{\pi}{4},\ P(0<\theta<\theta_C) > 0.5,\ \delta \leq 2R(\csc\frac{\pi}{4} - 1) \simeq 0.8R,$$
$$\delta = \delta_m \simeq 0.4R\ \ . \tag{9}$$

B. General Calculation of the UEG Force as a Function of the Equivalent Gravitational Mass

The additional gravitational force of attraction, as per the UEG theory, produced due to the light radiation from one star (source star) acting upon the other star (target star) in a binary system, would be directed towards a suitable center of gravity (CG). The distance δr (see Fig.2) of this CG from the center of the target star is determined by the effective gravitational mass m_g of the target star and the inertial mass m of the source star.

$$\frac{m}{m_g} = \frac{\delta r}{2R+\delta} = \frac{\delta r}{d};\ m_g >> m,\ \delta r << R, d\ . \tag{10}$$

The magnitude of the attraction upon an elemental mass δm, at a given location of the target star, is to be determined by the energy density $W_\mathcal{T}$ reaching at the particular location due to radiation from the source star (presuming the target star was removed, or is transparent to the source-star's radiation). When the CG approaches the center of the target star (when $m_g \to \infty$, $m_g >> m$), we expect the total UEG force upon the target star to approach zero. In this limit, a mass-less source with any non-zero luminosity (finite or infinite) can not exert a non-zero force or acceleration upon a target of infinite mass. This condition may be empirically enforced by by redistribution of the energy-density function $W_\mathcal{T}(r,\theta\phi$ into a new, effective energy-density function $W_{\mathcal{T}e}(r,\theta,\phi)$ with appropriate symmetry. For a spherically symmetric target, this may be accomplished by simply using a constant $W_{\mathcal{T}e}$, which is an average of the $W_\mathcal{T}(r,\theta,\phi)$ over the entire target sphere. A less restrictive, general approach would be to define the effective energy density at a particular location as the average of the $W_\mathcal{T}(r,\theta,\phi)$ over all θ and ϕ, for the same radial distance r of the given location. For further generality, we will choose an effective energy density $W_{\mathcal{T}e}(r,\theta')$ which is cylindrically symmetric about the orbital axis (see Fig.2), calculated by taking an average of the $W_\mathcal{T}(r,\theta',\phi')$ over all ϕ'.

$$W_{\mathcal{T}e}(r,\theta,\phi) = W_{\mathcal{T}e}(r',\theta'),\ r'=r,\ \theta' = \cos^{-1}(\sin\theta\cos\phi),$$
$$W_{\mathcal{T}e}(r',\theta') = \frac{1}{2\pi}\int_{\phi'=0}^{2\pi} W_\mathcal{T}(r',\theta',\phi')d\phi'$$
$$= \frac{1}{2\pi}\int_{\phi'=0}^{2\pi} \frac{L}{4\pi c(r'^2+d^2+2r'd\sin\theta'\cos\phi')}. \tag{11}$$

The gravitational acceleration $\overline{E}_{gu}$ due to the UEG effect may now be expressed using the $W_{\mathcal{T}e}$ and the UEG constant γ.

$$\overline{E}gu(r,\theta,\phi) = -\hat{r}_c Egu(r,\theta,\phi) = -\hat{r}_c \gamma W_{\mathcal{T}} e(r,\theta,\phi)$$
$$= -\hat{r}_c \frac{1}{2\pi} \int_{\phi'=0}^{2\pi} \frac{Gm_u}{(r^2+d^2+2rd\cos\phi' \sqrt{1-\sin^2\theta\cos^2\phi})} d\phi',$$
$$m_u = \frac{\gamma L}{4\pi Gc}. \tag{12}$$

The total gravitational force $\overline{F}_u$ towards the source star can be calculated by multiplying the $-\hat{z}$ component of the acceleration $\overline{E}_{gu}$ with mass density ρ_{vm}, and integrating over the target sphere. An effective gravitational mass m_{ue} associated with the UEG force $\overline{F}_u$ may be defined, and related to the source star luminosity L or its associated UEG mass m_u. The m_u is the equivalent gravitational mass of the source star in the limit of operating in isolation, when the m_g of the target star approaches zero or the distance δ_{cg} approaches $-d/2$, which means the effective CG used in the evaluation of the UEG force is at the center of the source star.

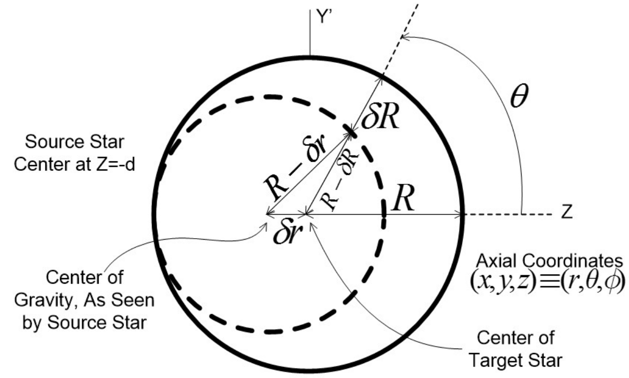
$$\overline{F}_u = -\hat{z}F_u = -\hat{z}\frac{Gm_{ue}m}{d^2},$$
$$F_u = \int_{r=0}^{R}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} (-\overline{E}_{gu}(r,\theta,\phi)\cdot\hat{z})\rho_{vm} r^2 \sin\theta d\phi d\theta dr$$
$$= \int_{r=0}^{R}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} [E_{gu}(r,\theta,\phi)\frac{r\cos\theta+\delta r}{\sqrt{r^2+\delta r^2+2r\delta r\cos\theta}}\rho_{vm}$$
$$\times r^2\sin\theta d\phi d\theta dr],\ \rho_{vm} = \frac{m}{(4/3)\pi R^3}. \tag{13}$$

