An axiomatic model of science

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Abstract

Here we present a mathematical model formalizing the practice of science in nature. Under the premise that science is the best methodology to understand the World, then presumably a formalization of such is its best mathematical framework. *Axiomatic science* is a significant improvement over the informal practice of science and it has numerous desirable properties. Axiomatic science is a model of science and of physics, and as such, it contains a 'science' part and a 'physics' part. Axiomatic science is able to derive the 'physics' part, including the laws of physics, using the 'science' part as the starting point. Axiomatic science thus explains the origins of the laws of physics as a theorem of the formal practice of science. Finally, axiomatic science is 'clarification tool par excellence' for the foundation of physics.

Contents

1 Introduction

Axiomatic science is a mathematical theory formalizing the practice of science in nature. Axiomatic science contains a 'science' part which describes the world brutely without models, patterns or laws, and a 'physics' part which derives the broadest patterns applicable to the brute description. Axiomatic science introduces (and requires the use of) 'natural models', in which the laws of physics are derived from the description of nature, as distinct from an 'artificial model', in which the description of nature (solutions) is/are obtained from the laws of physics. Axiomatic science is at least as general as informal science and it introduces very strong constraints on what it means for a theory to be scientific in the formal mathematical sense. For instance, all physical theories that are the product of informal science ought to be formalizable as theorems of axiomatic science, lest they would be provably unscientific.

Unlike a usual physical theory containing only a 'physics' part, axiomatic science, as it also contains a 'science' part, is unavoidably a more fundamental representation of objective reality (the purview of science), than any physical theory resulting from science. Consistent with its scope, axiomatic science proposes solutions to long and enduring problems regarding the foundation of physics. For instance; the problem of time and entropy, the origin of the appearance of a quantum collapse, identifying a preferred interpretation of quantum mechanics, as well as philosophical problems such as "why does the world obey the specific laws of physics that it does instead of basically any other?", and even "why are there laws of physics at all?". Axiomatic science explains why it can answer these questions, and also explains why physics is unable to do the same: quite simply, the solutions to these problems are found in the 'science' part and not the 'physics' part of the axiomatic framework.

Axiomatic science is constructed using the formalism of theoretic computer science including that of Turing machines and that of algorithmic information theory. This construction negates most, and quite probably all, objections to falsificationism from the philosophy of science. Specifically, the domain of axiomatic science is constructed precisely as the set of all formal statements that are necessarily true for all possible state of affairs of the World. Consequently, it is necessarily the case that no formal argument can successfully invalidate elements of its domain. Furthermore, as axiomatic science is sufficiently descriptive to account for all possible state of affairs, it is also necessarily the case that there exists no fact verifiable in the world which is outside its domain. Axiomatic science is universal in the computer theoretic sense and, intuitively, in the 'physical/experimental sense'.

Another key feature is that its properties are naturally transposed to the domain of physics. More precisely, one may qualify axiomatic science as a *super-tautology*. As a super-tautology, axiomatic science provides a bridge between epistemology and ontology. Indeed, there exists no formal method by which an observer can experimentally exceed the domain of axiomatic science, yet there exists no elements of the domain of science which cannot be verified experimentally in principle (i.e. given enough time and resources). Because of this transposition, it will be by practicing science within the setup of axiomatic science that we will derive the laws of physics in this framework, just as we identify them when we practice science in the World. However, in the present case, the laws of physics are derived not by experimentation but by formal proof and are derived without physical baggage, without conceptual ambiguity and in their full generally. For these reasons and because it is a formalization of the practice of science, axiomatic science is a candidate model to serve as a maximally fundamental description of nature.

Let us start with a teaser problem, then we will lay out the axiomatic basis of the model.

Which of the two logically implies the other: The egg, or the molecular theory of organic chemistry?

As the first step towards understanding axiomatic science, we seek to understand the relationship between the 'science' and the 'physics' part, the role played by the logical implication, by initial conditions, and by axioms. A peculiar demand of axiomatic science is to banish what we will call 'artificial models' in favor exclusively of what we will call 'natural models'. Let us first understand what we mean using examples, and then we will generalize the idea. Within the methodology of axiomatic science, the logical implication is used in the direction that the observations imply the theory. For instance,

1. The discovery of astronomical redshift implies (or at least gives credibility to) models accounting for a metric expansion of space.

2. The discovery of the cosmic microwave background (CMB) implies (or at least gives credibility to) Big Bang models.

3. The measured homogeneity of the temperature of the CMB implies (or at least gives credibility to) inflationary models.

4. The discovery of DNA implies (or at least gives credibility to) natural selection models regarding the evolution of life on Earth.

5. The observation of objects falling from trees imply (or at least give credibility to) the theory of gravitation.

In this paradigm, the observations form the basis of the logical argument. From now on, we will qualify such arguments as *natural*; in the sense that the conclusion logically follows from the observations. For natural models, the set of observations take the role of the axioms (the premise); they are the brute facts from which the model is logically implied.

Shockingly, with perhaps axiomatic science as the only exception, we find that no theory in physics is mathematically constructed as a natural model. Let us first investigate how a mathematical model of nature is typically constructed, and then explain why we believe it to be a fallacy — we will refer to it as the *artificial model fallacy*.

To produce an axiomatic physical theory, one first "adjust" how one describes nature by doing what amounts to an 'axiomatic re-organization/compression' of the data. Specifically, one finds a new set of axioms, different than brute observations, but nonetheless believed to account for at least some good portion of the observations, and then uses these axioms as the new starting point. This re-organization is in the ideal case logically equivalent to the brute facts. It is usually justified on the grounds that a more elegant or aesthetically pleasing model will be produced which would be preferred to the brute facts. For instance, at CERN, the LHC collision data produces about 25 petabytes of data annually (it is algorithmically quite inelegant), but the standard model reasonably fits in a few textbooks (comparatively, it is quite elegant). If one cares about elegance, this is quite an improvement! As another example, consider that about 100 tons of cosmic dust fall on earth every day, and that about 10-20 trillion drops of water fall on Earth in the same period, etc. That is a lot of events to log as data. But we can compress a good chunk of it by postulating that this simple formula $F = Gm_1m_2/r^2$ is a law of nature. We can compress an even bigger chunk of this data by adding a few more laws; such as aerodynamics laws, weather patterns, etc.

However, with this new admittedly more aesthetically pleasing axiomatic basis as a starting point, the logical argument has a new but *artificial* starting point and points in a new but *artificial* direction. Mathematically, it is now the model that implies the observations. For instance, in physics, it is common to re-organize the presentation of the previously enumerated statements as follows:

1. The metric expansion of the universe implies (predicts) the astronomical redshift.

2. The Big Bang implies (predicts) the CMB.

3. The inflationary period immediately after the Big Bang implies (predicts) the homogeneity of the CMB temperature.
4. Natural selection implies (predicts) the existence of an information bearing physical structure such that offsprings acquire the phenotypes of their parents (e.g. DNA).
5. Gravity implies (predicts) that objects will fall from trees, should their attachment fail.

With this re-organization, one elevates the axioms of the 'artificial model' above observations. Contrary to the direction of the natural argument, the artificial model, as it is axiomatic, now outranks observations, and consequently, the model becomes vulnerable to falsification. As far as the artificial model is concerned, its theorems are true statements, but as practitioners of science, we know that the model is overselling its salad. We are in fact quite aware that the theorems of the model are mere predictions, not necessarily true in the World, and we do welcome and even expect the discovery of confirmatory or refuting evidence of the model. Indeed, if the axiomatic re-organization of the raw data is not equivalent in the information-theoretic sense to the raw data (in practice it seldom is), then the model will eventually make incorrect predictions and will be falsified.

Axiomatic science exposes this axiomatic re-organization as a fallacy. Axiomatic science, as a framework, connects 'raw data' to 'laws of physics' without requiring a preliminary axiomatic re-organization of the raw data. Since axiomatic science is a formal theory, employing any kind of artificial models becomes strongly prohibited within the framework. For instance, if one holds an egg, then drops it on the floor, then whatever model of reality one holds, it is now constrained to account for a broken egg on the floor. The artificial argument (the model implies the broken egg) is a false implication: there exists no such implication as in all cases the model is simply falsified should it fail to account for the broken egg.

The central tenet of axiomatic science is to construct a framework consistent with the assumption that it is not the model that constrains the World; rather, it is the World that constrains the model. In other words, *the data implies the model, but the model never implies the data.* As a result, axiomatic science places the initial conditions, not at the Big Bang, but at the present because it is the present that holds the set of all constraining raw data. Even though causality can, in principle, be used as an artificial model for a subset of all observations, axiomatic science shuns it. Within the framework of axiomatic science, even something as common as assuming that the present is caused by the past cannot be done, as it is an artificial argument. Such assumption, if true, must be formally proven from the framework as a theorem (within the 'physics' part) before it can be adopted. Consequently, it is thus more fundamental within axiomatic science to state that the past is logically implied by the present and that the system's history is recoverable by forensic investigation and as a model of the raw data, than it is to say that the present is caused by the past; the latter being a special case abstraction of the former.

2 The axioms of science

The fundamental object of study of axiomatic science is not the electron, the quark or even the microscopic super-strings, but the experiment. An experiment represents an atom of verifiable knowledge.

Let $\{s_1, s_2, ...\}$ be the sentences of a language L with alphabet A.

Definition 1 (Experiment)**.** *An experiment p is a tuple comprising two sentences of L. The first sentence, h, is called the hypothesis. The second sentence,* TM*, is called the protocol. The protocol takes as input the hypothesis. We say that the experiment holds if* $\mathrm{TM}[h]$ *is defined, and fails otherwise. If p holds, we say that the protocol verifies the hypothesis.*

$$\mathrm{TM}[h] \begin{cases} = r & p \text{ holds} \\ \nexists & p \text{ fails} \end{cases} \tag{1}$$

Finally, r, also a sentence of L, is the result.

An experiment, so defined, is formally reproducible. Indeed, for the protocol TM to be a Turing machine, the protocol must specify all steps of the experiment including the complete inner workings of any and all instrumentation used for the experiment. The protocol must be described as an effective method equivalent to an abstract computer program. Should the protocol fail to verify the hypothesis, the entire experiment; the group including the hypothesis and the protocol with its complete description of all instrumentation, is falsified as a group.

The set of all such experiments are the programs that halt. The set includes all provable mathematical statements and it is universal in the computer theoretic sense.

Definition 2 (Domain)**.** *Let $\mathbb{D}$ be the domain* (Dom) *of axiomatic science. We can define $\mathbb{D}$ in reference to a universal Turing machine* UTM *as:*

$$\forall s \in L[\mathrm{UTM}[s] \text{ halts} \implies s \in \mathbb{D}] \tag{2}$$

Thus, $\mathbb{D} := \mathrm{Dom}[\mathrm{UTM}]$.

Definition 3 (Manifest)**.** *A manifest M is a subset of $\mathbb{D}$:*

$$M \subset \mathbb{D} \tag{3}$$

Definition 4 (Set of all manifests)**.** *Let $\mathcal{P}[A]$ denote the power set of A. Then the set of all manifests $\mathbb{W}$ is:*

$$\mathbb{W} := \mathcal{P}[\mathbb{D}] \tag{4}$$

Thus, $M \in \mathbb{W}$.

Assumption 1 (The fundamental assumption of science)**.** *The state of affairs of the World is describable as a set of reproducible experiments. Therefore, the state of affairs is describable as a manifest. Furthermore, to each state of affairs corresponds a manifest, and finally, the manifest is a complete description of the state of affairs.*

Axiom 1 (Existence of the reference manifest)**.** *As the World is in a given state of affairs, then there exists, as a brute fact, a manifest $\mathbb{M}$ which corresponds to its state.*

$$\exists!\mathbb{M} \tag{5}$$

$\mathbb{M}$ is called the 'reference manifest'.

Remark: The symbol M will denote any manifest in $\mathbb{W}$, whereas $\mathbb{M}$ specifically denotes the reference manifest corresponding to the present state of affairs and referenced in Axiom 1.

The manifest is how the world presents itself to us in the most direct, unmodelled, uninterpreted and in an uncompressed manner. Brutely knowing the manifest is how one perceives the world without understanding any patterns and without knowing any laws of physics.

As infinitely many manifests M can be constructed from the elements of $\mathbb{D}$, one may wonder why it is the reference manifest $\mathbb{M}$ that is actual, and not any other. This brings us to the next assumption.

Assumption 2 (The fundamental assumption of 'nature')**.** *One assumes that the reference manifest $\mathbb{M}$ is randomly selected from the set of all possible manifests $\mathbb{W}$ according to a probability distribution $\rho[M]$.*

With this assumption we abandon all hope, as difficulty as it may be, of there being a model which tells us why $\mathbb{M}$ and not M is actual. This assumption is most directly responsible for necessitating that any physical model be derived as a natural model. Essentially, it is the mathematical formulation of the intuitive notion that the state of affairs is not implied by the model, and therefore, the most one can say about it is simply that $\mathbb{M}$ is a randomly selected element of $\mathbb{W}$.

