# Axioms of science

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**Abstract**

Here we present a mathematical model formalizing the practice of science in nature. Under the premise that science is the best methodology to understand the World, then consequently a formalization of such is its best mathematical framework. *Axiomatic science* is a significant improvement over the informal practice of science and it has numerous desirable properties. Axiomatic science is a model of science and of physics, and as such it is able to derive the laws of physics purely as theorems of the practice. Finally, axiomatic science is clarification tool par excellence for the foundation of physics.

## 1 Introduction

Axiomatic science is at least as general as informal science. Consequently, all physical theories that are the product of informal science ought to be formalizable as theorems of axiomatic science, lest they would be provably unscientific. This introduces a very strong constraint on what it means for a theory to be scientific in the formal mathematical sense.

Axiomatic science is logically a more fundamental representation of objective reality (the purview of science), than any physical theory resulting from science. Consistent with its scope, it proposes solutions to long and enduring problems regarding the foundation of physics. For instance; the problem of time, the origin of the appearance of a quantum collapse, identifying a preferred interpretation of quantum mechanics, as well as philosophical problems such as "why does the world obey the specific laws of physics that it does instead of basically any other?", and even "why are there laws of physics at all?". Axiomatic science explains why it can answer these questions, and also explains why physics is unable to do the same: the solutions to these problems are found within the foundation of how the practice of science is defined along with its relevant assumptions and premises, and are not found within the laws of physics themselves. Said laws of physics are eventually derived as theorems of axiomatic science but long after these issues are resolved.

Axiomatic science is constructed using the formalism of theoretic computer science including that of Turing machines and that of algorithmic information theory. This construction negates most, and quite probably all, objections to falsificationism from the philosophy of science. Specifically, the domain of axiomatic science is constructed precisely as the set of all formal statements that are necessarily true for all possible state of affairs of the World. Consequently, it is necessarily the case that no formal argument can successfully invalidate elements of its domain. Furthermore, as axiomatic science is sufficiently descriptive to account for all possible state of affairs, it is also necessarily the case that there exists no fact verifiable in the world which is outside its domain. Axiomatic science is universal in the computer theoretic sense and, intuitively, in the 'physical/experimental sense'.

Another key feature is that its properties are naturally transposed to the domain of physics. More precisely, one may qualify axiomatic science as a *super-tautology*. As a super-tautology, axiomatic science provides a bridge between epistemology and ontology. Indeed, there exists no formal method by which an observer can experimentally exceed the domain of axiomatic science. Because of this transposition, it will be by practicing science within the setup of axiomatic science that we will derive the laws of physics in this framework, just as we identify them when we practice science in the World. However, in the present case, the laws of physics are derived not by experimentation but by formal proof and are derived without physical baggage, without ambiguity and in their full generally. For these reasons and because it is a formalization of the practice of science, axiomatic science is a candidate model to serve as a maximally fundamental description of nature. in fact, the laws of physics are derived as the equations that maximize natural information.

Let us start with a teaser problem, then we will lay out the axiomatic basis of the model.

## Which of the two logically implies the other: The egg, or the molecular theory of organic chemistry?

As the first step towards understanding axiomatic science, we seek to understand the role played by logical implication, by initial conditions, and by axioms. Perhaps surprisingly, axiomatic science is oblivious to the concept of philosophical causality, instead preferring in all cases the notion of logical implication to relate facts to one another. Another peculiar demand of axiomatic science is to banish what we will call "artificial" axioms in favor exclusively of what we will call "natural" axioms. Let us first understand what we mean using examples, and then we will generalize the idea. Within the methodology of axiomatic science, the logical implication is used in the direction that the observations imply the theory. For instance,

1. The discovery of astronomical redshift implies (or at least gives credibility to) models accounting for a metric expansion of space.

2. The discovery of the cosmic microwave background (CMB) implies (or at least gives credibility to) Big Bang models.

3. The homogeneity of the temperature of the CMB implies (or at least gives credibility to) inflationary models.

4. The discovery of DNA implies (or at least gives credibility to) natural selection models regarding the evolution of life on Earth.

