Confirmation of VŁ4 as sound

Abstract: Logic VŁ4 is defined as a bivalent classical logic that maps tautology correctly and hence is sound. Because paraconsistent, non bivalent, vector logics cannot map *non* tautology correctly, they are defined as *non* tautologous fragments of VŁ4 as a universal logic.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ▽, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

Löb's theorem also named the Gödel-Löb (GL) axiom is:

□(□p→p)→□p (1.1)

LET p, q, r, s: bivalent logic, multi-valued logic, modal logic, s.

#(#p>p)>#p ; `CTCT CTCT CTCT CTCT` (1.2)

Remark 1.2: Eq. 1.2 as rendered is *not* tautologous.

We define a bivalent logic, implying multivalues and modalities, as *not* implied by the GL axiom. (2.1)

((#(#p>p)>#p)=(s@s))>(p>(q&r)) ; `TTTT TTTT TTTT TTTT` (2.2)

We define a *non* bivalent logic, such as paraconsistent logic, implying multivalues and modalities, as implied by the GL axiom. (3.1)

(#p>p)>#p)=(s=s))>(p>(q&r)) ; T**F**T**F** T**F**TT T**F**T**F** T**F**TT (3.2)