$$\frac{m_{ue}}{m_u} = \frac{F_u d^2}{Gmm_u} = \frac{d^2}{(4/3)\pi R^3}\int_{r=0}^{R}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}[\frac{1}{2\pi}$$
$$\times \int_{\phi'=0}^{2\pi} \frac{1}{(r^2+d^2+2rd\cos\phi'\sqrt{1-\sin^2\theta\cos^2\phi})}d\phi']$$
$$\times \frac{r\cos\theta+\delta r}{\sqrt{r^2+\delta r^2+2r\delta r\cos\theta}} r^2\sin\theta d\phi d\theta dr. \tag{14}$$

C. UEG Force of Attraction for a Large Equivalent Gravitational Mass

It may not be possible to evaluate the above general formulas analytically, requiring numerical integration for any general values of the source mass m and the target equivalent gravitational mass m_g. However, when the m_g is very large compared to the m, the CG would be close to the target star center, with the distance δr much smaller than the target radius R.

$$\frac{m}{m_g} = \frac{\delta r}{2R+\delta} = \frac{\delta r}{d};\ \delta r << R, d,\ m_g >> m. \tag{15}$$



$$R-\delta r = \sqrt{(R-\delta R)^2 + (\delta r)^2 + 2\delta r(R-\delta R)\cos\theta}$$
$$\approx R-\delta R+\delta r\cos\theta,\ \delta R \approx \delta r(1+\cos\theta);\ \delta r, \delta R << R$$

FIG. 3.

The general formulas derived above can be simplified for this limiting case. For further simplicity of calculation, we would assume the energy density $W_{\mathcal{T}}$ to be approximately uniform, equal to that at the center of the target star. Under this assumption, the gravitational force due to a maximal spherical region around the center of gravity with radius $R-\delta r$ (spherical region with dashed boundary, see Fig.3) would result in zero total force due to symmetrical cancellation of contributions from its elemental parts. Contributions from only the thin boundary shell (see Fig.3) with thickness $\delta R = \delta r(1+\cos\theta)$ would be needed to calculate the total force. The expression (14) may be simplified under the above approximations, requiring integration only over the shell region of variable thickness $\delta R = \delta r(1+\cos\theta)$, at $r \simeq R$. The only non-trivial integration in (14) over θ may be analytically evaluated. The resulting simple analytical expression for the limiting case would be useful for conceptual understanding, as well as for approximate validation of the general results for large m_g.

$$\frac{m_{ue}}{m_u} \simeq \frac{3}{2}\int_{\theta=0}^{\pi}(\frac{dR}{R})\cos\theta\sin\theta d\theta$$
$$= \frac{3}{2}\int_{\theta=0}^{\pi}(\frac{\delta r}{R})(1+\cos\theta)\cos\theta\sin\theta d\theta = \frac{\delta r}{R}$$
$$= \frac{m}{m_g}(\frac{d}{R}),\ m_{ue} = \frac{m_u m}{m_g}(\frac{d}{R});\ \delta r, \delta R << d, R\ . \tag{16}$$

For a median distance $\delta r = 0.4R$, estimated for eclipsing close binaries in section IV A, the above limiting expression for the m_{ue} may be written as,

$$m_{ue} = \frac{m_u m}{m_g}(\frac{\delta+2R}{R}) = 2.4\frac{m_u m}{m_g}. \tag{17}$$

Based on the general derivation of section IV B, the ratio m_{ue}/m_u was computed using numerical integration

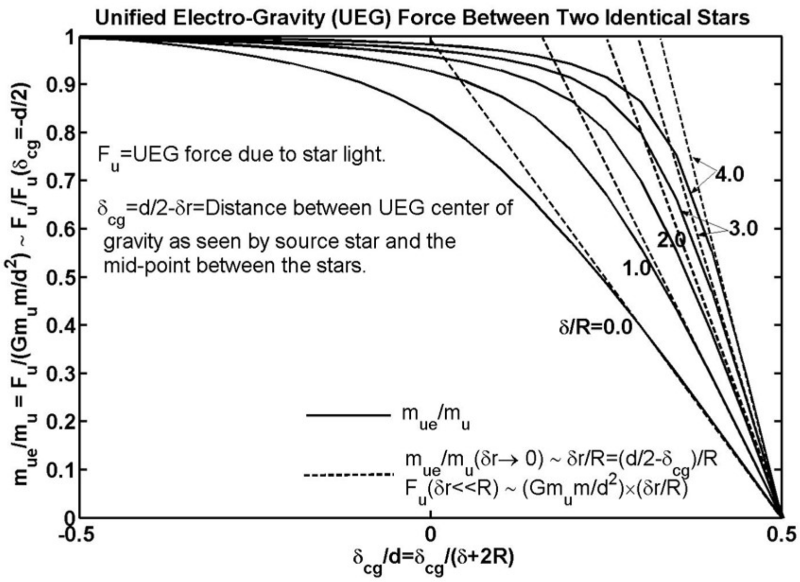


FIG. 4.