However, as dreadful as this assumption might be, it is the key to recover the corpus of physics. The first step is to interpret $\mathbb{M}$ as a carrier of information, and it is precisely because it is randomly selected from a larger set that this is possible. We briefly recall the mathematical theory of information of Claude Shannon: Specifically, $\mathbb{M}$ will be interpreted as a message randomly selected from the set $\mathbb{W}$. Therefore, and by using the Shannon definition of entropy, we can quantify the amount of information in the message $\mathbb{M}$ by using the Shannon definition of entropy, as follows:

Definition 5 (Natural Information)**.** *Under Assumption 2 and by using the definition of the Shannon entropy, we can introduce natural information. We define natural information as the information one gains by knowing which manifest is randomly selected from $\mathbb{W}$, according to the probability distribution $\rho[M]$. The entropy of natural information is defined as:*

$$S = -\sum_{M \in \mathbb{W}} \rho[M] \ln \rho[M] \tag{6}$$

We recall that in the informal case, one would re-organize/compress the raw data into a shorter more aesthetically pleasing and, hopefully, logically equivalent set of axioms, then call the set of axioms a model of the physical system. Intuitively, we understand that one attempted to maximize 'something' but precisely what (aesthetics?, elegance?, ... ?) is not quite clear mathematically when one does so only informally. This brings us to our next assumption:

Assumption 3 (The fundamental assumption of physics)**.** *The conservation equation that results from maximizing the entropy of natural information associated with the probability distribution $\rho[M]$ are the laws of physics.*

Axiomatic science reveals that the quantity which one attempted to maximize as one informally constructed an artificial model of the data, is, in actuality, the entropy of natural information. The problem of finding the laws of physics is thus reduced to what amounts to maximizing the entropy of natural information using $\mathbb{M}$ as the message and $\mathbb{W}$ as the set of possible messages. With these tools, we can solve for the laws physics without first having to re-organize the raw data as axioms, and thus without inadvertently producing an artificial model. Indeed, as the starting point is Axiom 1, the laws of physics will necessarily be derived by a natural argument hence they will be immunized against the pathologies present in artificial models. Ascent these problems, and because the laws of physics will be derived as theorems of axiomatic science, fundamental questions (e.g. "why are there laws of physics at all?", and other similarly fundamental questions) will have proposed solutions within the framework.

3 Technical introduction

To understand the relationship between natural information, entropy, statistical physics and why the laws of physics should admit conservation laws that can be expressed in such a manner, we will introduce geometric (or generalized/non-commutative) thermodynamics, but first, we will provide a recap of statistical physics, and then of algorithmic thermodynamics.

3.1 Recap: Statistical physics

As many readers will no doubt have guessed, we will make heavy use of the framework of statistical physics to maximize the entropy of natural information

Table 1: Typical thermodynamic quantities

Symbol	**Name**	**Units**	**Type**
$E(q)$	energy	[Joule]	extensive
$1/T = k_B\beta$	temperature	1/[Kelvin]	intensive
$\overline{E}$	average energy	[Joule]	macroscopic
$V(q)$	volume	[meter^3]	extensive
$p/T = k_B\gamma$	pressure	[Joule/(Kelvin-meter^3)]	intensive
$\overline{V}$	average volume	[meter^3]	macroscopic
$N(q)$	number of particles	[kg]	extensive
$-\mu/T = k_B\delta$	chemical potential	[Joule/(Kelvin-kg)]	intensive
$\overline{N}$	average number of particles	[kg]	macroscopic

under suitable priors. Generally speaking, in statistical physics, we are interested in the distribution that maximizes the Boltzmann entropy,

$$S = -k_B \sum_{q \in \mathbb{Q}} \rho[q] \ln \rho[q] \tag{7}$$

subject to the fixed macroscopic quantities (the statistical priors). The solution to this maximization problem is the Gibbs ensemble. Typical thermodynamic quantities are shown in Table 1.

Taking these quantities as examples, the partition function (Gibbs ensemble) becomes:

$$Z = \sum_{q \in \mathbb{Q}} e^{-\beta E[q] - \gamma V[q] - \delta N[q]} \tag{8}$$

The probability of occupation of a micro-state (Gibbs measure) is:

$$\rho[q] = \frac{1}{Z} e^{-\beta E[q] - \gamma V[q] - \delta N[q]} \tag{9}$$

The average values are:

$$\overline{E} = \sum_{q \in Q} \rho[q] E[q] \tag{10}$$

$$\overline{V} = \sum_{q \in Q} \rho[q] V[q] \tag{11}$$

$$\overline{N} = \sum_{q \in Q} \rho[q] N[q] \tag{12}$$

and the variance for each quantity is:

$$\overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{13}$$

$$\overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{14}$$

$$\overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{15}$$

The entropy can be obtained from the partition function and is given by:

$$S = k_B \left(\ln Z + \beta\overline{E} + \gamma\overline{V} + \delta\overline{N} \right) \tag{16}$$

The laws of thermodynamics can be recovered by taking the following derivatives:

$$\left.\frac{\partial S}{\partial \overline{E}}\right|_{\overline{V},\overline{N}} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial \overline{V}}\right|_{\overline{E},\overline{N}} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial \overline{N}}\right|_{\overline{E},\overline{V}} = -\frac{\mu}{T} \tag{17}$$

and grouping them as follows:

$$d\overline{E} = TdS - pd\overline{V} + \mu d\overline{N} \tag{18}$$

This is the equation of the state of the system.

3.2 Recap: Algorithmic thermodynamics

Many authors[1, 2, 3, 4, 5, 6, 7, 8, 9] have discussed the similarity between the Gibbs entropy $S = -k_B \sum_{q\in\mathbb{Q}} \rho[q] \ln \rho[q]$ and the entropy in information theory $H = -\sum_{q\in\mathbb{Q}} \rho[q] \log_2 \rho[q]$. Furthermore, the similarity between the halting probability Ω and the Gibbs ensemble of statistical physics has also been studied[10, 11, 12, 8]. First let us introduce Ω. Consider the following sum:

$$\Omega = \sum_{p \text{ halts}} 2^{-|p|} \tag{19}$$

Here, $|p|$ denotes the length of p, a computer program. The sum represents the probability that a random program will halt on a universal Turing machine. The Chaitin's construction[2] (a.k.a. Ω, halting probability, Chaitin's constant) is defined for a universal Turing machine as a sum over its domain (the set of programs that halts for it) where the term $2^{-|p|}$ acts as a special probability distribution which guarantees that the value of the sum, Ω, is between zero and one[13]. Knowing Ω is enough to know the programs that halt and those that do not for the universal Turing machine it is defined for. Since the halting problem is unsolvable, Ω must, therefore, be non-computable. In fact, Ω's connection to the halting problem guarantees that it is algorithmically random, normal and incompressible.

It is possible to calculate some small (always finite) quantity of bits of Ω. As such, Calude[14] calculated the first 64 bits of Ω_U for some universal Turing machine U as:

$$\Omega_U = 0.000000100000010000011 0..._2 \tag{20}$$

Running the calculation for a handful of bits is certainly possible, however, any finitely axiomatic systems will eventually run out of steam and hit a wall. Calculating the digits of π, for instance, will not hit this kind of limitation. For π, the axioms of arithmetic are sufficiently powerful to compute as many bits as we wish to calculate, limited only by the physical resources of the computers at our disposal. To understand why this is not the case for Ω, we have to realize that solving Ω requires solving problems of arbitrarily higher complexity, the complexity of which always eventually outclasses the power of any finitely axiomatic system.

In 2002, Tadaki[8] suggested augmenting Ω with a multiplication constant D, which acts as a decompression term on Ω.

$$\begin{array}{lcl} \text{Chaitin construction} & \rightarrow & \text{Tadaki ensemble} \\ \Omega = \sum\limits_{q \text{ halts}} 2^{-|q|} & \rightarrow & \Omega_D = \sum\limits_{q \text{ halts}} 2^{-D|q|} \end{array} \tag{21}$$

With this change, Tadaki argued that the Gibbs ensemble compares to the Tadaki ensemble as follows:

$$\begin{array}{ll} \text{Gibbs ensemble} & \text{Tadaki ensemble} \\ Z = \sum\limits_{x \in X} e^{-\beta E[x]} & \Omega_D = \sum\limits_{q \text{ halts}} 2^{-D|q|} \end{array} \tag{22}$$

Interpreted as a Gibbs ensemble, the Tadaki construction forms a statistical ensemble where each program corresponds to one of its micro-state. The Tadaki ensemble admits the following quantities; the prefix code of length $|q|$ conjugated with D. As a result, it describes the partition function of a system which maximizes the entropy subject to the constraint that the average length of the codes is some constant $\overline{|q|}$;

$$\overline{|q|} = \sum_{q \text{ halts}} |q| 2^{-D|q|} \tag{23}$$

The entropy of the Tadaki ensemble is proportional to the average length of prefix-free codes available to encode programs:

$$S = k_B \left(\ln \Omega + D\overline{|q|} \ln 2 \right) \tag{24}$$

The constant $\ln 2$ comes from the base 2 of the halting probability function instead of base e of the Gibbs ensemble.

John C. Baez and Mike Stay[12] took the analogy further by suggesting a connection between algorithmic information theory and thermodynamics, where the characteristics of the ensemble of programs are equivalent to thermodynamic observables. In algorithmic thermodynamics, one extends Ω with algorithmic quantities to obtain:

$$\begin{aligned} &\text{Baez-Stay ensemble} \\ &\Omega' = \sum_{q \text{ halts}} 2^{-\beta E[q] - \gamma V[q] - \delta N[q]} \end{aligned} \tag{25}$$

Noting its similarities to the Gibbs ensemble of statistical physics (8), these authors suggest an interpretation where $E[q]$ is the expected value of the logarithm of the program's runtime, $V[q]$ is the expected value of the length of the program, and $N[q]$ is the expected value of the program's output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper);

"

1. $T = 1/\beta$ is the *algorithmic temperature* (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.
2. $p = \gamma/\beta$ is the *algorithmic pressure* (analogous to pressure). This measures the trade-off between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount while holding the number of programs in the ensemble and their mean output fixed.
3. $\mu = -\delta/\beta$ is the *algorithmic potential* (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

"

–John C. Baez and Mike Stay

From equation (25), they derive analogs of Maxwell's relations and consider thermodynamic cycles, such as the Carnot cycle or Stoddard cycle. For this, they introduce the concepts of *algorithmic heat* and *algorithmic work*. Finally, we note that other authors have suggested other alternative mappings in other but related contexts[10, 9].

3.3 Applicability to axiomatic science

Comparing the axioms of science (Section 2) to the very similar computer theoretic setup for algorithmic thermodynamics, it is clear that algorithmic thermodynamics will play a significant role. In fact, axiomatic science defines experiments as protocols verifying an hypothesis, which is analogous to a program halting for an input. With algorithmic thermodynamics, we now have an algorithmic analog to statistical physics, a framework already familiar to physics and capable of producing conservation equations, that can be applied to our axiomatic model of science. What is left to do is to apply said framework to the axioms of science in such a way that the general conservation equations is mathematically the same as a law of physics. Then under Axiom 1 and Assumption 1, as the manifest is the brute description of the state of affairs, then the general conservation law resulting from the framework will be its maximally informative natural model (i.e. the conservation equation for which the entropy of natural information is maximized). This gives us the 'physics' part as a theorem of the 'science' part.

Let us now summarize the relevant prior literature as it will help us introduce the solution that we propose.

3.4 Hint 1: Seth Lloyd

In 2002, Lloyd[15] calculated the total number of bits available for computation in the universe, as well as the total number of operations that could have occurred since the universe's beginning. Seth Lloyd's paper was published 17 years ago and has received approximately 520 citations. His calculations stand to this day and they are considered to be uncontroversial.

For both quantities (the quantity of bits stored in the universe and the quantity of operations made on those bits), Lloyd obtains the number $\approx 10^{122} k_B[\text{bit}]$. This number is consistent with other approaches; for instance, the Bekenstein-Hawking entropy[16, 17] of the cosmological horizon (also $\approx 10^{122} k_B[\text{bit}]$), and the entropy of the holographic surface at the cosmological horizon suggested by Susskind[18] (also $\approx 10^{122} k_B[\text{bit}]$).

