Abstract

In this article, we use the sieve of Eratosthenes to prove the Oppermann Conjecture.

X is a large number. We use \( p_i \) for all the primes, 2, 3, 5, 7, 11, 13, ..., \( i=1,2,3,\ldots \).

Let \( p_m \) be the greatest prime which satisfies \( p_m \leq X \). Here we have, \( p_m \leq X \) and \( p_{m+1} > X + 1 \), or \( p_m < X \) and \( p_{m+1} \geq X + 1 \).

By using the sieve of Eratosthenes up to the first \( m \) primes, \( p_i \), \( i=1,\ldots,m \), the remaining numbers in the period of \( (X^2-X, X^2) \) is equaled to \( [X \prod_{i=1}^{m} (1 - \frac{1}{p_i})] \). 

Here \([a]\) means the greatest integer less than or equal to \( a \).

Obviously there is at least one remaining number in this period.

It is also a prime number.

Similarly the remaining numbers in the period of \( (X^2, X^2 + X) \) is also equaled to \( [X \prod_{i=1}^{m} (1 - \frac{1}{p_i})] \). 

Obviously there is at least one remaining number in this period too and it is also a prime number.

Thus we prove the Oppermann Conjecture.