5. Objects falling from trees imply (or at least give credibility to) the theory of gravitation.

In this paradigm, the observations form the basis of the logical argument. From now on, we will qualify such arguments as *natural*; in the sense that the conclusion logically follows from the observations. For natural arguments, the set of observations take the role of the axioms (the premise); they are the brute facts from which the model is logically implied.

Shockingly, with perhaps axiomatic science as the only exception, we find that no theory in physics is mathematically constructed as a natural argument. Let us first investigate how a mathematical model of nature is typically constructed, and then explain why we believe it leads to problems.

To produce an axiomatic physical theory, one first "adjust" how one describes nature by doing what amounts to an 'axiomatic re-organization/compression' of the data. Specifically, one finds a new set of axioms, different than brute observations, but nonetheless believed to account for at least some good portion of the observations, and then uses these axioms as the new starting point. This re-organization is in the ideal case logically equivalent to the brute facts. It is usually justified on the grounds that a more elegant or aesthetically pleasing model will be produced which would be preferred to the brute facts. For instance, at CERN, the LHC collision data produces about 25 petabytes of data annually (it is algorithmically quite inelegant), but the standard model reasonably fits in a few textbooks (comparatively, it is quite elegant). If one cares about elegance, this is quite an improvement! As another example, consider that about 100 tons of cosmic dust fall on earth every day, and that about 10-20 trillion drops of water fall on Earth in the same period, etc. That is a lot of events to log as data. But we can compress a good chunk of it by postulating that this simple formula $F = Gm_1m_2/r^2$ is a law of nature. We can compress an even bigger chunk of this data by adding a few more laws; such as aerodynamics laws, weather patterns, etc.

However, with this new admittedly more aesthetically pleasing axiomatic basis as a starting point, the logical argument has a new but *artificial* starting point and points in a new but *artificial* direction. Mathematically, it is now the model that implies the observations. For instance, in physics, it is common to re-organize the presentation of the previously enumerated statements as follows:

1. The metric expansion of the universe implies (predicts) the astronomical redshift.

2. The Big Bang implies (predicts) the CMB.
3. The inflationary period immediately after the Big Bang implies (predicts) the homogeneity of the CMB temperature.
4. Natural selection implies (predicts) the existence of an information bearing physical structure such that offsprings acquire the phenotypes of their parents (e.g. DNA).
5. Gravity implies (predicts) that objects will fall from trees, should their attachment fail.

With this re-organization, one elevates the axioms of the 'artificial model' above observations. Contrary to the direction of the natural argument, the artificial model, as it is axiomatic, now outranks observations, and consequently, the model becomes vulnerable to falsification. As far as the artificial model is concerned, its theorems are true statements, but as practitioners of science, we know that the model is overselling its salad. We are in fact quite aware that the theorems of the model are mere predictions, not necessarily true in the World, and we do welcome and even expect the discovery of confirmatory or refuting evidence of the model. Indeed, if the axiomatic re-organization of the raw data is not equivalent in the information-theoretic sense to the raw data (in practice it seldom is), then the model will eventually make incorrect predictions and will be falsified.

Axiomatic science exposes this axiomatic re-organization as a fundamental pathology and corrects it. Axiomatic science, as a framework, connects 'raw data' to 'laws of physics' without requiring a preliminary axiomatic re-organization of the raw data. Since axiomatic science is a formal theory, employing any kind of artificial argument becomes strongly prohibited within the framework. For instance, if one holds an egg, then drops it on the floor, then whatever model of reality one holds, it is now constrained to account for a broken egg on the floor. The artificial argument (the model implies the broken egg) is a false implication: there exists no such implication as in all cases the model is simply falsified should it fail to account for the broken egg.

The central tenet of axiomatic science is to construct a framework consistent with the assumption that it is not the model that constrains the World; rather, it is the World that constrains the model. In other words, *the data implies the model, but the model never implies the data.* As a result, axiomatic science places the initial conditions, not at the Big Bang, but at the present because it is the present that holds the set of all constraining raw data. Even though causality can, in principle, be used as an artificial model for a subset of all observations, axiomatic science shuns it. Within the framework of axiomatic science, even something as common as assuming that the present is caused by the past cannot be done, as it is an artificial argument. Such assumption, if true, must be formally proven from the framework as a theorem before it can be adopted. Consequently, it is thus more fundamental within axiomatic science to state that the past is logically implied by the present and that the system's history is recoverable by forensic investigation and as a model of the raw data, than it is to say that the present is caused by the past; the latter being a special case abstraction of the former.