How did Lloyd derive these numbers? First, he calculated the value for these quantities while ignoring the contribution of gravity and he obtained $\approx 10^{90} k_B[\text{bit}]$. It is only by including the degrees of freedom of gravity that the number $\approx 10^{122} k_B[\text{bit}]$ is obtained, which he does in the second part. As we are interested in the totals, we will go directly to the calculations that include the contribution of gravity. We state Lloyd's main result and note that the details of the calculation can be reviewed in his paper. Lloyd obtains a relation between time and number of operations for the universe:

$$\#\text{ops} \approx \frac{\rho_c c^5 t^4}{\hbar} \approx \frac{t^2 c^5}{G\hbar} = \frac{1}{t_p^2} t^2 \tag{26}$$

where ρ_c is the critical density and t_p is the Planck time and t is the age of the universe. With present-day values of t, the result is $\approx 10^{122} k_B$[bit]. Lloyd concludes that his results are consistent with the Bekenstein bound and the holographic principle. He states:

> "Applying the Bekenstein bound and the holographic principle to the universe as a whole implies that the maximum number of bits that could be registered by the universe using matter, energy, and gravity is $\approx \frac{c^2 t^2}{l_p^2} = \frac{t^2}{t_p^2}$."

which is also $\approx 10^{122} k_B$[bit]. A particularly interesting consequence of this result is that these relations appear to imply conservation of both information and operations in space-time (the numerical quantity of 10^{122} is obtained by summing over all available degrees of freedom in space-time). So with this hint, we are now looking for a fundamental relationship between entropy, information, operations, and... space-time.

3.5 Hint 2: Entropy and space-time

A relation between entropy and space-time has been anticipated (or at least hinted at) since probably the better part of four decades. The first hints were provided by the work of Bekenstein[19, 20, 21] regarding the similarities between black holes and thermodynamics, culminating in the four laws of black hole thermodynamics. The temperature, originally introduced by analogy, was soon augmented to a real notion by Hawking[16] with the discovery of the Hawking temperature derivable from quantum field theory on curved space-time. We note the discovery of the Bekenstein-Hawking entropy, connecting the area of the surface of a horizon to be proportional to one fourth the number of elements with Planck area that can be fitted on the surface: $S = k_B c^3/(4\hbar G)A$.

We mention Ted Jacobson[22] and his derivation of the Einstein field equation as an equation of state of a suitable thermodynamic system. To justify the emergence of general relativity from entropy, Jacobson first postulated that the energy flowing out of horizons becomes hidden from observers. Next, he attributed the role of heat to this energy for the same reason that heat is energy that is inaccessible for work. In this case, its effects are felt, not as "warmth", but as gravity originating from the horizon. Finally, with the assumption that the heat is proportional to the area A of the system under some proportionality constant η, and some legwork, the Einstein field equations are eventually recovered.

Recently, Erik Verlinde[23] proposed an entropic derivation of the classical law of inertia and of classical gravity. He compared the emergence of such laws to that of an entropic force, such as a polymer in a warm bath. Each law is emergent from the equation $TdS = Fdx$, under the appropriate temperature and a posited entropy relation. His proposal has encouraged a plurality of attempts to reformulate known laws of physics using the framework of statistical physics. Visser[24] provides, in the introduction to his paper, a good summary of the literature on the subject. The ideas of Verlinde have been applied to loop quantum gravity ([25]), the Coulomb force ([26]), Yang-Mills gauge fields ([27]), and cosmology ([28, 29, 30]). Some criticism has, however, been voiced[31, 32, 33, 34, 35], including by Visser[24].

Even more recently, a connection between entanglement entropy and general relativity has been supported by multiple publications[36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53].

Finally, we mention the body of work of George Ellis regarding the evolving block universe hypothesis detailed in [54, 55, 56] and the connection between space-time events, general relativity and quantum mechanics.

We are now ready to attempt a solution.

3.6 Attempt 1: Finding a specific system of statistical physics with suitable properties

Our first series of attempts could be grouped under a simple concept: In each case, we attempted to construct a specific system of statistical physics having a double interpretation; one, as a system of algorithmic thermodynamics admitting a conservation relation involving bits and operations, and second, that said conservation relation be interpretable as a physical system of space-time (i.e. as a system described by general relativity).

Finding a specific system of statistical physics means attributing a definition to the functions of the thermodynamic observables that are used in the partition function. This is the approach used by Ted Jacobson and Erik Verlinde in the context of connecting general relativity and classical relativity, respectively, to entropy. In each of their paper, the degrees of freedom of space are assumed to be quadratic (i.e. they grow as an area law). Consequently, the thermodynamic observables are quadratic degrees of freedom. Attempting to expand upon these ideas, we have produced over a dozen draft papers regarding the emergence of many physical laws, including a toy model of a cosmology emergent from quadratic degrees of freedom. However, in the end, we felt that there was a general problem with this approach.

The problem with this approach is that any results that we would obtain would be specific to the artificially constructed partition function. Even if it had the desired properties, one would still have to justify why this specific partition function and not another happens to be the one which describes the World. Specifically, we were unable to justify by natural argument why the degrees of freedom of space would be quadratic. The model, constructed as a specific system of statistical physics would thus remain artificial.

Furthermore, we were missing out on the full potential of statistical physics. Indeed, statistical physics is able to produce conservation equations on the broadest of scales. As a typical example, we refer to the fundamental relation of thermodynamics involving the conservation of energy over a change in thermodynamic observables:

$$d\overline{E} = TdS - pd\overline{V} \tag{27}$$

This relation applies to all systems of statistical physics (provided they admit $\overline{V}$ and $\overline{E}$ as thermodynamic observables and are at equilibrium) regardless of the specific system under investigation.

To capture this generality into our natural model, our final solution was not to search for a specific system of statistical physics, but instead to increase the generality of thermodynamics; in the present case, with a non-commutative algebra applied to the thermodynamic observables. In this generalization, which we call *geometric thermodynamics*, the extremely general conservation relation above becomes a special case of an even more general conservation relation that, surprisingly, has the suitable properties.

We will now introduce geometric thermodynamic.

4 Geometric Thermodynamics

We identified the potential to generalize statistical physics with a non-commutative algebra as we attempted to create thermodynamic cycles that are consistent with the symmetries of space-time. By doing so, we realized that such cycles could be produced if the relevant thermodynamic observables obeyed a non-commutative algebra. With this insight, we have "reverse engineered" the type of partition function along with a suitable microscopic object of study which would eventually produce cycles with the suitable properties.

To understand in more detail, let us investigate a hypothetical cycle involving a number of thermodynamic observables. Lets name them $\overline{X}$, $\overline{Y}$ and $\overline{Z}$. Such quantities would be extensive, have the meter as their unit, and would be conjugated to a Lagrange multiplier $\tilde{k}$ having the inverse units (m^{-1}). The equation of state of such a system would be:

$$\tilde{k}^{-1}dS = d\overline{X} + d\overline{Y} + d\overline{Z} \tag{28}$$

For a change over the quantities $\overline{X}$, $\overline{Y}$ and $\overline{Z}$ to be consistent with the symmetries of Euclidean space, one would expect that the change in entropy along two paths of equal distance, say a path going in a straight line from $(0, 0, 0)$ to $(0, 5, 0)$ and a path going in a straight line from $(0, 0, 0)$ to $(3, 4, 0)$, to be equal. Indeed, the Euclidean distance along either path is the same: in this case, 5 meters. Since the paths are related to one another via rotation of the frame of reference, the entropic cost of the transformation should only depend on the Euclidean length of the path, and not on the orientation of the frame of reference.

One can enforce this property by demanding that the thermodynamic observables obey a suitable non-commutative algebra. Let's see with an example. As the first step, we add generators of the Clifford algebra $(\sigma_1, \sigma_2, \sigma_3)$ to each quantity. We get:

$$\tilde{k}^{-1} dS = d\overline{X}\sigma_1 + d\overline{Y}\sigma_2 + d\overline{Z}\sigma_3 \tag{29}$$

The second step is to verify that the entropy conforms to the Euclidean distance. We can investigate if this is the case by squaring the equation of state. We obtain:

$$\begin{aligned} \tilde{k}^{-2}(dS)^2 =& \sigma_1^2 (d\overline{X})^2 +_2^2 (d\overline{Y})^2 + \sigma_3^2 (d\overline{Z})^2 \\ &+ (\sigma_1\sigma_2 + \sigma_2\sigma_1) d\overline{X} d\overline{Y} + (\sigma_1\sigma_3 + \sigma_3\sigma_1) d\overline{X} d\overline{Z} + (\sigma_2\sigma_3 + \sigma_3\sigma_2) d\overline{Y} d\overline{Z} \end{aligned} \tag{30}$$

In the case were σ_1, σ_2 and σ_3 are commutative, the cross terms $\sigma_1\sigma_2 + \sigma_2\sigma_1$, $\sigma_1\sigma_3 + \sigma_3\sigma_1$ and $\sigma_2\sigma_3 + \sigma_3\sigma_2$ do not cancel, but if they are, say matrices, with the following properties:

$$\sigma_1^2 = 1 \tag{31}$$

$$\sigma_2^2 = 1 \tag{32}$$

$$\sigma_3^2 = 1 \tag{33}$$

$$\sigma_1\sigma_2 + \sigma_2\sigma_1 = 0 \tag{34}$$

$$\sigma_1\sigma_3 + \sigma_3\sigma_1 = 0 \tag{35}$$

$$\sigma_2\sigma_3 + \sigma_3\sigma_2 = 0 \tag{36}$$

(We note that a matrix representation of these generators is simply the Pauli matrices.)

Then, the cross terms cancel and we obtain:

$$\tilde{k}^{-2}(dS)^2 = (d\overline{X})^2 + (d\overline{Y})^2 + (d\overline{Z})^2 \tag{37}$$

The resulting equation of state has the mathematical form of the Euclidean distance $d^2 := \tilde{k}^2(dS)^2$. The entropy, as demanded, is invariant under rotation of the frame of reference. As we will see, if one uses the flexibility of geometric algebra, one can generalize this argument to space-times of any dimensions, any signature, and even including arbitrarily curved space-times.

For instance, a thermodynamic system of special relativity would have $\overline{X}$, $\overline{Y}$, $\overline{Z}$ and $\overline{T}$ as its thermodynamic quantities. The equation of state, using the generators γ_0, γ_1, γ_2 and γ_3 is:

$$\tilde{k}^{-1} dS = \tilde{k}^{-1} f d\overline{T}\gamma_0 + d\overline{X}\gamma_1 + d\overline{Y}\gamma_2 + d\overline{Z}\gamma_3 \tag{38}$$

Here, both $\tilde{k}$ and f are Lagrange multipliers. $\overline{T}$ is a extensive quantity with units s and it is conjugated with f having units s^{-1}. Squaring the equation of state gives:

$$\begin{aligned}\tilde{k}^{-2}(dS)^2 =&\tilde{k}^{-2}f^2(d\overline{T})^2\gamma_0^2 + (d\overline{X})^2\gamma_1^2 + (d\overline{Y})^2\gamma_2^2 + (d\overline{Z})^2\gamma_3^2 \\ &+ \tilde{k}^{-2}f^2 d\overline{T}d\overline{X}(\gamma_0\gamma_1 + \gamma_1\gamma_0) + \tilde{k}^{-2}f^2 d\overline{T}d\overline{Y}(\gamma_0\gamma_2 + \gamma_2\gamma_0) + \tilde{k}^{-2}f^2 d\overline{T}d\overline{Z}(\gamma_0\gamma_3 + \gamma_3\gamma_0) \\ &+ d\overline{X}d\overline{Y}(\gamma_1\gamma_2 + \gamma_2\gamma_1) + d\overline{X}d\overline{Z}(\gamma_1\gamma_3 + \gamma_3\gamma_1) \\ &+ d\overline{Y}d\overline{Z}(\gamma_2\gamma_3 + \gamma_3\gamma_2) \end{aligned} \tag{39}$$

The equation of state has the Lorentz symmetries provided the generators have the following non-commutative properties:

$$\gamma_0^2 = 1 \tag{40}$$
$$\gamma_1^2 = -1 \tag{41}$$
$$\gamma_2^2 = -1 \tag{42}$$
$$\gamma_3^2 = -1 \tag{43}$$
$$\gamma_0\gamma_1 + \gamma_1\gamma_0 = 0 \tag{44}$$
$$\gamma_0\gamma_2 + \gamma_2\gamma_0 = 0 \tag{45}$$
$$\gamma_0\gamma_3 + \gamma_3\gamma_0 = 0 \tag{46}$$
$$\gamma_1\gamma_2 + \gamma_2\gamma_1 = 0 \tag{47}$$
$$\gamma_1\gamma_3 + \gamma_3\gamma_1 = 0 \tag{48}$$
$$\gamma_2\gamma_3 + \gamma_3\gamma_2 = 0 \tag{49}$$
$$\tag{50}$$

(We note that a matrix representation of these generators is simply the Dirac matrices.)