for different distances $\delta_{cg} = d/2 - \delta r$ of the CG (as seen by the source star) from the mid point between the source and target stars, for a fixed value of the surface-to-surface distance δ between the two stars. The computed ratio m_{ue}/m are plotted in Fig.4 as a function of the normalized distance δ_{cg}/R, for selected values of the normalized parameter δ/R. The computed results in the Fig.4 are compared with the limiting values of the m_{ue}/m_u, analytically derived above in section IV C. The general and the limiting plots in the Fig.4 are shown to validate each other when δ_r approaches small values (δ_{cg}/d approaches 0.5), or equivalently when the target-star's gravitational mass m_g is much larger than the source-star's inertial mass m, as expected. The effective UEG mass m_{ue} of the source star approaches its maximum value m_u when the CG moves to the center of the source star, or δ_{cg}/d approaches -0.5, as expected. The m_{ue} starts to drop significantly lower than the m_u after the CG moves beyond the mid point between the source and target stars ($\delta_{cg} > 0$), closer toward the target star. Note that the total effective gravitational mass m_{ge} of the source star is the sum of the inertial mass m and the effective UEG mass m_{ue}.

In a binary system, the same normalized plots of the Fig.4 would be applicable to model force from the first upon the second star, as well as from the second upon the first. In the former case, the m_{ue}, m_u, m and m_{ge} would be associated with the first star (referred to with a subscript 1), but the m_g would be associated with the second star (referred to with a subscript 2), where as for the later case the associations would be reversed. The final solutions for the m_{ue1}, m_{ge1}, m_{ue2}, m_{ge2} may be established by equating $m_{g2} = m_{ge2} = m_{ue2} + m_2$ and $m_{g1} = m_{ge1} = m_{ue1} + m_1$. The process would be simpler when the two partner stars are identical, in which case the results of Fig.4 may be used for a final solution of the m_{ue} of each star, by enforcing the additional condition $m_g = m_{ge} = m_{ue} + m$ for mutual balance. For this condition, $\delta r/R = m/m_g = 1/(m_{ue}/m + 1)$ and therefore δ_{cg}/d would be directly dependent on the ratio m_{ue}/m, for a given parameter δ/R. Accordingly, for a given δ/R, the Fig.4 essentially provides m_{ue}/m_u as a function of m_{ue}/m, from which m_{ue}/m or $m_{ge}/m = m_{ue}/m + 1$ can be deduced for different values of m_u/m. These results are shown in Fig.5, for selected values of δ/R. These results are verified with those similarly deduced from the limiting expressions of m_{ue}/m_u in (16) applicable for sufficiently large m_{ue}/m, as follows.

The limiting expression for the m_{ue} in (16) is approximately equal to the $m_{ge} = m_{ue} + m$ because $m_{ue}/m >> 1$, or $m_{ue} >> m$. Accordingly, (16) may be solved for the symmetry condition $m_{ue} \simeq m_{ge} = m_g$, as prescribed earlier for a binary star with two identical partners.

$$m_g = m_{ue} + m \simeq \sqrt{\frac{d}{R} m_u m} \; ,$$
$$\frac{m_g}{m} = \frac{m_{ue}}{m} + 1 \simeq \sqrt{\frac{d m_u}{R m}} . \tag{18}$$

For the estimated $\delta = 0.4R$ in (9,17), we get,

$$m_g = m_{ue} + m \simeq \sqrt{2.4 m_u m} \; ,$$
$$\frac{m_g}{m} = \frac{m_{ue}}{m} + 1 \simeq \sqrt{\frac{2.4 m_u}{m}}, \; \delta = \delta_m = 0.4R \; . \tag{19}$$

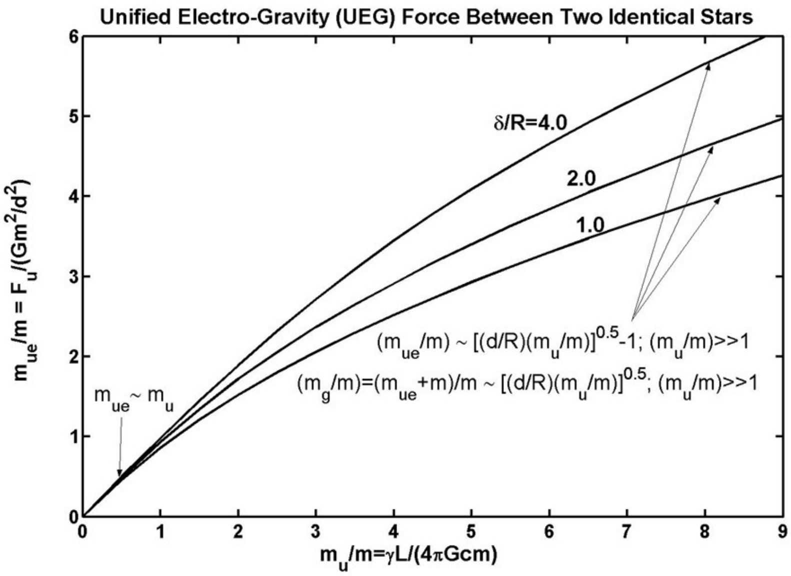


FIG. 5.

Note that the above limiting relation of (19) is in similar form as derived from the energy source model in section III, (7), for large m_u/m. This is a strong correlation between the two models, indicating a strong validation of the associated UEG theory. The factor d/R in (18) is functionally equivalent to the factor α deduced in (7). The expected value of $\alpha = 2.85$ in (7) (see section III A) is somewhat larger than the estimated factor $(d/R) \simeq 2.4$ in (19). Note that the factor (d/R) was estimated in section IV A, (9), based on a visual condition of orbital eclipsing where the R is the visual radius of a star, whereas the relation (18) was deduced from orbital dynamics where the R is the core radius of the star. Accordingly, the actual (d/R) for use in (18,19) should be somewhat larger than the estimate of 2.4 from (9), which would be consistent with the $\alpha = 2.85$ expected from the energy source model. Further, the above expected value of $\alpha = 2.85$ from section III A is valid only for a typical star like sun, with variations around this value for high and low intensity stars, which may be expected to track similar variations of the (d/R) factor, leading to the expected confirmation between the two models.

