Mapping the Real Numbers in the Closed Interval \([0, 1]\) to the Natural Numbers

**Abstract.** We exhibit an explicit mapping of the real numbers in \([0, 1]\) to the natural numbers.

There is a problem in defining the real numbers in \([0, 1]\) as the set of all unending decimal expansions. That’s vague. Most non-algebraic real numbers cannot be explicitly referenced.

We need a new definition.

Each real number \(S_{\text{subscript}}\) in the closed interval \([0, 1]\) is:

the limit \(m \to \infty\) \(n = 1\) to \(m\), \(0 \leq a_n \leq 9\) \(\sum a_n / 10^n = S_{\text{subscript}}\).

\(S_{\text{subscript}}\) is the limit of partial decimal sums.

In a terminating decimal the \(S_{\text{subscript}}\) subscript is the negative integer form of the decimals.

\(.14533778 \to S_{-14533778} \quad .1453 \to S_{-1453} \quad \text{For zero} \ .0 \to S_{0}\)

An unending decimal expansion \(S_{\text{subscript}}\) subscript is from a part of the decimal that can be listed.

\(.14533778\ldots \rightarrow S_{14533778} \quad .45226\ldots \rightarrow S_{45226}\)

Each \(S_{\text{subscript}}\) represents an unique decimal expansion. For non-repeating unending decimal expansions only the listed part, which generates the subscript of \(S_{\text{subscript}}\) is known.

Unending decimal expansions, whose listed parts start similarly can be distinguished as follows:

\(.14533778\ldots\) differs from \(.1453\ldots\) in the \(3778 + 8 = 3786\text{th}\) decimal position.

\(.335443\ldots\) differs from \(.335\ldots\) in the \(443 + 6 = 449\text{th}\) decimal position.

and so forth.

Decimals that start with \(.0\ldots\) are explicitly listed until non-zero decimals are reached. then the \(S_{\text{subscript}}\) subscript is the mirror image of the decimal reflected through the decimal point.

\(.000715\ldots \rightarrow S_{-517000} \quad .000715 \rightarrow S_{-517000}\)

Decimals that contain a series of \(0's\) are explicitly listed until non-zero decimals are reached.

\(.7000157\ldots \rightarrow S_{-7000157} \quad .7000157 \rightarrow S_{-7000157}\)

All the decimal expansions in \([0, 1]\) can be mapped to the natural numbers.

\(0 \to S_0 = .0, 1 \to S_1 = .1, 2 \to S_2 = .2, 3 \to S_3 = .3, 4 \to S_4 = .4, 5 \to S_5 = .5, 6 \to S_6 = .6, \ldots\)

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