We also pose $c := \tilde{k}^{-1}f$, then, the equation of state is:

$$\tilde{k}^{-2}(dS)^2 = c^2(d\overline{T})^2 - (d\overline{X})^2 - (d\overline{Y})^2 - (d\overline{Z})^2 \tag{51}$$

Geometric thermodynamics is quite easy to construct, yet it is incredibly powerful. In the general case, one begins by defining an arbitrary non-commutative basis as follows: $\forall\mu \in \{0, ..., n-1\}$ and $\forall\nu \in \{0, ..., n-1\}$, then

$$g_{\mu\nu} := \frac{1}{2}(e_\mu e_\nu + e_\nu e_\mu) \tag{52}$$

A matrix g can then be defined as:

$$g_{\mu\nu} = \begin{array}{cc} \begin{array}{cccccc} \scriptstyle\mu=0 & \scriptstyle\mu=1 & \scriptstyle\mu=2 & \scriptstyle\mu=3 & \cdots & \scriptstyle\mu=n-1 \end{array} & \\ \begin{pmatrix} g_{00} & g_{10} & g_{20} & g_{30} & \cdots & g_{(n-1)0} \\ g_{01} & g_{11} & g_{21} & g_{31} & \cdots & g_{(n-1)1} \\ g_{02} & g_{12} & g_{22} & g_{32} & \cdots & g_{(n-1)2} \\ g_{03} & g_{13} & g_{23} & g_{33} & \cdots & g_{(n-1)3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{0(n-1)} & g_{1(n-1)} & g_{2(n-1)} & g_{3(n-1)} & \cdots & g_{(n-1)(n-1)} \end{pmatrix} & \begin{array}{c} \scriptstyle\nu=0 \\ \scriptstyle\nu=1 \\ \scriptstyle\nu=2 \\ \scriptstyle\nu=3 \\ \vdots \\ \scriptstyle\nu=n-1 \end{array} \end{array} \tag{53}$$

One then defines n thermodynamic observables using this geometric basis. The priors defined in equation (10), for example $\overline{E} = \sum_{q\in\mathbb{Q}} E[q]\rho[q]$ are now simply multiplied with a generator e_i of the geometric algebra, yielding n equations. $\forall i \in \{0, ..., n-1\}$:

$$\overline{X_i}e_i = \sum_{q\in\mathbb{Q}} (X_i[q]e_i)\,\rho[q] \tag{54}$$

Then, by maximizing the entropy with these priors as the constraints and by using the method of the Lagrange multipliers, one will obtain a generalized non-commutative thermodynamics conservation relation, in lieu of equation (18).

$$dS = \kappa d\overline{X}_0 e_0 + \cdots + \kappa d\overline{X}_{n-1} e_{n-1} \tag{55}$$

We note that had we instead selected a geometric algebra such that the generators are commutative, then one would recover, as a special case, the traditional conservation relation of energy found in statistical physics. Explicitly, posing the properties of the generators $e_0, ..., e_{n-1}$ to be commutative:

$$e_i^2 = 1 \tag{56}$$

$$e_i e_j = e_j e_i \tag{57}$$

one obtains the relation $dS = \kappa d\overline{X}_0 + ... + \kappa\overline{X}_{n-1}$, which is of the same mathematical form as equation (18). Therefore, geometric thermodynamic is indeed a generalization of statistical physics.

4.1 Outline

Our goal will be to derive the geometric partition function $\mathbf{Z}$ (Theorem 1), the geometric probability measure $\boldsymbol{\rho}[q]$ (Theorem 2) the geometric entropy $\mathbf{S}$ (Theorem 3) and the geometric equation of state $d\mathbf{S}$ (Theorem 4). Then, to find a scalar expression for $d\mathbf{S} \to dS$ and show that it equates the curvature of space-time (Theorem 5). Finally, we will show how the Feynman path integral and how quantum field theory (QFT) are special cases of geometric thermodynamics, in (Section 4.6) and (Section 4.8) respectively.

4.2 Results

To apply geometric thermodynamics to the axioms of science, we will pose our final assumption:

Assumption 4 (The fundamental assumption of 'geometric substance')**.** *We equip an experiment p with the observables of geometric thermodynamics. We will call such an experiment a geometric event* $\mathbf{q}$*.*

We have opted to use the boldface (for instance $\boldsymbol{\rho}$, $\mathbf{q}$) as our notation to designate that the quantity is a vector of the geometric algebra. We will prefix the name of such quantities with the term *geometric*. Geometric quantities contain basis elements within their expression so as to enforce the suitable non-commutative relation between the quantities of the expression.

First, let us define a space-time event:

Definition 6 (Space-time event)**.** *A space-time event q is an $(m+n)$-tuple.*

$$q := (\underbrace{X_0, ...X_{n-1}}_{\text{time terms}}, \underbrace{X_n, ...X_{n+m-1}}_{\text{space terms}}) \tag{58}$$

The quantities X_i are elements of $\mathbb{R}$. We pose immediately that the first n terms denote the 'time dimensions' and the next m terms denote the 'space dimensions'. A space-time event can be represented algebraically using geometric algebra:

$$\mathbf{q} := \underbrace{X_0 e_0 + ... + X_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{X_n e_n + ... + X_{n+m-1} e_{n+m-1}}_{\text{space terms}} \tag{59}$$

Now, let us define a geometric event:

Definition 7 (Geometric event)**.** *A geometric event $\mathbf{q_g}$ is a generalization of a space-time event. A geometric event is a multi-vector of the geometric algebra. For example,*

$$\mathbf{q_g} := \mathbf{q} + \underbrace{A + Be_0e_1 + Ce_0e_2 + ... + De_0e_1e_2 + ...}_{\text{geometric terms}} \tag{60}$$

Without loss of generality and to keep the steps of the proof to manageable lengths, we will derive the results using space-time events. The geometric terms can be trivially added to all the results that follow.

Definition 8 (Set of events)**.** *We define a set of events $\mathbb{Q}$ as:*

$$\left(\mathbb{Q} \subset \{q : q \in \mathbb{R}^{n+m}\}\right) \wedge |\mathbb{Q}| < 2^{\aleph_1} \tag{61}$$

The quantity $n+m$ denotes the number of dimensions of the events. The notation $|\mathbb{Q}| < 2^{\aleph_1}$ indicates that we will be dealing with countable sets, and thus the sum notation of the entropy will be used instead of the differential form.

Definition 9 (Interval)**.** *Let* $\mathbf{q}_1$ *and* $\mathbf{q}_2$ *be events written using the geometric algebra:*

$$\mathbf{q}_1 := t_1 e_t + x_1 e_x + y_1 e_y + z_1 e_z \tag{62}$$
$$\mathbf{q}_2 := t_2 e_t + x_2 e_x + y_2 e_y + z_2 e_z \tag{63}$$

Then, the interval between these events is defined in reference to the familiar metric tensor g*:*

$$(\mathbf{q}_1 - \mathbf{q}_2)^2 = ((t_1 - t_2)e_t + (x_1 - x_2)e_x + (y_1 - y_2)e_y + (z_1 - z_2)e_z)^2 \tag{64}$$
$$= ((\Delta t)e_t + (\Delta x)e_x + (\Delta y)e_y + (\Delta z)e_z)^2 \tag{65}$$
$$\begin{aligned}=&(\Delta t)e_t(\Delta t)e_t + (\Delta t)e_t(\Delta x)e_x + (\Delta t)e_t(\Delta y)e_y + (\Delta t)e_t(\Delta z)e_z\\ &+ (\Delta x)e_x(\Delta t)e_t + (\Delta x)e_x(\Delta x)e_x + (\Delta x)e_x(\Delta y)e_y + (\Delta x)e_x(\Delta z)e_z\\ &+ (\Delta y)e_y(\Delta t)e_t + (\Delta y)e_y(\Delta x)e_x + (\Delta y)e_y(\Delta y)e_y + (\Delta y)e_y(\Delta z)e_z\\ &+ (\Delta z)e_z(\Delta t)e_t + (\Delta z)e_z(\Delta x)e_x + (\Delta z)e_z(\Delta y)e_y + (\Delta z)e_z(\Delta z)e_z\end{aligned} \tag{66}$$
$$\begin{aligned}=&(\Delta t)^2 e_t^2 + (\Delta x)^2 e_x^2 + (\Delta y)^2 e_y^2 + (\Delta z)^2 e_z^2\\ &+ \Delta t \Delta x(e_t e_x + e_t e_x) + \Delta t \Delta y(e_t e_y + e_y e_t) + \Delta t \Delta z(e_t e_z + e_z e_t)\\ &+ \Delta x \Delta y(e_x e_y + e_y e_x) + \Delta x \Delta z(e_x e_z + e_z e_x)\\ &+ \Delta y \Delta z(e_y e_z + e_z e_y)\end{aligned} \tag{67}$$
$$= \sum_{\mu\nu} g_{\mu\nu} \Delta X_\mu \Delta X_\nu \tag{68}$$

where $\forall \mu \in \{0,1,2,3\} \forall \nu \in \{0,1,2,3\} \left[g_{\mu\nu} = \frac{1}{2}(e_\mu e_\nu + e_\nu e_\mu) \right]$.

We will derive our ensemble of events using the following definition for the entropy:

Definition 10 (Geometric entropy)**.**

$$\mathbf{S} = -\sum_{q \in \mathbb{Q}} \boldsymbol{\rho}[q] \ln \boldsymbol{\rho}[q] \tag{69}$$

Definition 11 (Physical quantities of the partition function)**.** *As we derive the partition function for an ensemble of events, two physical quantities will be introduced as Lagrange multipliers. They are: 1) the entropic repetency* $\tilde{k}$ *(normally the repetency is represented by the symbol* $\tilde{\nu}$*, but we already use the symbol* ν *extensively for the indices of our basis; therefore, we opt to use* $\tilde{k}$ *here to avoid ambiguity); and 2) the entropic frequency* f*. Specifically,* $\tilde{k} = k/2\pi = 1/\lambda$*, where* k *is the wave-number and* λ *is the wavelength.*

These quantities are the conjugated variables to a distance x and time t, respectively. By convention, we prefix the Lagrange multipliers with the word

Table 2: The physical quantities of the geoemtric ensemble

Symbol	**Name**	**Units**	**Type**
$x(q)$	space	[meter]	extensive
$\tilde{k}$	entropic repetency	[1/meter]	intensive
$\overline{x}$	thermal space	[meter]	macroscopic
$t(q)$	time	[second]	extensive
f	entropic frequency	[1/second]	intensive
$\overline{t}$	thermal time	[second]	macroscopic
$c := f/\tilde{k}$	entropic speed	[meter/second]	intensive

"entropic", and its averaged conjugated quantity will be prefixed with the word "thermal". $\tilde{k}$ and f are both intensive properties, whereas x and t are extensive. Indeed, a process taking 1 min followed by a process taking 2 min takes a total of 3 min (extensive). For the x quantity; walking 1 meter followed by walking 2 meters implies one has walked a total of 3 meters (extensive). Adding or removing clocks from a group of clocks ticking at a frequency f (say once per second) has no impact on the frequency of the other elements of the group (intensive). The same argument applies to the entropic repetency (intensive). The units of $\tilde{k}$ are m^{-1}, the units of x are the meters, the units of t are the seconds, and the units of f are s^{-1}. Finally, we define the quantity $c := f/\tilde{k}$. These quantities are summarized in Table 2.

To derive the ensemble of events, we will assume the following is permitted: Instead of creating an ensemble of M (a manifest) selected over $\mathbb{W}$ (the set of all manifests), we create n ensembles of p (an experiment) selected over $\mathbb{D}$ (the domain of science). In this case, the ensemble $M \in \mathbb{W}$ is the grand canonical ensemble to n canonical ensembles $p \in \mathbb{D}$. At any point, should we prefer to work with $M \in \mathbb{W}$, rather than with n systems of $p \in \mathbb{D}$, we can redress to a grand-canonical ensemble simply by introducing $\mu N(M)$ as a thermodynamic observable in the grand-canonical ensemble and summing $M \in \mathbb{W}$ instead of $p \in \mathbb{D}$. Specifically, the assumption is that $\mu N(M)$ is a valid thermodynamic observable of a manifest.

As this assumption is about experiments, and geometric events are experiments equipped with additional structure, then we will also inherit this assumption for geometric events. With geometric events, we will now create a canonical ensemble by summing over $q \in \mathbb{Q}$.

Definition 12 (Ensemble of events)**.** *The probability measure of an ensemble of events maximizes the entropy under the constraints of the macroscopic priors defined in Table 2 expressed in the geometric algebra of events. Specifically,* $\forall i \in \{0, ..., n+m-1\}$ *the priors to the ensemble are:*

$$\overline{X}_i e_i = \sum_{q \in \mathbb{Q}} (X_i[q] e_i)\, \boldsymbol{\rho}[q] \tag{70}$$

The functions $X_i[q]$ are maps $X_i : \mathbb{Q} \to \mathbb{R}$ where $X_i[q]$ returns the value of the i^{th} element of the $(m+n)$-tuple of q. There are $m+n$ priors. The terms $\overline{X}_i e_i$ are averages.

Note: we will prove Theorem 1 and 2 together.

Theorem 1 (Geometric partition function)**.** *The partition function of the ensemble of events is:*

$$\mathbf{Z} = \sum_{q \in \mathbb{Q}} \exp\left(-f \underbrace{(X_0[q]e_0 + ... + X_{n-1}[q]e_{n-1})}_{\text{time terms}} - \tilde{k} \underbrace{(X_n[q]e_n + ... + X_{n+m-1}[q]e_{n+m-1})}_{\text{space terms}} \right) \tag{71}$$

where $\tilde{k}$ and f are Lagrange multipliers.