It may be noted, for a general case of gravitation between two different bodies, and therefore for the special case of two identical bodies as well, the basic theory of Newton's universal gravitation may need to be reviewed and revised in consistency with Newton's laws of mechanics, when one or both of the bodies are radiating. The UEG effects due to radiation as modeled above may be represented in terms of a revised equivalent gravitational constant G_u, substituting for the universal constant G for non-radiating bodies. The G_u is in general different from the G, and is different for two specific bodies dependent on their individual radiation, inertial mass and separation distance. This is presented in Appendix A, drawing particular attention to interesting special conditions that may arise in section A 2.

V. DERIVATION AND VALIDATION OF THE NEW MLR

In accordance with our equivalence modeling, the mass-term in the MLR of (1) is actually equal to the m_g for the orbital model of section IV, or the m_e for the energy-source model of section III. And, the luminosity L in the MLR (1) is proportional to the m_u as defined in (2). In other words, the MLR of (1) actually provides the relationship between (m_g, m_e) and m_u. This data, together with the theoretical relationship of Fig.5 between $m_{ue}/m = m_g/m - 1$ and m_u/m, may be used to deduce a new MLR (mass(actual inertial)-luminosity relation), based on the orbital model of section IV, as plotted in Fig.6 for $d/R = 2.85$.

Similarly, an alternate new MLR may be derived, using the theoretical relationship (7) between the equivalent mass m_e and m_u, based on the energy-source model of section III, and the (m_g, m_e) to m_u relationship of the MLR (1). This new MLR is plotted in Fig.6 for the parameter $\alpha = 2.85$, for comparison with its alternate MLR derived from the orbital model. As discussed earlier in section IV, the parameters α and (d/R) from the two models are functionally equivalent, and are estimated to have comparable values. The two new MLR's from the

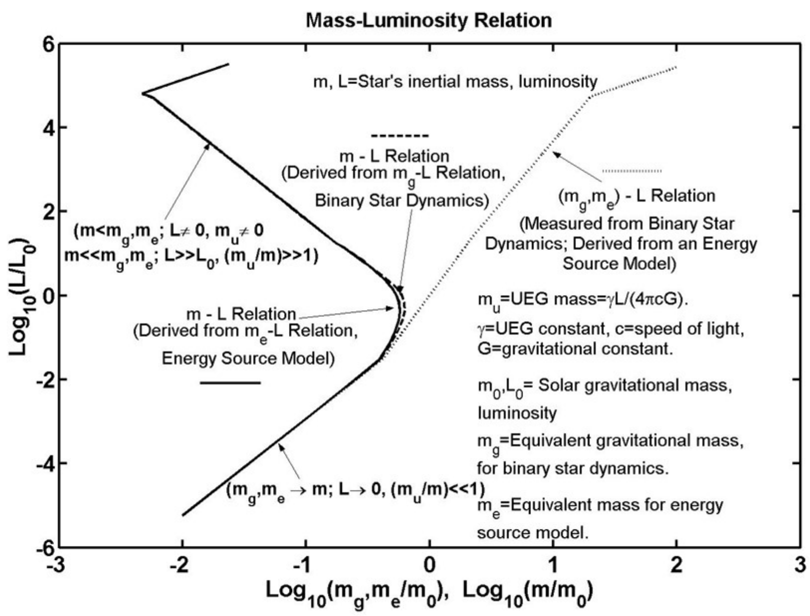


FIG. 6.

two independent models are seen to closely follow each other, but somewhat deviate in the region of solar-mass stars. Such agreement between the functional trends of the two models validate the two models as well as the associated UEG theory, over the entire range of low to high-luminosity stars. Note that the actual inertial mass deduced in a new MLR in Fig.6 is significantly less than what was believed (m_g or m_e) based on the existing MLR of (1) [4, 7]. Interestingly, for the mid-region of the MLR of Fig.6, the inertial mass m reduces for increasing luminosity, which may appear counter-intuitive based on a energy-source model [7] using Newtonian gravitation. The significantly different trends of the new MLR, as compared to the existing MLR (1), may prompt review of existing models of stellar evolution [12], which is beyond the scope of the present work.

The new MLR may be expressed in approximated analytical forms for high luminosity, using the relation between the normalized variables (m_g/m_0, m_e/m_0) and $L/L_0 = m_u/m_{u0}$ from (1) and the limiting relationship of (7,19) similarly normalized. For low luminosity with negligible m_u, the new MLR would remain approximately unchanged from the existing MLR (1).

$$\begin{aligned} \frac{L}{L_0} &\simeq 0.23(\frac{m}{m_0})^{2.3},\ m < 0.43m_0, \\ &\simeq (1.5)^{-(\frac{2.0}{1.5})}(0.46\alpha\frac{m}{m_0})^{-(\frac{3.5}{1.5})},\ 16 < \frac{L}{L_0} < 64000, \\ &\simeq (3200)^2(0.46\alpha\frac{m}{m_0}),\ \frac{L}{L_0} > 64000. \end{aligned} \tag{20}$$

The parameter α is borrowed from the energy source model of (7), which is equivalent to the factor (d/R) from the orbital model of (18), and is expected to be about 2.85 for solar-mass stars with possible variation toward larger values for more luminous stars, as discussed in the end of section IV. The parameters $m_0 = m_{e0} = m_{g0}$, L_0 and m_{u0} refer to the solar mass (equivalent gravitational, not inertial), luminosity and UEG mass, respectively, and the factor 0.46 represents the expected ratio m_{u0}/m_{e0}.