## 2 The axioms of science

The fundamental object of study of axiomatic science is not the electron, the quark or even the microscopic super-strings, but the experiment. An experiment represents an atom of verifiable knowledge.

Let $\{s_1, s_2, ...\}$ be the sentences of a language $L$ with alphabet $A$.

**Definition 1** (Experiment)**.** *An experiment $p$ is a tuple comprising two sentences of $L$. The first sentence, $h$, is called the hypothesis. The second sentence,* TM*, is called the protocol. The protocol takes as input the hypothesis. We say that the experiment holds if* $\mathrm{TM}[h]$ *is defined, and fails otherwise. If $p$ holds, we say that the protocol verifies the hypothesis.*

$$\mathrm{TM}[h] \begin{cases} = r & p \text{ holds} \\ \nexists & p \text{ fails} \end{cases} \tag{1}$$

*Finally, $r$, also a sentence of $L$, is the result.*

An experiment, so defined, is formally reproducible. Indeed, for the protocol TM to be a Turing machine, the protocol must specify all steps of the experiment including the complete inner workings of any and all instrumentation used for the experiment. The protocol must be described as an effective method equivalent to an abstract computer program.

The set of all such experiments are the programs that halt. The set includes all provable mathematical statements and it is universal in the computer theoretic sense.

**Definition 2** (Domain)**.** *Let $\mathbb{D}$ be the domain* (Dom) *of axiomatic science. We can define $\mathbb{D}$ in reference to a universal Turing machine* UTM *as:*

$$\forall s \in L[\mathrm{UTM}[s] \text{ halts} \implies s \in \mathbb{D}] \tag{2}$$

*Thus,* $\mathbb{D} := \mathrm{Dom}[\mathrm{UTM}]$.

**Definition 3** (Manifest)**.** *A manifest $M$ is a subset of $\mathbb{D}$:*

$$M \subset \mathbb{D} \tag{3}$$

**Definition 4** (Set of all manifests)**.** *Let $\mathcal{P}[A]$ denote the power set of $A$. Then the set of all manifests $\mathbb{W}$ is:*

$$\mathbb{W} := \mathcal{P}[\mathbb{D}] \tag{4}$$

*Thus,* $M \in \mathbb{W}$.

**Assumption 1** (The fundamental assumption of science)**.** *The state of affairs of the World is describable as a set of reproducible experiments. Therefore, the state of affairs is describable as a manifest. Furthermore, to each state of affairs corresponds a manifest, and finally, the manifest is a complete description of the state of affairs.*

**Axiom 1** (Existence of the reference manifest)**.** *As the World is in a given state of affairs, then there exists, as a brute fact, a manifest* $\mathbb{M}$ *which corresponds to its state.*

$$\exists!\mathbb{M} \tag{5}$$

$\mathbb{M}$ *is called the 'reference manifest'.*

Remark: The symbol $M$ will denote any manifest in $\mathbb{W}$, whereas $\mathbb{M}$ specifically denotes the reference manifest corresponding to the present state of affairs and referenced in Axiom 1.

The manifest is how the world presents itself to us in the most direct, unmodelled, uninterpreted and in an uncompressed manner. Brutely knowing the manifest is how one perceives the world without understanding any patterns and without knowing any laws of physics.

As infinitely many manifests $M$ can be constructed from the elements of $\mathbb{D}$, one may wonder why it is the reference manifest $\mathbb{M}$ that is actual, and not any other. This brings us to the next assumption.

**Assumption 2** (The fundamental assumption of nature)**.** *One assumes that the reference manifest* $\mathbb{M}$ *is randomly selected from the set of all possible manifests* $\mathbb{W}$ *according to a probability distribution* $\rho[M]$.