Theorem 2 (Geometric probability measure)**.** *The probability measure $\boldsymbol{\rho}(q)$ is:*

$$\boldsymbol{\rho}(q) = \mathbf{Z}^{-1} \exp\left(-f \underbrace{(X_0[q]e_0 + ... + X_{n-1}[q]e_{n-1})}_{\text{time terms}} - \tilde{k} \underbrace{(X_n[q]e_n + ... + X_{n+m-1}[q]e_{n+m-1})}_{\text{space terms}} \right) \tag{72}$$

Proof. We will now prove Theorem 1 and Theorem 2. One obtains the partition function $\mathbf{Z}$ with the usual method of the Lagrange multipliers.

1. The constraints, are:

$$I = \sum_{q \in \mathbb{Q}} \boldsymbol{\rho}[q] \tag{73}$$

   where I is the identity element, and

$$\forall i \in \{0, ..., n+m-1\} \left[\overline{X}_i e_i = \sum_{q \in \mathbb{Q}} (X_i[q]e_i)\, \boldsymbol{\rho}[q] \right] \tag{74}$$

2. The function to maximize is:

$$\mathbf{S} = -\sum_{q \in \mathbb{Q}} \boldsymbol{\rho}[q] \ln \boldsymbol{\rho}[q] \tag{75}$$

3. The Lagrange equation is:

$$\boldsymbol{\mathcal{L}} = \left(-\sum_{q\in\mathbb{Q}} \boldsymbol{\rho}[q] \ln \boldsymbol{\rho}[q] \right) - \lambda \left(\sum_{q\in\mathbb{Q}} \boldsymbol{\rho}[q] - I \right) - \sum_{i=0}^{n+m-1} \left(\lambda_i \sum_{q\in\mathbb{Q}} \left(X_i[q] e_i \right) \boldsymbol{\rho}[q] - \overline{X}_i e_i \right) \tag{76}$$

where λ and the set of λ_i are Lagrange multipliers.

4. Maximizing $\boldsymbol{\mathcal{L}}$ with respect to $\boldsymbol{\rho}[q]$ is done by taking its derivative and posing it equal to $\mathbf{0}$.

$$\frac{\partial \boldsymbol{\mathcal{L}}}{\partial \boldsymbol{\rho}[q]} = \mathbf{0} \tag{77}$$

where $\mathbf{0}$ is the null vector. Here we will be dealing with a number of vector derivatives and related identities. Specifically, we require scalar-by-vector and vector-by-vector. From *The Matrix Cookbook* by [57], these derivatives behave as we would expect them to from what we know of derivatives involving scalar variables. Then using the appropriate corresponding identities, one easily gets:

$$\frac{\partial \boldsymbol{\mathcal{L}}}{\partial \boldsymbol{\rho}[q]} = -\ln \boldsymbol{\rho}[q] - I - \lambda I - \sum_{i=0}^{n+m-1} \left(\lambda_i X_i[q] e_i \right) = \mathbf{0} \tag{78}$$

5. Solving for $\boldsymbol{\rho}(q)$ in (Equation 78) one obtains:

$$\boldsymbol{\rho}(q) = \exp(-I - \lambda I) \exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \tag{79}$$

6. From the constraint $I = \sum_{q\in\mathbb{Q}} \boldsymbol{\rho}(q)$, we can find the expression for $\exp(-I - \lambda I)$ as follows:

$$I = \sum_{q\in\mathbb{Q}} \boldsymbol{\rho}(q) \tag{80}$$

$$= \exp(-I - \lambda I) \sum_{q\in\mathbb{Q}} \exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \tag{81}$$

$$\implies I \left(\sum_{q\in\mathbb{Q}} \exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \right)^{-1} = \exp(-I - \lambda I) \tag{82}$$

7. We then define the inverse of the left term as the partition function:

$$\mathbf{Z} := \sum_{q\in\mathbb{Q}} \exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right) \tag{83}$$

8. Finally, we write $\boldsymbol{\rho}(q)$ using $\mathbf{Z}$. We obtain:

$$\boldsymbol{\rho}(q) = \mathbf{Z}^{-1} \exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right) \tag{84}$$

Out of isotropic considerations, we pose $\forall j \in \{0, ..., n-1\}[\lambda_j = f]$ and $\forall k \in \{n, ..., n+m-1\}[\lambda_k = \tilde{k}]$, and we obtain (Equation 71) and (Equation 72). □

Theorem 3 (Geometric entropy)**.** *The entropy of the ensemble of events is:*

$$\mathbf{S} = \ln \mathbf{Z} + \underbrace{f\overline{X}_0 e_0 + ... + f\overline{X}_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{\tilde{k}\overline{X}_n e_n + ... + \tilde{k}\overline{X}_{n+m-1} e_{n+m-1}}_{\text{space terms}} \tag{85}$$

We interpret the entropy as the information one gains by knowing which event q was randomly selected from the set of events $\mathbb{Q}$ under the probability distribution $\boldsymbol{\rho}[q]$.

Proof. Replacing $\boldsymbol{\rho}[q]$ in the definition for the entropy $\mathbf{S}$ (Equation 69) with the probability distribution for the ensemble of events ($\boldsymbol{\rho}[q]$ in Equation 72), one obtains:

$$\mathbf{S} = -\sum_{q\in\mathbb{Q}} \left[\mathbf{Z}^{-1} \exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right) \ln\left(\mathbf{Z}^{-1} \exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right)\right)\right] \tag{86}$$

With a few rearrangements, one obtains:

$$\mathbf{S} = -\mathbf{Z}^{-1} \sum_{q\in\mathbb{Q}} \left[\exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right) \left(\ln \exp\left(-\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right) - \ln \mathbf{Z}\right)\right] \tag{87}$$

The logarithm of the exponential of a matrix is equal to the matrix $\ln \exp \mathbf{A} = \mathbf{A}$. Therefore,

$$\mathbf{S} = -\mathbf{Z}^{-1} \sum_{q \in \mathbb{Q}} \left[\exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \left(\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) - \ln \mathbf{Z} \right) \right] \tag{88}$$

$$= -\mathbf{Z}^{-1} \sum_{q \in \mathbb{Q}} \left[\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \right] + \mathbf{Z}^{-1} \ln \mathbf{Z} \sum_{q \in \mathbb{Q}} \left[\exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \right] \tag{89}$$

From definition (71), $\mathbf{Z} = \sum_{q \in \mathbb{Q}} \left[\exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \right]$. Therefore,

$$\mathbf{S} = -\mathbf{Z}^{-1} \sum_{q \in \mathbb{Q}} \left[\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \right] + \ln \mathbf{Z} \tag{90}$$

From definition (70), the average of a quantity $\overline{X}_i e_i = \sum_{q \in \mathbb{Q}} (X_i[q] e_i) \boldsymbol{\rho}[q]$. Therefore,

$$\mathbf{S} = \ln \mathbf{Z} + \sum_{i=0}^{n+m-1} \overline{X}_i e_i \tag{91}$$

$\square$

Theorem 4 (Geometric equation of state)**.** *The equation of state of the ensemble of events is:*

$$d\mathbf{S} = \underbrace{f d\overline{X}_0 e_0 + ... + f d\overline{X}_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{\tilde{k} d\overline{X}_n e_n + ... + \tilde{k} d\overline{X}_{n+m-1} e_{n+m-1}}_{\text{space terms}} \tag{92}$$

Proof. The differential form extends easily from $\mathbf{S}$ to $d\mathbf{S}$ using partial derivatives to recover the applicable laws of thermodynamics, then grouping them in the form of an equation of state as per introductory statistical physics. $\square$

4.3 Space-time entropy

We note that geometric entropy $\mathbf{S}$ is not an element of the reals, but a vector of the geometric algebra. This situation is undesirable —one would prefer a definition of entropy that is a real number— and, in fact, it will be by resolving this problem that we will obtain the interesting physics connecting the entropy to the curvature of space-time.

Theorem 5 (Space-time entropy)**.** *Due to the peculiar non-commutative properties of the algebra of events, the multi-vector* d**S** *becomes a real number simply by squaring (taking the geometric product of* d**S** *with itself) the equation of state:*

$$(d\mathbf{S})^2 = \left(\underbrace{f d\overline{X}_0 e_0 + ... + f d\overline{X}_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{\tilde{k} d\overline{X}_n e_n + ... + \tilde{k} d\overline{X}_{n+m-1} e_{n+m-1}}_{\text{space terms}} \right)^2 \tag{93}$$

Squaring d**S** *does not change the meaning of the equation nor the entropy, yet it nonetheless erases base-specific information making* d**S** *a scalar as desired. To help us interpret the squared equation, we now divide each side of the above equation by* $\tilde{k}^2$*. Finally, we identify the term* $\tilde{k}^{-2}(d\mathbf{S})^2$ *as the space-time interval* $(ds)^2$*. Then, the equation of state is easily recognized as the interval of general relativity (the metric) in* $m+n$ *space. Let* $(ds)^2 := \tilde{k}^{-2}(d\mathbf{S})^2$*, then:*

$$\underbrace{(ds)^2}_{\text{interval}} = \left(\underbrace{c d\overline{X}_0 e_0 + ... + c d\overline{X}_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{d\overline{X}_n e_n + ... + d\overline{X}_{n+m-1} e_{n+m-1}}_{\text{space terms}} \right)^2 \tag{94}$$

Proof. Expanding the power of two to the right-hand side of the equation and rearranging, and by using the definition of the interval (Definition 9) it is straightforward to recover the definition of the metric g;

$$\underbrace{\tilde{k}^{-2}(d\mathbf{S})^2}_{\text{entropic distance}} = \underbrace{(ds)^2}_{\text{interval}} = \underbrace{\sum_{\mu\nu} g_{\mu\nu} d\overline{X}_\mu d\overline{X}_\nu}_{\text{metric}} \tag{95}$$

□

4.4 Einstein field equations

With access to a generally curved space-time expressed by a metric g, deriving the Einstein field equations can be done straightforwardly by appealing to the principle of stationary action. The action $\mathcal{A}$ is defined as (here we use $\mathcal{A}$ for the action instead of S to avoid confusing the symbol with that of the entropy):

$$\mathcal{A} = \int \mathcal{L} d^{(4)} V \tag{96}$$

In curved space-time, the 4-volume element $d^{(4)}V$ is given by:

$$d^{(4)}V = \sqrt{-g}d^4x \tag{97}$$

where g is the determinant of the metric tensor matrix. We then take the Ricci scalar as the simplest curvature invariant which produces a scalar, and we pose $\mathcal{L} := R$. We then obtain:

$$\mathcal{A} = \int R\sqrt{-g}d^4x \tag{98}$$

which, up to a multiplication constant, we recognize as the Hilbert-Einstein action.

4.5 Sketch: Nambu-Goto action

We will provide a sketch of the derivation of the Nambu-Goto action, relevant to string theory, from the geometric equation of state and then we will discuss the results.

Extending the geometric equation of state from space-time events to geometric events, one obtains:

$$\begin{aligned} d\mathbf{S} = &\underbrace{fd\overline{X_0}e_0 + ... + fd\overline{X}_{n-1}e_{n-1}}_{\text{time terms}} \\ &+ \underbrace{\tilde{k}d\overline{X}_n e_n + ... + \tilde{k}d\overline{X}_{n+m-1}e_{n+m-1}}_{\text{space terms}} \\ &+ \underbrace{\nu dA_{01}e_0e_1 + \nu dA_{02}e_0e_2 + ... + pdV_{012}e_0e_1e_2 + ...}_{\text{geometric terms}} \end{aligned} \tag{99}$$

The quantities of this equation of state are relativistic invariants. Keeping the area terms and crossing out the time, space and higher-dimensional geometric terms, one obtains the area elements as invariants, as follows:

$$d\mathbf{S}_A = \underbrace{\nu dA_{01}e_0e_1 + \nu dA_{02}e_0e_2 + ...}_{\text{invariant area terms}} \tag{100}$$

One can then construct an action consistent with the area terms of the geometric event being invariant. Under the relation $dA = d^2\Sigma\sqrt{-g}$, one obtains the action $\mathcal{A}$ of an invariant area:

$$\mathcal{A} = \frac{T}{c}\int d^2\Sigma\sqrt{-g} \tag{101}$$

An invariant area is a *world-sheet*.

One can assign the usual parameters σ and τ as the coordinates of the area. Then, one then obtains the induced metric as:

$$g_{\tau\sigma} = \eta_{\mu\nu}\frac{\partial X^\mu}{\partial \tau}\frac{\partial X^\nu}{\partial \sigma} \tag{102}$$

Finally, one rewrites (101) as

$$\mathcal{A} = -\frac{T}{c}\int d^2\Sigma\sqrt{\left(\frac{\partial X^\mu}{\partial \tau}\frac{\partial X^\nu}{\partial \sigma}\right)^2 - \left(\frac{\partial X^\mu}{\partial \tau}\right)^2\left(\frac{\partial X^\nu}{\partial \sigma}\right)^2} \tag{103}$$

which is the Nambu-Goto action, here derived as a special case of the geometric equation of state.