VI. DISCUSSION AND CONCLUSIONS

The UEG theory applied for stellar dynamics in a binary star system is found to be consistent with that for stellar energy-source model, as per comparison of the resulting mass-luminosity relation (MLR) with the existing MLR [3, 4, 7] from orbital measurement of binary stars as well as from the Eddington model of stellar energy-source. The "mass" in the existing MLR was assumed to be the inertial mass of a star which is equal to the gravitational mass as per the conventional Newtonian gravity, but it needs to be modified as per the UEG theory in different manners for the binary-star dynamics and for the stellar energy-soure model. However, the associated "equivalent-mass" parameters in the two cases happen to exhibit the same functional trend, with respect to actual inertial mass and luminosity (or the UEG mass) of the star. Accordingly, the existing MLR derived from the binary-star measurement and the Eddington's energy-source model were seen to agree with each other. This coincidence historically removed any doubt about the validity of the existing theories of [3, 4, 7]. We now know that mass term in the existing MLR is not really the actual mass (inertial) as has been assumed for long. The new MLR between the star's actual inertial mass and the light output is derived and plotted using the new UEG theory. As per the new theory, the inertial mass of the sun or any other solar-mass star is actually about half of what has been believed all along! The sun's gravitational force acting upon us on the earth is contributed in approximately equal parts by its inertial mass as by the UEG force due to its luminosity. This is a significant new finding, fundamentally changing our common understanding of the basic nature of gravity.

The effective gravitational mass of an individual star in a binary system, consisting of two stars of comparable mass and light intensity, would in general be different from that when the star operates as a single isolated star applying gravitational force on a nearby planet of significantly smaller mass, when the additional UEG effects are included. However, for observed binary systems with approximately two solar-mass stars, the equivalent gravitational mass in the binary-star system happened to be approximately equal to that of an isolated solar-mass star (equivalent gravitational mass of sun in our solar system), as well as equal to the equivalent mass needed in an energy-source model to produce its light output close to the solar luminosity. This coincidence of the data for the solar-mass stars in the existing MLR with the observed gravitation of the sun in our solar system, also helped in removing any further doubt in the validity of the Newtonian gravitation. This, together with the other coincidence of the two MLRs mentioned earlier, may be considered interesting "conspiracies of nature" in the history of science, which possibly allowed to hide the new UEG theory of gravitation without suspicion for so long, till now!

Further, Einstein's equivalence principle in the general theory of relativity [6], together with Newton's third law of action-reaction equality [10], can be shown to require the gravitational mass of a source body to be equal to its inertial mass, with the gravitational constant G assumed be universally applicable to all bodies and locations. This fundamental requirement seemed to rule out possibility of the source gravitational mass of a radiating body to be any different from its inertial mass, also contributing to hiding the possibility of an additional UEG force for radiating bodies, without suspicion, as discussed above. However, all issues are shown in appendix A to be resolved, in full consistency with fundamental mechanical principles, if an equivalent gravitational constant G_u is allowed to be in general different for different pairs of gravitating bodies accounting for any additional UEG force due to radiation. This may limit the scope of the equivalence principle of the general relativity [6] to gravitation only between non-radiating bodies in a strictly "free-space" medium with no radiation-energy content, which the principle was fundamentally intended to. This is a significant development, revising the Newton's law of universal gravitation and Einstein's principle of equivalence, extended to gravitation between general radiating bodies.

Starting with the remarkable success of the UEG theory in particle physics [1, 2], the further validation of the UEG theory in the present work of stellar orbital and energy-source physics to model the MLR, should now establish significant confidence in the theory, unifying its application in the small (elementary particles) as well as large (solar system and binary stars) dimensions, and spherically symmetric (elementary particle and an isolated single star) as well as asymmetric (binary star) structures.

Appendix A: UEG Theory of Gravitation for General Radiating Bodies, and Conservation of Momentum and Energy

The UEG theory of gravitation in a binary system, as modeled in this paper, expresses the gravitational accelerations in terms of new effective gravitational masses $m_{g1} = m_1 + m_{ue1}$ and $m_{g2} = m_2 + m_{ue2}$ of the two bodies in the system. This may lead to review of different concepts of mass, which may determine not only the gravitational field produced by a body, but also the mechanics of the body's motion in terms of its inertial mass, momentum and energy. In the process, we may need to identify the mass of a given body in distinct forms, which may or may not be equal under general conditions of a radiating body.

The gravitational mass of a body, which acts like a source or cause of the gravitational field the body produces in the surrounding medium, may be referred to as the source gravitational mass, m_g. As per the UEG theory of binary stars, the $m_g = m + m_{ue}$ is the source gravitational mass of a radiating body, which is clearly different from that, $m_g = m$, without the radiation. As per conventional theory of gravitation and mechanics (Newtonian or relativistic), in the absence of any radiation, a unique mass parameter m defines not only the body's gravitational field but also its inertial as well as internal energy ($E = mc^2$). In the presence of radiation, we need to review if the change in the body's source gravitational mass $m_g = m + m_{ue} \neq m$ may also change the body's inertia or energy. If any such change may violate certain fundamental principles, the UEG theory may have to be properly revised.