With this assumption we abandon all hope, as hard as it may be, of there being a model which tells us why $\mathbb{M}$ and not $M$ is actual. This assumption is most directly responsible for necessitating that any physical model be derived as a natural argument. Essentially, it is the mathematical formulation of the intuitive notion that the state of affairs is not implied by the model, and therefore, the most one can say about it is simply that $\mathbb{M}$ is a randomly selected element of $\mathbb{W}$.

However, as dreadful as this assumption might be, it is the key to recover the corpus of physics. The first step is to interpret $\mathbb{M}$ as a carrier of information, and it is precisely because it is randomly selected from a larger set that this is possible. We briefly recall the mathematical theory of information of Claude Shannon: Specifically, $\mathbb{M}$ will be interpreted as a message randomly selected from the set $\mathbb{W}$. Therefore, and by using the Shannon definition of entropy, we can quantify the amount of information in the message $\mathbb{M}$ by using the Shannon definition of entropy, as follows:

**Definition 5** (Natural Information). *Under Assumption 2 and by using the definition of the Shannon entropy, we can introduce natural information. We define natural information as the information one gains by knowing which manifest is randomly selected from $\mathbb{W}$, according to the probability distribution $\rho[M]$. The entropy of natural information is defined as:*

$$S = -\sum_{M \in \mathbb{W}} \rho[M] \ln \rho[M] \tag{6}$$

We recall that in the informal case, one would re-organize/compress the raw data into a shorter more aesthetically pleasing and, hopefully, logically equivalent set of axioms, then call the set of axioms a model of the physical system. Intuitively, we understand that one attempted to maximize 'something' but precisely what (aesthetics?, elegance?, ... ?) is not quite clear mathematically when one does so only informally. Now here is the kicker:

**Assumption 3** (The fundamental assumption of physics). *The conservation equation that results from maximizing the amount of natural information associated with the probability distribution $\rho[M]$ are the laws of physics.*

Axiomatic science reveals that the quantity which one attempted to maximize as one informally constructed an artificial model of the data, is, in actuality, natural information. The problem of finding the laws of physics is thus reduced to what amounts to maximizing natural information using $\mathbb{M}$ as the message and $\mathbb{W}$ as the set of possible messages. With these tools, we can solve for the laws physics without first having to re-organize the raw data as axioms, and thus without inadvertently producing an artificial model. Indeed, as the starting point is Axiom 1, the laws of physics will necessarily be derived by a natural argument will be immunized against the pathologies present in artificial models. Ascent these pathologies, and because the laws of physics will be derived as theorems of axiomatic science, questions of extreme fundamentality (e.g. "why are there laws of physics at all?", and other similarly fundamental questions) can now be answered within the framework.

# 3 Technical introduction

To understand the relationship between natural information, entropy, statistical physics and why the laws of physics should admit conservation laws that can be expressed in such a manner, we will introduce geometric (or generalized/non-commutative) thermodynamics, but first, we will provide a recap of statistical physics, and then of algorithmic thermodynamics.

## 3.1 Recap: Statistical physics

As many readers will no doubt have guessed, we will make heavy use of the framework of statistical physics to maximize the entropy of natural information

| Symbol | Name | Units | Type |
|---|---|---|---|
| $E(q)$ | energy | [Joule] | extensive |
| $1/T = k_B\beta$ | temperature | 1/[Kelvin] | intensive |
| $\overline{E}$ | average energy | [Joule] | macroscopic |
| $V(q)$ | volume | [meter$^3$] | extensive |
| $p/T = k_B\gamma$ | pressure | [Joule/(Kelvin-meter$^3$)] | intensive |
| $\overline{V}$ | average volume | [meter$^3$] | macroscopic |
| $N(q)$ | number of particles | [kg] | extensive |
| $-\mu/T = k_B\delta$ | chemical potential | [Joule/(Kelvin-kg)] | intensive |
| $\overline{N}$ | average number of particles | [kg] | macroscopic |

Table 1: Typical thermodynamic quantities

under suitable priors. Generally speaking, in statistical physics, we are interested in the distribution that maximizes the Boltzmann entropy,

$$S = -k_B \sum_{q\in\mathbb{Q}} \rho[q] \ln \rho[q] \tag{7}$$

subject to the fixed macroscopic quantities (the statistical priors). The solution to this maximization problem is the Gibbs ensemble. Typical thermodynamic quantities are shown in Table 1.