4.6 Sketch: Feynman path integral

In the previous subsections we have used geometric thermodynamics to describe the relationship between entropy and static geometry. The action was then added on top of the geometry to define the dynamics of the system. Now, we instead wish to include the dynamics directly as a system of geometric thermodynamics. We re-state the geometric partition function previously obtained:

$$\mathbf{Z} = \sum_{q\in\mathbb{Q}}\exp\left(-f\underbrace{(X_0[q]e_0+...+X_{n-1}[q]e_{n-1})}_{\text{time terms}}-\tilde{k}\underbrace{(X_n[q]e_n+...+X_{n+m-1}[q]e_{n+m-1})}_{\text{space terms}}-\underbrace{A-Be_0e_1+...}_{\text{geometric terms}}\right) \tag{104}$$

This construction uses the basis of a Clifford algebra $Cl_{n-1,n+m-1}(\mathbb{R})$ and sums over microstates that are constructed out of all elements of the basis. This construction uses length elements X_0 to X_{n-1} with units meter conjugated to $\tilde{k}$ with units 1/meters, as well as time elements X_n to X_{n+m-1} with units seconds conjugated to f with units 1/seconds. How do we convert this static description to a dynamical description? First, we will show that the Feynman path integral is a special case of the framework.

To recover the Feynman path integral, one changes the bulk description, the domain and the basis of the geometric partition function, but one nonetheless retain its mathematical form. Instead of using $X_0[q]$ with units meters conjugated to $\tilde{k}$ with units 1/meters, we define the bulk using $S[p]$ with units Joules-seconds conjugated to $1/\hbar$ with units 1/Joules-seconds. Then we redefine the domain of the sum from $q \in \mathbb{Q} \to p \in \mathbb{P}$ (now a path p element of the set of all paths $\mathbb{P}$). What about the imaginary term i of the sum? We recall that the Clifford algebra $Cl_{0,1}(\mathbb{R})$ generates the complex numbers. This can easily be seen from the Clifford algebra $\{1, e_0\}$ as its basis defines $e_0e_0 := -1$, and thus $e_0 = i$. With linear combinations, the basis elements generates the complex numbers. For instance $a + be_0$ using the notation $i = e_0$ becomes $a + bi$. Finally, one replaces the sum with an integral. With these replacements, the equation describes a dynamics associated with the system. Furthermore, maximizing the entropy acquires the meaning of erasing path information. The sum now represents the physical notions of action/paths (instead of static-geometries), but since the mathematical form of the sum remains the same then geometric thermodynamics remains applicable as a mathematical framework.

We use these replacements to reduce the geometric partition function to the Feynman path integral by using $Cl_{0,1}(\mathbb{R})$ instead of $Cl_{n-1,n+m-1}(\mathbb{R})$. Using only the element e_0 of the basis, and the aforementioned replacements, ones defines a statistical prior as follows:

$$\langle S\rangle e_0 = \int S[p] e_0 \rho[p] dp \tag{105}$$

By maximizing the entropy using the method of the Lagrange multipliers, one obtains the following geometric partition function:

$$\mathbf{Z} = \int \exp\left(e_0 S[p]/\hbar\right) dp \tag{106}$$

Finally, using the notation $e_0 = i$, one can clearly see an equation of the same mathematical form as the Feynman path integral:

$$\mathbf{Z} = \int \exp\left(iS[p]/\hbar\right) dp \tag{107}$$

This connection between geometric thermodynamics and quantum mechanics is even more remarkable when we apply it to QFTs. Let us do that now.

4.7 Recap: The similarly between QFT and statistical physics

In a review by McCoy[58], the similarity between the formalism of quantum field theory and statistical field theory are clearly illustrated. Using the Feynman path integral formulation, a QFT over a field ϕ, action S and observables $\langle O_1\rangle, \langle O_2\rangle, ..., \langle O_n\rangle$ is defined as follows:

$$\langle O_j\rangle = \frac{1}{Z}\int O_j e^{iS[\phi]/\hbar} d\phi, \text{ where } Z = \int e^{iS[\phi]/\hbar} d\phi \tag{108}$$

The mathematical form of this equation is very similar to statistical physics. Indeed, in statistical physics:

$$\overline{X_i} = \frac{1}{Z}\sum_{q\in\mathbb{Q}} X_i[q] e^{-E[q]/k_b T}, \text{ where } Z = \sum_{q\in\mathbb{Q}} e^{-E[q]/k_b T} \tag{109}$$

From the similarities, one notices that the quantum fluctuations in QFT, whose intensity is proportional $\hbar$, is the analogue of the thermal fluctuations whose intensity is set by $k_B T$ in thermodynamics. Finally, one notes the presence of the imaginary unit i, absent from the statistical physics formulation. Let us explore the similarities further in the following subsection.

4.8 Quantum Field Theory

Let us now consider the case where one adds a real scalar $\alpha A[\phi]$. Such a QFT would be given by:

$$\langle O_j \rangle = \frac{1}{Z} \int O_j e^{iS[\phi]/\hbar - \alpha A[\phi]} d\phi, \text{ where } Z = \int e^{iS[\phi]/\hbar - \alpha A[\phi]} d\phi \tag{110}$$

An exponential term of the type $\alpha A + Si$ is not often used in a QFT, but for purposes of generality we will add it here. Such a term is permitted because the identity $e^{a+bi} = e^a e^{bi}$ implies that the real term will simply be absorbed into the normalization constant. Such a real number would set the path degeneracy, if any.

An equation of this type can be easily obtained using geometric thermodynamics using the following geometric basis $\{1, e_0\}$, where $e_0 e_0 = -1$. One starts with two geometric statistical priors:

$$\langle S \rangle e_0 = \int S[\phi] e_0 \rho[\phi] d\phi \tag{111}$$

$$\langle A \rangle = \int A[\phi] \rho[\phi] d\phi \tag{112}$$

Then maximizing the entropy using the method of the Lagrange multipliers (as done in Theorem 2), one obtains the following probability measure:

$$\rho[\phi] = \frac{1}{Z} e^{-\lambda_0 S[\phi] e_0 - \lambda_1 A[\phi]} \tag{113}$$

where λ_0 and λ_1 are Lagrange multipliers. Then by setting $\lambda_0 := -1/\hbar$, $-\lambda_1 := \alpha$, and by writing e_0 as i, one obtains:

$$\rho[\phi] = \frac{1}{Z} e^{iS[\phi]/\hbar - \alpha A[\phi]} \tag{114}$$

Using $\rho[\phi]$ to re-construct an average over an arbitrary observable O_j, one obtains the original path integral formulation of the QFT:

$$\langle O_j \rangle = \frac{1}{Z} \int O_j e^{iS[\phi]/\hbar - \alpha A[\phi]} d\phi \tag{115}$$

With this result, one can interpret the QFT as a system of (geometric) statistical physics where the entropy was maximized to erase path information. Here, the presence of the imaginary term i is merely a consequence of the chosen geometric algebra.

This is not the end of the story however. This connection between QFT, geometry and statistical physics can be exploited to produce a purely geometric interpretation of QFT. To better see the connection, let us consider the next example.

4.9 Towards a geometric interpretation of Quantum Field Theory

Now with the connection between $Cl_{0,1}(\mathbb{R})$ and QFT established, one may investigate what occurs when geometric algebras of higher dimensions are used in lieu of $Cl_{0,1}(\mathbb{R})$. For instance, one may investigate a QFT constructed using the quaternions (represented by the Clifford algebra $Cl_{0,2}(\mathbb{R})$). The existence of quaternionic quantum theories was hypothesis by Von Neumann and specific examples were later constructed explicitly (although in a context quite different that the Feynman path integral formulation). In the quaternionic case and using the Feynman path integral, the QFT would adopt the following form:

$$Z = \int e^{\frac{1}{\hbar}(iS_i[\phi]+jS_j[\phi]+kS_k[\phi])-\alpha A[\psi]} d\phi \tag{116}$$

where $1, i, j, k$ are the generators of the quaternions with the suitable non-commutative properties. Here also geometric thermodynamics is sufficiently powerful to recover the quaternionic QFT as a special case. One starts with the priors defined over the basis $\{1, e_0, e_1, e_0e_1\}$. In this case there are four:

$$\langle S_i\rangle e_0 = \int S_i[\phi]e_0\rho[\phi]d\phi \tag{117}$$

$$\langle S_j\rangle e_1 = \int S_j[\phi]e_0\rho[\phi]d\phi \tag{118}$$

$$\langle S_k\rangle e_0e_1 = \int S_k[\phi]e_0e_1\rho[\phi]d\phi \tag{119}$$

$$\langle A\rangle = \int A[\phi]\rho[\phi]d\phi \tag{120}$$

Finally, by maximizing the entropy (erasing path information) one obtains the quaternionic QFT in the same manner as we have just done for $Cl_{0,1}(\mathbb{R})$.

What interpretation are we to give to a quaternionic QFT? Well, here is the interesting part. We recall Euler's formula:

$$e^{ix} = \cos x + i\sin x \tag{121}$$

In a complex QFT, we can use Euler's formula to convert the term e^{iS} to $\cos S + i \sin S$. In this case, it is clear that the integral/sum is performed over a plurality of complex numbers. Said complex numbers are in fact responsible for defining the interference associated with adding the amplitude of each path either constructively or destructively. Let us now see what happens with quaternions.

Euler's formula can be extended to the quaternions. Indeed,

$$e^{a+bi+cj+dk} = e^{a}\left(\cos\sqrt{b^2+c^2+d^2} + \frac{\sin\sqrt{b^2+c^2+d^2}}{\sqrt{b^2+c^2+d^2}}(ib+jc+kd)\right) \tag{122}$$

In the quaternionic basis, summing such numbers also involves constructive and destructive combinations of amplitudes. However, the interference pattern is obviously much more complex with quaternions than it is with merely the complex numbers. In essence, we have traded simplicity in the formulation of the action (now only a linear combination of scalars over a non-commutative basis) for more complexity in the interface pattern governing the sum of the amplitudes associated to each path. The sum now involves *quaternionic interference* instead of *complex interference*.

4.10 Geometric interference patterns

We recall the Taylor expansion of the definition of the exponential form:

$$e^{X} = \sum_{n=0}^{\infty} \frac{X^n}{n!} = 1 + X + \frac{X^2}{2!} + \frac{X^3}{3!} + \dots \tag{123}$$

Using this expansion, we can obtain the interface pattern governing the summation of the paths amplitude for a QFT defined using any geometric basis. For instance, a general multivector of $Cl_{1,3}(\mathbb{R})$ with basis:

$$\begin{aligned}
&\{1,\\
&\ e_0, e_1, e_2, e_3,\\
&\ e_0e_1, e_0e_2, e_0e_3, e_1e_2, e_1e_3, e_2e_3,\\
&\ e_0e_1e_2, e_0e_1e_3, e_1e_2e_3,\\
&\ e_0e_1e_2e_3\}
\end{aligned} \tag{124}$$

is $v = a + be_0 + ce_1 + de_2 + fe_3 + ge_0e_1 + ... + ze_0e_1e_2e_3$. Here v can be replaced in the Taylor expansion for e^X which will produce the *geometric interference* pattern for the QFT whilst the action is a linear geometric multivector.

4.11 A General Geometric QFT

As we will now see, constructing a QFT using geometric thermodynamics clarifies the path towards generalization. Let us provide an explicit construction in the case of the complexification of the space-time algebra, $\mathbb{C} \times Cl_{1,3}(\mathbb{R})$ (i.e. in the Dirac algebra). In this basis, the most general geometric QFT that can be constructed is:

$$\langle O_j \rangle = \frac{1}{Z} \int O_j e^{\mathbf{S}} d\phi \tag{125}$$

where

$$\begin{aligned}
\mathbf{S} =& S[\phi] \\
&+ e_0 S_0[\phi] + e_1 S_1[\phi] + e_2 S_2[\phi] + e_3 S_3[\phi] \\
&+ e_0 e_1 S_{01}[\phi] + e_0 e_2 S_{02}[\phi] + e_0 e_3 S_{03}[\phi] + e_1 e_2 S_{12}[\phi] + e_1 e_3 S_{13}[\phi] + e_2 e_3 S_{23}[\phi] \\
&+ e_0 e_1 e_2 S_{012}[\phi] + e_0 e_1 e_3 S_{013}[\phi] + e_1 e_2 e_3 S_{123}[\phi] \\
&+ e_0 e_1 e_2 e_3 S_{0123}[\phi] + \\
&+ i S_i[\phi] \\
&+ i e_0 S_{i0}[\phi] + i e_1 S_{i1}[\phi] + i e_2 S_{i2}[\phi] + i e_3 S_{i3}[\phi] \\
&+ i e_0 e_1 S_{i01}[\phi] + i e_0 e_2 S_{i02}[\phi] + i e_0 e_3 S_{i03}[\phi] + i e_1 e_2 S_{i12}[\phi] + i e_1 e_3 S_{i13}[\phi] + i e_2 e_3 S_{i23}[\phi] \\
&+ i e_0 e_1 e_2 S_{i012}[\phi] + i e_0 e_1 e_3 S_{i013}[\phi] + i e_1 e_2 e_3 S_{i123}[\phi] \\
&+ i e_0 e_1 e_2 e_3 S_{i0123}[\phi]
\end{aligned} \tag{126}$$

Let us now investigate how geometric QFT connect to conventional QFT.

