Oppermann Conjeture

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Abstract

In this article, we use the sieve of Eratosthenes to prove the Oppermann Conjecture.

X is a large number. We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

Let p_m be the greatest prime which satisfies $p_m \leq X$. Here we have, $p_m \leq X$, and $p_{m+1} > X + 1$, or $p_m < X$, and $p_{m+1} \geq X + 1$,

By using the sieve of Eratosthenes up to the first m primes, p_i, i=1,...m, the remaing numbers in the period of $(X^2 - X, X^2)$ is equaled to $[X \prod_{i=1,...,m}(1 - 1/p_i)] \geq [\frac{X}{p_m} \prod_{i=2,...,m}(\frac{p_i - 1}{p_{i-1}})] \geq 1$.

Here $[a]$ means the greatest integer less than or equal to a.

Obviously there is at least one remaining number in this period.

It is also a prime number.

Similarly the remaing numbers in the period of $(X^2, X^2 + X)$ is also equaled to $[X \prod_{i=1,...,m}(1 - 1/p_i)] \geq [\frac{X}{p_m} \prod_{i=2,...,m}(\frac{p_i - 1}{p_{i-1}})] \geq 1$.

Obviously there is at least one remaining number in this period too and it is also a prime number.

Thus we prove the Oppermann Conjecture.