As per the UEG model, the m_{ue} is the additional source gravitational mass due to radiation from a given body, as seen by the other body in a particular binary system. The total gravitational acceleration, a'_{12} or a'_{21}, produced as per the new UEG theory by one body, 1 or 2, and experienced at the location of the other body, 2 and 1, respectively, is expressed as (see Fig.7):

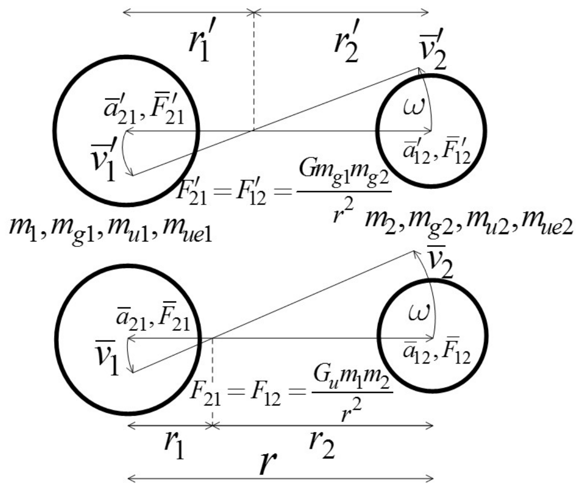


FIG. 7.

$$a_{12}' = \frac{Gm_{g1}}{r^2},\ a_{21}' = \frac{Gm_{g2}}{r^2},$$
$$m_{g1} = m_1 + m_{ue1},\ m_{g2} = m_2 + m_{ue2}. \tag{A1}$$

Useful limiting conditions may be recognized. The mue of a radiating body would be less than or equal to its maximum value mu, which would occur when the radiating body operates effectively in isolation, as the most dominant body with negligible or no gravitational influence from any surrounding body. In the other limit, mue of the radiating body would be zero, and consequently the source gravitational mass of the body would be equal to its Newtonian mass ($mg = m + mue = m$), when it operates in the presence of a nearby dominant body, with the Newtonian mass of the source body much smaller than that of the dominant body. Further, when the two bodies are sufficiently far apart from each other, the mue of each body would be approximately equal to their respective maximum values, mu.

$$m_{ue2} \to 0\ (m_{u2} \neq 0),\ m_{g2} \to m_2,\ m_{ue1} \to m_{u1},$$
$$m_{g1} \to (m_1 + m_{u1});\ m_1 >> m_2.$$
$$m_{ue1} \to m_{u1},\ m_{ue2} \to m_{u2};\ r \to \infty. \tag{A2}$$

The above accelerations (A1) may be expressed in special coordinates (primed) (see Fig.7), which may be referred to as the UEG coordinates of the particular binary system, with reference origin located between the two bodies, at distances r_1' and r_2' from the bodies 1 and 2, respectively. This would be different from the coordinates (un-primed) used in a conventional modeling of the binary system using Newtonian gravity, where the respective distances of the reference origin would have been $r_1 = r \times m_2/(m_1 + m_2)$ and $r_2 = r \times m_1/(m_1 + m_2)$.

The accelerations a_{21}' and a_{12}' experienced by the bodies 1 and 2, and the respective velocities v_1' and v_2' along the individual circular orbits, may be expressed in terms of their common angular velocity ω and the radial distances r_1' and r_2'. The v_1' and v_2' are directed opposite with respect to each other.

$$a_{12}' = \omega^2 r_2',\ a_{21}' = \omega^2 r_1',$$
$$\omega^2(r_1' + r_2') = \omega^2 r = a_{12}' + a_{21}' = \frac{G(m_{g1}+m_{g2})}{r^2},$$
$$\omega = \sqrt{\frac{G(m_{g1}+m_{g2})}{r^3}},$$
$$P_2 = m_{g2}v_2' = m_{g2}\omega r_2' = \frac{m_{g2}a_{12}'}{\omega} = \frac{Gm_{g1}m_{g2}}{r^2\omega},$$
$$P_1 = m_{g1}v_1' = m_{g1}\omega r_1' = \frac{m_{g1}a_{21}'}{\omega} = \frac{Gm_{g1}m_{g2}}{r^2\omega},$$
$$P_1 = P_2,\ \ \overline{P}_1 = m_{g1}\overline{v}_1',\ \overline{P}_2 = m_{g2}\overline{v}_2',$$
$$\overline{P}_1 = -\overline{P}_2,\ \overline{P} = \overline{P}_1 + \overline{P}_2 = 0\ . \tag{A3}$$

It is shown that total momentum $\overline{P} = \overline{P}_1 + \overline{P}_2$ of the binary system would be conserved as zero, if the momentum $\overline{P}_i$, $i = 1, 2$, of an individual body is defined as $\overline{P}_i = m_{gi}\overline{v}_i$, *not* $\overline{P}_i = m_i\overline{v}_i$ as conventionally defined in Newtonian mechanics. In other words, the inertial mass m_I of a radiating body may no longer be equal to its conventional inertial mass m, but could now be equal to its source gravitational mass mg. Accordingly, the UEG theory would not only change the source gravitational mass mg, but also the inertial mass m_I of a radiating body to be equal to $m_I = mg = m + mue$. The inertial mass m_I would be equal to the conventional inertial mass m, only when there is no radiation ($mu \to 0$).