4.12 From geometric to conventional QFT

The geometric action can be written as a conventional action by the application of mathematical transformations such that the basis elements $\{1, e_0, e_1, e_2, e_3, ..., e_0 e_1, ...\}$ are removed from the expression of $\mathbf{S}$. To achieve this, one first groups the terms of the action $\mathbf{S}$ by the grades of the algebra, then rewrites each groups as the square root of the geometric product of the group.

As an example, consider the multivector $\mathbf{v} := A + e_0 B + e_1 C + e_0 e_1 D$ with basis $\{1, e_0, e_1, e_0 e_1\}$ having 3 grades and the following non-commutative relation:

$$\frac{1}{2}(e_\nu e_\mu + e_\mu e_\nu) = \delta_{\nu\mu} \tag{127}$$

Then we can rewrite $\mathbf{v}$ as follows:

$$\mathbf{v} := V + e_0X_0 + e_1X_1 + e_0e_1U \tag{128}$$
$$= \pm\sqrt{V^2} \pm \sqrt{(e_0X_0 + e_1X_1)^2} \pm \sqrt{(e_0e_1U)^2} \tag{129}$$
$$= \pm V \pm \sqrt{X_0^2 + X_1^2} \pm iU \tag{130}$$

As another example, consider the case where the basis is non-orthonormal, such that the non-commutative relation is:

$$\frac{1}{2}(e_\nu e_\mu + e_\mu e_\nu) = g_{\nu\mu} \tag{131}$$

In this case $\mathbf{v}$ can be rewritten as:

$$\mathbf{v} := V + e_0X_0 + e_1X_1 + e_0e_1U \tag{132}$$
$$= \pm\sqrt{V^2} \pm \sqrt{(e_0X_0 + e_1X_1)^2} \pm \sqrt{(e_0e_1U)^2} \tag{133}$$
$$= \pm V \pm \sqrt{g_{00}X_0X_0 + g_{01}X_0X_1 + g_{10}X_1X_0 + g_{11}X_0X_0} \pm iD \tag{134}$$

Now, we are getting close. Consider as a final example and using a subalgebra of the Dirac algebra, an action of the form:

$$\mathbf{S} = ie_0S_0[\phi] + ie_1S_1[\phi] + ie_2S_2[\phi] + ie_3S_3[\phi] + iV[\phi]; \tag{135}$$

Which can be rewritten as:

$$\mathbf{S} = \pm i\sqrt{\eta^{\mu\nu}S_\mu[\phi]S_\nu[\phi]} + iV[\phi] \tag{136}$$

One can rewrite $\mathbf{S}$ as the result of an integral of some Lagrangian density $L[\phi]$. One then rewrites $\mathbf{S}$ as:

$$\mathbf{S} = \frac{i}{\hbar}\int\int\int\int\left(\pm\sqrt{\hbar^2\eta^{\mu\nu}\partial S_\mu[\phi]\partial S_\nu[\phi]} - V[\phi]\right)dxdydzdt \tag{137}$$

Finally, postulating that $S_\nu[\phi]$ and $S_\mu[\phi]$ are linear and first-order in ϕ such that $S_n[\phi] \to \hbar\phi[x,y,z,t]_n$, one obtains:

$$\mathbf{S} = \frac{i}{\hbar}\int\int\int\int\left(\pm\sqrt{\eta^{\mu\nu}\partial\phi[x,y,z,t]_\mu\partial\phi[x,y,z,t]_\nu} - V[\phi]\right)dxdydzdt \tag{138}$$

which is the scalar free field QFT.

With this example, we have recover a QFT as a system of (geometric/non-commutative) statistical physics; itself simply a generalisation of (commutative) statistical physics. Is QFT simply generalized/non-commutative statistical physics?

5 Discussion

We will now apply axiomatic science to unresolved problems within the foundation of physics. As stated in the introduction, unlike artificial models containing only a 'physics' part, axiomatic science contains both a 'science' part and a 'physics' part. Axiomatic science is thus able to explain the origins of the laws of physics using science as the starting point, whereas for an artificial model, the origin of such laws is a blind spot. It will then be within this newly illuminated blind spot that we will find the solutions proposed by axiomatic science.

As a disclaimer, we state that in this paper, we are only beginning to scratch the surface of axiomatic science. Therefore our intention is not to completely resolve all of these problems, but rather to present the case made by axiomatic science and also to encourage future research.

5.1 Interpretation of quantum mechanics

What is generally considered 'physical/quantum randomness'; that is the post-measurement random selection of an eigenstate, is not an add-on in axiomatic science, but its very foundation. The quantum measurement problem is thus identified and corrected within the 'science' part of the framework. First, let us recall what the quantum measurement is.

In quantum physics, the unitary evolution of the wave function is deterministic, but the notion breaks down if measurements are occurring in the system (or are performed on the system). In the Von Neumann scheme, a measurement of the second kind, for a quantum object with wave function $|\psi\rangle$ and a quantum apparatus with wave function $|\phi\rangle$, is defined as:

$$|\psi\rangle|\phi\rangle \to \sum_n c_n|\chi_n\rangle|\phi_n\rangle \tag{139}$$

After the measurement the system is in one of eigenstates $|\chi_n\rangle$ with probability $|c_n|^2$. That the otherwise deterministic unitary evolving system adopts the "un-deterministic" initiative to collapse itself randomly in one of multiple states after measurement is quite the mystery. In fact, nothing in quantum physics predicts that such a thing would occur. Consequently, the notion of the measurement is introduced into quantum mechanics, formally, as a full-blown axiom not derivable from the Schrodinger equation itself, or from any of the other axioms of quantum mechanics.

The theory of quantum decoherence is a modern take on Von Neumann measurements. De-coherence from the environment $|e\rangle$ is introduced as follows:

$$|\psi\rangle|\phi\rangle|e\rangle \to \sum_n c_n|\chi_n\rangle|\phi_n\rangle|e_n\rangle \tag{140}$$

Under contact with an environment having multiple degrees of freedom, any interference pattern normally observable from $|\psi\rangle$ will be smudged by the environment beyond the ability of instruments to detect it. De-coherence explains why a quantum superposition of eigenstates is unlikely to be observed macroscopically as interaction with the environment very quickly causes the system to evolve towards a classical probability distribution. However, decoherence is ultimately of no help in regards to explaining why one eigenstate out of many is randomly selected for the system to be in post-measurement.

The measurement postulate, as a law, is derived empirically and it is introduced purely so that quantum physics predicts a single macroscopic world (not a superposition of many worlds), consistent with observations. The two primary competing interpretations of this phenomenon are a) the Copenhagen interpretation and b) the Everett many-worlds interpretation, but there exists at least half a dozen others. None of these interpretations are, however, considered satisfactory by mainstream physics and thus the question remains unsettled; in the first case the collapse is simply postulated but no mechanisms are generally accepted for it, and in the second case it is postulated that the observer becomes coupled with a specific result of the measurement causing the appearance of collapse, but no mechanism to account for this coupling is generally accepted either. The interpretational problem is retained in all extensions of quantum theory from the Dirac equation to quantum field theory, etc. As one is generally free to apply any of the compatible interpretations to quantum theory, deciding which one is correct, if any, is often criticized as a non-falsifiable problem.

Axiomatic science proposes to address the problem from the other direction. The primary idea is that the quantum measurement is quantified, in axiomatic science, by natural information. We recall that in Shannon's theory of information, entropy quantifies the amount of information one gains by knowing which message is randomly selected from a set of possible messages. In the present case, axiomatic science postulates that the reference manifest (Axiom 1), describing the state of affairs of the world (Assumption 1), is randomly selected from the set of all possible manifests (Assumption 2). Natural information is then quantified by the entropy associated with the message (Definition 5).

In axiomatic science, this is the setup. We call this first part 'science' and we soon derive the 'physics' part. As an analogy, consider that one must define first-order logic before one can define set theory. Here, science is to physics what first-order logic is to set theory. Now that we have defined the 'setup of reality' (i.e. the 'science' part), we can then derive the physical laws which apply to it. This is made possible by Assumption 3 and 4, the fundamental assumptions of physics and of 'substance', respectively. Finally, the conservation law of a suitable framework of statistical physics, in the present proposed case, geometric thermodynamics, is obtained as the fundamental relation of the system.

So how and why do the laws of physics acquire a quantum measurement problem? Here is the culprit: To derive the laws of physics using 'science' as the starting point, one must at some step of the proof maximize the entropy of natural information. Specifically, Axiom 1 postulates the existence of the reference manifest, granting it a special status. But, one sums over all possible manifests in $\mathbb{W}$. In the sum, the reference manifest $\mathbb{M}$ does not have a special status, and its statistical weight is given by $\rho(\mathbb{M})$ just as if it were any other manifest M. As the sum over $\mathbb{W}$ is required to derive the laws of physics from 'science', we note the surprising consequence that the reference manifest is not sufficient by itself to derive the laws of physics. The ability to list, in principle, all alternative manifests to the reference manifest is also required.

Maximizing the entropy of natural information has the consequence of erasing natural information. This renders natural information unavailable to the laws of physics, derived afterwards and as a consequence of the erasure. Consequently, one who uses said derived laws of physics to search for solutions (manifests) will unavoidably encounter this entropy within the solutions. Indeed, post erasure, the role of natural information is clarified: essentially, natural information represents the total amount of information about nature that cannot be derived by the laws of physics. Using the terminology *observer* common in physics, we would say that the observer 'sees' the reference manifest as natural information, but the laws of physics derived by the practice of science, 'sees' natural entropy in lieu of natural information. Consequently, there is an information gap between what is known to the observer (the reference manifest) and what is derivable by the laws of physics (the set of all manifests). The gap is precisely the sum-total of all quantum measurements required to connect the predictions of physics to the actual manifest, ergo, natural information.

The quantum measurement problem is acquired as a problem only when one constructs an artificial model of nature because such models are blind to the 'science' part, and thus cannot account for the origin of the laws of physics. We recall that an artificial model is produced when one postulates the laws of physics, then solves for manifests, and a natural model is produced when one postulates the manifests then solves for the laws of physics. In the artificial case, one obtains a plurality of manifests as possible solutions; only one of which is the reference manifest. Since one intuitively expects that the laws of physics ought to explain the actual world and only the actual world, one may then become baffled as to why the laws of physics do not produce the reference manifest as their only solution, but instead a plurality of manifests. The culprit is identified by axiomatic science: natural information was erased to derive the laws of physics.

So, why do we need to erase natural information to recover the laws of physics? The fundamental motivation is to release the laws of physics from the shackles of natural information in order to facilitate formulating the broadest possible patterns about nature. Indeed, one cannot form a pattern from a single existing candidate, unless one invents hypothetical candidates to fix the pattern. For example, I can say "I am a physicist, but I could have been a doctor instead", or I could say "I measured the spin up, but it could have been down". Although neither violates the laws of physics, in reality, one happened and the other didn't. It is precisely because natural information is erased from the laws of physics that the claim 'both alternatives are possible, but only one happened' can be made. Consequently, the laws of physics will recover both alternatives as possible solutions but would be unable to determine which of the two occurred without observing the manifest directly.

To better understand why the laws of physics are not derivable without erasing natural information, it helps to attempt the following challenge; can we derive the laws of physics without erasing natural information, perhaps in such a way that the theory remains aware of the reference manifest? One can try, but one will not recover the laws of physics; instead, one will obtain a manifest theory:

Definition 13 (Manifest theory)**.** *A manifest theory is a program p that outputs $\mathbb{M}$ when ran on a universal Turing machine. Thus,*

$$\mathrm{UTM}(p) = \mathbb{M} \tag{141}$$

We further qualify a manifest theory as elegant if it is the shortest program that outputs the reference manifest when ran on said universal Turing machine.

The manifest theory is pure computation with no insight or patterns. Contrary to the solutions of the laws of physics, for the manifest theory, all alternatives (e.g., I being a doctor instead of a physicist) are impossible simply because they did not happen and therefore will not be outputted by the program. Consequently, the manifest theory does not distinguish between "could have happened, but didn't" and "did not happen/cannot happen".

In contrast, the laws of physics, explicitly derived by maximizing natural entropy, are the widest and broadest patterns one can formulate about nature. The patterns (law of physics) emerge as a fundamental relation, precisely because we erase the shackles of natural information which would otherwise fix the theory to a manifest theory.

