An Explicit mapping of the Real Numbers to the Set of Natural Numbers

by Jim Rock

Abstract. By showing that Cantor's diagonal technique that lists the real numbers does not even work to show the proper cardinality of the rational numbers, we refute his proof technique. To explicitly reference them the real numbers are represented as the limit of partial decimal sums. We show that the real numbers are a denumerable set. We create a new (Level Set Theory), where all infinite sets have the same cardinality as the set of natural numbers.

The positive integers can be put in one to one correspondence with the terminating decimal fractions in the open interval (0, 1). $1 \to .1,\ 2 \to .2,\ \ldots,\ 10 \to .01,\ \ldots$ Each terminating decimal is the mirror image reflection through the decimal point of a positive integer. The mapping does not include any repeating decimal fractions. From this mapping the set of all rational numbers would appear to be uncountable. Cantor's diagonal technique that attempts to list the real numbers to prove them uncountable is invalid.

The problem is in defining the real numbers as the set of all infinite decimal expansions. That's vague.
Most non-algebraic real numbers cannot be explicitly referenced. We need a new definition.
Let p be an integer. Each real number S is:
the limit $m \to \infty$ $n = 1$ to m, $0 \le a_n \le 9$ $\sum p + a_n / 10^n = S$.

Make an infinite row of all the algebraic numbers. Each algebraic number has a column above it with that number having all the irrational algebraic numbers as exponents, leaving out duplicates. This infinite grid is matched to the natural numbers by a diagonalization technique. Let the grid element in row ***a*** and column ***b*** be **(*a, b*)**.

Then $\boldsymbol{0 \to (1, 1)\ 1 \to (2, 1)\ 2 \to (1, 2)\ 3 \to (3, 1)\ 4 \to (2, 2)\ 5 \to (1, 3)\ 6 \to (4, 1)\ 7 \to (3, 2)\ 8 \to (2, 3)\ 9 \to (1, 4) \ldots}$

The real numbers just created are a denumerable set. We use the six arithmetic processes (addition, subtraction, multiplication, division, exponentiation, and root extraction) on them repeatedly in any sequence to create all possible new transcendental numbers from the original set of real numbers. This new set of real numbers is denumerable.

This new set of real numbers is listed as an infinite row with columns above it of each row element having all the irrational numbers from the row just created as exponents, leaving out duplicates. The diagonalization process is then repeated. All possible additional transcendental numbers are created from this set by applying the six arithmetic processes. This enlarged set of real numbers is still denumerable.

This process of creating ever larger new sets of real numbers can be continued indefinitely. Eventually, every real number (S as defined above) will be generated. We conjecture that no other transcendental (trigonometric, logarithmic, power series or other) functions need be performed to generate the real numbers. Each time after the six arithmetic processes are performed the new denumerable set of real numbers just created is added as a row in a composite infinite grid, leaving out all duplicates. The diagonalization technique is applied to the composite infinite grid to map all the real numbers to the natural numbers.

A new non-hierarchical (Level Set Theory) is created when we let $1/\chi_0 = 0$.
Then the limit $x \to \infty$ $f(x) = \chi_0 \leftrightarrow$ the limit $x \to \infty$ $1/f(x) = 0$.
The limit $n \to \infty$ $n = \chi_0$, and the limit $n \to \infty$ $2^n = 2^{\chi_0} = \chi_0$. Thus, $|2^{\chi_0}| = |\chi_0|$.
The limit $n \to \infty$ $n^n = \chi_0^{\chi_0} = \chi_0$. Thus, $|\chi_0^{\chi_0}| = |\chi_0|$.
There is no hierarchy of infinites. The limit $n \to \infty$ $2^{2^n} = 2^{2^{\chi_0}} = \chi_0$. $|2^{2^{\chi_0}}| = |\chi_0|$.

Explore the detailed proofs and fascinating consequences of $|2^{\chi_0}| = |\chi_0|$ in https://arxiv.org/abs/1002.4433
Addressing mathematical inconsistency: Cantor and Gödel refuted by J. A. Perez.