$$m_{I1} = m_{g1} = m_{ue1} + m_1 \neq m_1;$$
$$m_{I1} = m_1,\ m_{ue1} = m_{u1} \to 0.$$
$$m_{I2} = m_{g2} = m_{ue2} + m_2 \neq m_2;$$
$$m_{I2} = m_2,\ m_{ue2} = m_{u2} \to 0. \tag{A4}$$

To be consistent with the conserved momentum, the gravitational force may also have to be defined as the product of the corresponding gravitational acceleration and the new inertial mass m_I, not the conventional mass m. With this new definition of the gravitational force, the vector force, $\overline{F}_{12}'$, produced by one body acting upon the other would be equal in magnitude, but oppositely directed, to that, $\overline{F}_{21}'$, when the source and target bodies are interchanged. This would satisfy the Newton's third law of action and reaction, resulting in the total force in the system equal to zero, as should be expected.

$$F_{12}' = m_{g2}a_{12}' = \frac{Gm_{g1}m_{g2}}{r^2},$$
$$F_{21}' = m_{g1}a_{21}' = \frac{Gm_{g2}m_{g1}}{r^2},$$
$$F_{12}' = F_{12}',\ \overline{F}_{12}' = -\overline{F}_{21}',\ \overline{F}_{12}' + \overline{F}_{21}' = 0. \tag{A5}$$

Clearly, the Newton's third law would not work if the mass of the target body were its conventional mass m, for radiating bodies with $m \neq m_g$ in general, resulting in a total non-zero force for the total system, which would also violate the principle of conservation of momentum.

$$F'_{12} = m_2 a'_{12} = \frac{G m_{g1} m_2}{r^2},\ F'_{21} = m_1 a'_{21} = \frac{G m_{g2} m_1}{r^2},$$
$$F'_{12} \neq F'_{12}\ \overline{F}'_{12} \neq -\overline{F}'_{21},\ \overline{F}'_{12} + \overline{F}'_{21} \neq 0. \quad \text{(A6)}$$

The consistency of momentum and its conservation may be extended to the kinetic energy of the body, so that the energy may also be conserved as would be desired. Accordingly, the kinetic energy may also need to use the new inertial mass m_I . That is, the kinetic energy KE (non-relativistic) would be equal to $KE = (1/2)m_I v^2 = (1/2)(m + m_{ue})^2 \neq (1/2)mv^2$. This may lead to a fundamental dilemma. Extending the treatment of energy to special relativity, this may lead to a fictitious rest energy of $E_0 = m_I c^2$, which is different from the actual rest energy $E_0 = mc^2$ of the body when the radiation is turned off. This is contrary to the understanding, that the intrinsic energy of the body should not be different, if the radiation is suddenly turned on or off. However, the above new definitions for the gravitational mass, inertial mass, momentum, kinetic energy, as well as the rest mass (though appears non-physical), could still be mathematically consistent with each other, as shown in the above equations, in reference to the local UEG reference coordinates (primed coordinates). The above dilemma of rest energy may perhaps be ignored, by considering the rest mass simply as a reference value, and any difference between the total and the reference rest energy may still be consistently used for modeling and "book-keeping" of the kinetic energy.

1. Revised UEG model for General Radiating Bodies

The above modeling of gravitation and inertia may be consistently used, as discussed above, but only in a hypothetical situation of having the only two bodies in an ideal empty space, in complete isolation from any other bodies. The model may be untenable in a real physical situation with other surrounding bodies. If the above model is followed for a general multi-body system, each pairing of the multi-body system would be associated with two inertial masses for the two individual members of the pairing, but any particular body would in general carry a different inertial mass when it is associated with a different pairing. Consider an arbitrary three-body system, where any two pairings of the system would share a common member. The common member would be associated with a different inertial mass, and therefore different momentum and energy for a given velocity, in modeling the gravitational force it experiences from the other two different bodies of the two pairings. Clearly, such non-unique values of the momentum and energy for the same particular body would not be fundamentally sensible, warranting suitable revision of the UEG model in order to reestablish order and consistency.

In order that the momentum or energy of a given body be uniquely defined and conserved, they must be proportional to the body's conventional mass m which remains unique under general conditions. This would be the case, if we require the gravitational force between two bodies to be proportional to the mass m of each body. We would maintain the same relative acceleration $a = a' = a'_{12} + a'_{21}$ between the two bodies in a binary system, defined in a new coordinate system (unprimed), such that the angular speed ω remains unchanged. In other words, the results of orbital periodicity from the original UEG model still remain valid through the following proposed revision.

$$a' = a'_{12} + a'_{21} = \frac{G(m_{g1} + m_{g2})}{r^2},$$
$$F_{12} = \frac{G_u m_1 m_2}{r^2} = F_{21},$$
$$a_{12} = \frac{F_{12}}{m_2} = \frac{G_u m_1}{r^2},\ a_{21} = \frac{F_{21}}{m_1} = \frac{G_u m_2}{r^2},$$
$$a = a_{12} + a_{21} = \frac{G_u(m_1 + m_2)}{r^2}, \quad \text{(A7)}$$

$$\omega = \frac{a}{r^2} = \frac{a'}{r^2},\ a = a',$$
$$G_u = G\frac{m_{g1} + m_{g2}}{m_1 + m_2} = G\frac{m_1 + m_{ue1} + m_2 + m_{ue2}}{m_1 + m_2}. \quad \text{(A8)}$$

The new model would satisfy all the Newton's law's of motion, as well as conservation of momentum and energy, with reference origin for orbital motion of the two stars same as that from Newtonian gravitation ($r_1 = r \times m_2/(m_1 + m_2)$, $r_2 = r \times m_1/(m_1 + m_2)$). However, each pair of bodies would now be associated with a different equivalent gravitational constant G_u, which is different from the Newtonian gravitational constant G. This is a significant new development, which may warrant review of orbital motions of two- and general multi-body systems, which may include radiating (stellar) and/or non-radiating (dark star or planets) elements.