Picking the laws of physics as our explanatory tool of choice, rather than the manifest theory, is a choice made by physicists who prefers understanding the world by patterns rather than by brute computation. This 'preference' is formalized implicitly by Assumption 2, the fundamental assumption of 'nature'. Indeed, when one is presented with a world that exists brutely, one is free to assume that it is randomly selected from the set of all possible worlds, provided that one can list all the possible worlds (having access to a universal Turing machine or general intelligence is required for this step). One identifies all alternatives to the present state of nature, then formulates a law which holds for all alternatives and including the present state. In fact, the laws of physics are revealed by axiomatic science as the broadest patterns one can formulate about nature, regardless of the specific manifest which happens to brutely exist. In this sense, the laws of physics have a special place as the broadest mathematical patterns compatible with a 'lawless nature'.

In axiomatic science, only the reference manifest exists brutely. Listing the other manifests as hypothetical alternatives, a step necessary to identify a pattern, is an algorithmic operation performed on a 'chalkboard' and does not grant the status of existence to these alternative manifests. This is where the distinction between the interpretation of quantum mechanics offered by axiomatic science and the others, occurs. For axiomatic science neither the collapse of the wave-function interpretation, nor the many-world interpretation are acceptable interpretations, as there is never a situation where more than one solution has the property of existence. Axiomatic science correctly predicts that one solution is actual, the reference solution, knowable to the 'observer' as the reference manifest, but that any pattern identified from the erasure of natural information will not know which; that is, unlike the 'observer', the laws of physics sees natural entropy in lieu of natural information.

Here, we want to produce a more rigorous derivation of the above-stated interpretation of quantum mechanics.

Theorem 6 (QM interpretation proposed by axiomatic science)**.** *Axiomatic science states that there is no collapse (thus it rejects the Copenhagen interpretation), and also that the system was never in a superposition of many-worlds to begin with (thus it rejects the many-world interpretation). Axiomatic science states that all alternative worlds are mathematical creations used to facilitate the formulation of the laws of physics as patterns, and thus, have no ontological properties. Axiomatic science predicts the discrepancy between what is observed, and what the laws of physics offers as solutions, without the introduction of ad hoc postulates, and quantifies the discrepancy using natural information.*

Proof. First, let us investigate standard quantum mechanics (QM). QM is an artificial model. As such, the laws of physics are postulated, then solutions are found. Due to this construction, there is ambiguity regarding the ontological status of its solutions and the model is open to falsification. For instances, let us suppose a quantum theory QM, with input h, and solved for solutions. Here, h may be interpreted as a description of the specific physical system QM is solved for. One may write:

$$\text{Solve}(\text{QM}, h) = \alpha_1 \left|\text{solution}_1\right\rangle + \alpha_2 \left|\text{solution}_2\right\rangle + \alpha_3 \left|\text{solution}_3\right\rangle + \dots \tag{142}$$

However, experimentalists report to us that they observe only one element of $\{\left|\text{solution}_1\right\rangle, \left|\text{solution}_2\right\rangle, \left|\text{solution}_3\right\rangle, \dots\}$ with probability $\alpha_1\alpha_1^*$, $\alpha_2\alpha_2^*$, $\alpha_3\alpha_3^*$, $\dots$, respectively. The quantum measurement problem is concerned with explaining the discrepancy: solving for a plurality of possible solutions is not the same as reporting one solution randomly selected from the set of solutions. Thus, the predictions do not match the observations. One then asks: are the other solutions real, and if so, where do they "go" post-observation? Were they never real to begin with? etc.

In axiomatic science, the existential qualifier is granted to the reference manifest $\mathbb{M}$ by Axiom 1, but no other manifest M receives this "award". In fact, the other non-existing manifests are not derived by observation, but by enumeration on a 'chalkboard' using a Turing machine. Thus, the set of all manifests is a "man-made/intellectual" construction. Knowing which manifest is the reference from the set of all possible manifests defines natural information. Now, to derive the laws of physics, one then erases natural information as one maximizes natural entropy. Consequently, the laws of physics do not represent the natural world, but the plurality of all possible world (not too far off, but still not identical). Incorrectly thinking that they do is the source of the confusion regarding quantum mechanical interpretations.

The discrepancy between what is reported by observers (the reference manifest) and was is obtained by solving the laws of physics (the set of all possible manifests) is not a surprise worthy of its own postulate, but instead a theorem fully predicted and derivable from axiomatic science.

Specifically, in axiomatic science, one can solve the implied physical laws for solutions, but one solution will not be like the others:

$$\text{Solve}(\text{AS}, h) = \alpha_0 |\mathbb{M}\rangle + \alpha_1 |M_1\rangle + \alpha_2 |M_2\rangle + \alpha_3 |M_3\rangle + \dots \tag{143}$$

Finally, solving the same problem but with added natural information NI, one gets exactly what is observed:

$$\text{Solve}(\text{AS}, h, \text{NI}) = |\mathbb{M}\rangle \tag{144}$$

At no point is there confusion about what the other non-existing solutions are or aren't, and at no point is a quantum measurement postulate need to be inserted into the axiomatic foundation. Finally, with axiomatic science, experimentalist no longer surprise us when they report observing only one element of the set of solutions, as this is exactly what we expect them to tell us in the first place.

□

5.2 Entropy in a QFT

We have seen that a QFT can be derived as a system of (geometric) statistical physics. In (geometric) statistical physics, a QFT is defined in reference to the probability measure which maximizes the entropy associated with the selection of one geometry from a set of possible geometries. Specifically, one defines a set of geometric statistical priors, then maximizes the entropy under these priors and by using the method of the Lagrange multipliers, finally, one obtains the probability measure associated with the QTF. The partition function sums the probability amplitudes constructively or destructively according to the geometric interference pattern of the QFT. Like for any system of (geometric) statistical physics, an entropy is assigned to the system via an equation of state. In the case of the QFT, this entropy quantifies the informational departure of the QFT from the classical theory (the entropy quantifies the gain in information from knowing which path or geometry was selected from the set of possible geometries). Here also we see the same pattern; the quantum theory requires the erasure of natural information to be derived. Thus, starting from the QFT and attempting to derive solutions, one unavoidably encounters this entropy within the solutions.

5.3 Time

If the world exists as a brute manifest (Axiom 1), why does an observer believe he/she has a future and a past? Recall that both the past and the future would be associated to different state of affairs of the World and thus would be described by their own manifests M (Assumption 1), different than $\mathbb{M}$, and thus having no ontological properties. Why then is the observer not 'stuck' in a singular and static $\mathbb{M}$? How and why does time appears to flow from the 'past' to the 'future'?

We also recall that in the introduction we stated that something as 'trivial' as postulating that the present is caused by the past cannot be done in axiomatic science as it is an artificial argument; the past or the future must be derived as logically implied by the 'raw data' as a natural argument before it can be integrated into the framework.

Geometric thermodynamics offers the past and the future as natural arguments. Specifically, it suggests a purely entropic model of space-time in which the build-up of natural information determines the structure of space-time (its geometric substance). Let's see how this transpires in the details. So far, we have introduced a non-commutative extension to statistical physics granting it the ability to associate an entropy to arbitrary space-times. Generally speaking, statistical physics connects a set Q of 'microstates' (a.k.a the microscopic description) to a set of functions on Q (a.k.a. the bulk state) by the use of Lagrange multipliers, and under the principle of maximum entropy. In the present case, the 'microscopic' object of study is the space-time event, the bulk state is the space-time curvature, and the Lagrange multiplier is the speed of light. A partition function of such events can be constructed by using generators of the Clifford algebra. These generators ascribe geometric properties to the equation of state of such a system. Finally, the Einstein field equations are obtained by applying the principle of stationary action to the equation of state.

We state immediately that the geometric equation of state does not attribute an entropy to space (this is done by the Bekenstein-Hawking entropy and requires a horizon), but to space-time. In space-time, like in space, the role of the entropy will be to hide a region of it from the observer. As we will argue, in space-time, the hidden region defines the future of the observer. Specifically, the future is hidden from the present by entropy. It is as a result of this entropy that the observer can attribute a future to itself.

A change in the entropy of natural information is quantified by $\tilde{k}^{-2}(d\mathbf{S})^2$ which we have equated to the space-time interval $(ds)^2$ (in Theorem 4). We are already familiar with the replacement $(ds)^2 = c^2(d\tau)^2$, where τ is the *proper time*. We will now call $\tilde{k}^{-2}(d\mathbf{S})^2$ the *entropic distance*. Consequently, a change in proper time will always be quantified by a change in entropy. The entropic distance quantifies the informational departure of some hypothesized future or past manifest from the reference manifest. The near future, with a short entropic distance, is very similar to the present, and the far future, with a long entropic distance, is proportionally dissimilar. The arrow of time is upgraded to an arrow of space-time.

Let us begin by comparing the Euclidean case to the Minkowski case. In Euclidean space, the geometric equation of state is:

$$\tilde{k}^{-2}(d\mathbf{S})^2 = (dx)^2 + (dy)^2 + (dz)^2 \tag{145}$$

In Minkowski space-time, the geometric equation of state is:

$$\tilde{k}^{-2}(d\mathbf{S})^2 = (dx)^2 + (dy)^2 + (dz)^2 - c^2(dt)^2 \tag{146}$$

Finally, in an arbitrary space-time, the geometric equation of state is:

$$\tilde{k}^{-2}(d\mathbf{S})^2 = \sum_{\mu,\nu} g_{\mu\nu} d\overline{X}^{\mu} d\overline{X}^{\nu} \tag{147}$$

In the Euclidean case, we note that there is no nullsurface. All events (points) have a non-zero entropic distance from the origin. The case is degenerate. In the Minkowski space-time and the general space-time, however, we do note the presence of a nullsurface (light cone) on which all events have zero entropic distance from the origin. The presence of this nullsurface and the fact that it is a surface of zero entropic distance will allow us to define an extended present using entropy.

Definition 14 (Present)**.** *In space-times with a nullsurface, we can define an extended present for the observer as the set of all events of zero entropic distance:*

$$0 = \tilde{k}^{-2}(dS)^2 \tag{148}$$

Definition 15 (Past/Future)**.** *We can further define a position in time away from the present and quantified by the entropy as:*

$$ds = \pm\sqrt{\tilde{k}^{-2}(d\mathbf{S})^2} \tag{149}$$

We note that if the geometric equation of state admits a nullsurface, then the entropy will be constant along the surface, and the entropic distance between all events on the surface will be zero. We thus conjecture that all events on the nullsurface are known/knowable to the observer because they have an entropic distance of zero from it and thus none of these events are hidden by entropy. The information available in the reference manifest is the information available on the nullsurface. Consequently, when we point a telescope towards the sky, the information we have access to is what's on the nullsurface.

With entropic definitions for the past, the present and the future, it now becomes attractive to picture the world as an evolving block[56]. Here the relation $ds = \sqrt{\tilde{k}^{-2}(d\mathbf{S})^2}$ quantifies the informational departure of a future event from the present. As stated, this entropy hides from an observer knowledge of future events. This entropy is eliminated if and when the gap between the observer and the entropically-distant event is reduced to 0 (that is when the observer moves forward in time until the event occurs). Thus, the future is hidden from the present by entropy. The past is attributed to an opposite description: Indeed, a reduction in entropy as measured from the present characterizes the past. The past is, therefore 'over-determined' by the present.

Definition 16 (Flow of time)**.** *For any reference manifests, time appears to flow because geometric thermodynamics attributes a positive entropic distance to a region in space-time (the future) and a negative entropic distance to another (the past).*

With time now 'apparently flowing', we can understand why the arrow points towards the future:

Definition 17 (Arrow of time)**.** *The gradient of space-time entropy always peaks in the direction of the observer's future in its frame of reference. Therefore, a space-time system seeking to statistically increase its entropy is powered to evolve forward in time by entropy. Indeed at thermodynamic equilibrium, the entropic frequency does become an entropic power as* $k_B T f = P$.

What is the role of the speed of light? In geometric thermodynamics, the speed of light is not fundamental, but instead emergent (in the same sense that thermodynamic pressure and temperature are emergent) from statistical physics. The speed of light, usually taken as an axiom in special relativity, is here defined as the ratio of the Lagrange multipliers of the partition function $c := f/\tilde{k}$. The speed of light here fulfills a role similar to the role fulfilled by the temperature in standard thermodynamics. Essentially, the speed of light is the "temperature" of space-time. The speed of light is a property emergent from the random selection of events from a larger set under the principle of maximum entropy. The speed of light is constant in an ensemble of space-time events at equilibrium for the same reason that the temperature is constant in a system at thermodynamic equilibrium. Indeed, when f is the inverse of the Planck time, and $\tilde{k}$ the inverse of the Planck length, we get c, the speed of light:

$$f/\tilde{k} = \left(\sqrt{\frac{c^3}{\hbar G}}\sqrt{\frac{\hbar G}{c^5}}\right)^{-1} = c \tag{150}$$

The speed of light appears constant because all observers on the nullsurface agree as to the results of their measurements (guaranteed because all events have zero entropic distance). Consequently, a theory of 'apparent communication' (information synchronicity along the nullsurface) replacing literal communications, follows from the speed of light being emergent. Indeed, communication (light) has an apparent speed because full measurement agreement between all observers is only achieved on the nullsurface and distances on this surface are measured proportionally to the speed of light.

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