2. Revised UEG Model for Gravitation in a Binary System Under Special Conditions

We may evaluate the equivalent gravitational constant G_u, under useful special conditions:

Case I: For the ideal case of a binary system with equal partner stars, the magnitudes of all the accelerations a_{12}, a_{21}, a'_{12} and a'_{21}, in the primed as well the unprimed coordinates would be equal.

$$G_u = G\frac{m+m_{ue}}{m},\ m_{ue1}=m_{ue2}=m_{ue},\ m_1=m_2=m,$$
$$a_{12}=a_{21}=a'_{12}=a'_{21}$$
$$=\frac{Gm_g}{r^2}=\frac{G(m+m_{ue})}{r^2}=\frac{G_u m}{r^2}. \tag{A9}$$

Case II: For a binary star system where the individual stars are sufficiently far apart, using the limiting condition of (A2) in (A7,A8), we have,

$$G_u = G\frac{m_1+m_{u1}+m_2+m_{u2}}{m_1+m_2},$$
$$m_{ue1}=m_{u1},\ m_{ue2}=m_{u2},\ r\to\infty. \tag{A10}$$

Case III: In the solar system, the gravitational acceleration between the sun (body 1) and any of its planets (body 2) may be modeled by considering the sun the most dominant body, as per the limiting condition (A2). In this case, the Newtonian gravitational constant G may be substituted by an effective gravitational constant G_u, which is larger than the G by the factor $m_{g1}/m_1=(m_1+m_{u1})/m_1$.

$$G_u \simeq G\frac{m_1+m_{u1}}{m_1}=G\frac{m_{g1}}{m_1},\ m_{ue1}=m_{u1},\ m_1>>m_2,$$
$$(m_1+m_{ue1})=m_{g1}>>m_{g2}=(m_2+m_{ue2}),$$
$$a_{21}\simeq\frac{G_u m_2}{r^2}=\frac{Gm_{g1}m_2}{m_1 r^2}\neq\frac{Gm_2}{r^2},$$
$$a_{12}\simeq\frac{G_u m_1}{r^2}=\frac{Gm_{g1}}{r^2}. \tag{A11}$$

Case IV: In contrast to the case III, the gravitation between a dominant luminous star (body 1) and a dominant massive but dark star (dark body 2) with little or no radiation, where $m_1<<m_2$, $m_{ue1}>>m_1,m_2$, would lead to an interesting situation. The dark star would exert much more acceleration on the luminous star, compared to that expected from its mass alone as per Newtonian gravity. It is as if the dark star has "acquired" the UEG mass m_{ue1} of the luminous star. Whereas, the luminous star would exert much less acceleration on the dark body, compared to that expected from its large effective UEG mass m_{ue1}. This concept may be referred to as "inversion." This results in complete opposite effect from what would be expected from a conventional stellar model, where a larger mass (source gravitational), and therefore a larger gravitational acceleration exerted upon the darker body, would be normally assumed (incorrectly) to be associated with the more luminous body. Although we establish only the limiting condition, as stated above (see (A12)), similar inversion conditions would apply also for any general unequal binary-star systems, where the more luminous star has lower mass. In light of this significant new result, some old controversies in astronomy such as the Algol Paradox [13, 14] in close binary systems and the mystery of the Sirus star system [15, 16] may need to be reevaluated.

$$G_u\simeq\frac{Gm_{ue1}}{m_2},\ m_{ue1}>>m_2>>m_1,\ m_{ue2}=0,$$
$$a_{21}=\frac{G_u m_2}{r^2}\simeq\frac{Gm_{ue1}}{r^2}>>\frac{Gm_2}{r^2},$$
$$a_{12}=\frac{G_u m_1}{r^2}\simeq\frac{Gm_{ue1}m_1}{m_2 r^2}<<\frac{Gm_{ue1}}{r^2}=a_{21}. \tag{A12}$$

Case V: Consider gravitation between a dominant massive body (body 1) with no or very little radiation, and a radiating body (body 2) of relatively small inertial or UEG mass compared to the body 1. This situation would be applicable to a planet like our earth, with a satellite or moon which may be naturally or artificially lighted. The gravitational force and acceleration in this case may be adequately modeled by using the conventional Newtonian gravitation, with $G_u\simeq G$. This is independent of the light radiation from the satellite or moon, even if its UEG mass m_{u2} or m_{ue2} is significant compared to its inertial mass m_2, or equivalently its source gravitational mass $m_{g2}=m_2+m_{ue2}$ is significantly larger than its inertial mass m_2, as long as the m_{u2}, m_{ue2}, m_2 and m_{g2} are sufficiently smaller compared to the mass m_1 of the dominant body. In other words, the source gravitational mass m_{g2}, which should have proportionately increased the gravitational force as per the unrevised UEG model in (A5), would no longer determine the gravitational force in the revised model of (A7,A8), under the present conditions. Instead, the gravitation is modeled by the simple Newtonian gravitation, with the force proportional to the inertial mass m_2 not the source gravitational mass m_{g2}.

$$G_u\simeq G,\ m_1>>m_2,m_{u2},m_{ue2};$$
$$m_{u1}=m_{ue1}<<m_1\ . \tag{A13}$$

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