Taking these quantities as examples, the partition function (Gibbs ensemble) becomes:

$$Z = \sum_{q\in\mathbb{Q}} e^{-\beta E[q]-\gamma V[q]-\delta N[q]} \tag{8}$$

The probability of occupation of a micro-state (Gibbs measure) is:

$$\rho[q] = \frac{1}{Z} e^{-\beta E[q]-\gamma V[q]-\delta N[q]} \tag{9}$$

The average values are:

$$\overline{E} = \sum_{q\in Q} \rho[q] E[q] \tag{10}$$

$$\overline{V} = \sum_{q\in Q} \rho[q] V[q] \tag{11}$$

$$\overline{N} = \sum_{q\in Q} \rho[q] N[q] \tag{12}$$

and the variance for each quantity is:

$$\overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{13}$$

$$\overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{14}$$

$$\overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{15}$$

The entropy can be obtained from the partition function and is given by:

$$S = k_B \left( \ln Z + \beta \overline{E} + \gamma \overline{V} + \delta \overline{N} \right) \tag{16}$$

The laws of thermodynamics can be recovered by taking the following derivatives:

$$\left.\frac{\partial S}{\partial \overline{E}}\right|_{\overline{V},\overline{N}} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial \overline{V}}\right|_{\overline{E},\overline{N}} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial \overline{N}}\right|_{\overline{E},\overline{V}} = -\frac{\mu}{T} \tag{17}$$

and grouping them as follows:

$$d\overline{E} = TdS - pd\overline{V} + \mu d\overline{N} \tag{18}$$

This is the equation of the state of the system.

## 3.2 Recap: Algorithmic thermodynamics

Many authors[1, 2, 3, 4, 5, 6, 7, 8, 9] have discussed the similarity between the Gibbs entropy $S = -k_B \sum_{q \in \mathcal{Q}} \rho[q] \ln \rho[q]$ and the entropy in information theory $H = -\sum_{q \in \mathcal{Q}} \rho[q] \log_2 \rho[q]$. Furthermore, the similarity between the halting probability $\Omega$ and the Gibbs ensemble of statistical physics has also been studied[10, 11, 12, 8]. First let us introduce Omega. Consider the following sum:

$$\Omega = \sum_{p \text{ halts}} 2^{-|p|} \tag{19}$$

Here, $|p|$ denotes the length of $p$, a computer program. The sum represents the probability that a random program will halt on a universal Turing machine. The Chaitin's construction (a.k.a. $\Omega$, halting probability, Chaitin's constant) is defined for a universal Turing machine as a sum over its domain (the set of programs that halts for it) where the term $2^{-|p|}$ acts as a special probability distribution which guarantees that the value of the sum, $\Omega$, is between zero and one[13]. Knowing $\Omega$ is enough to know the programs that halt and those that do not for the universal Turing machine it is defined for. Since the halting problem is unsolvable, $\Omega$ must, therefore, be non-computable. In fact, $\Omega$'s connection to the halting problem guarantees that it is algorithmically random, normal and incompressible.

It is possible to calculate some small (always finite) quantity of bits of $\Omega$. As such, Calude[14] calculated the first 64 bits of $\Omega_U$ for some universal Turing machine $U$ as:

$$\Omega_U = 0.0000001000000100000110..._2 \tag{20}$$

Running the calculation for a handful of bits is certainly possible, however, any finitely axiomatic systems will eventually run out of steam and hit a wall. Calculating the digits of $\pi$, for instance, will not hit this kind of limitation. For $\pi$, the axioms of arithmetic are sufficiently powerful to compute as many bits as we wish to calculate, limited only by the physical resources of the computers at our disposal. To understand why this is not the case for $\Omega$, we have to realize that solving $\Omega$ requires solving problems of arbitrarily higher complexity, the complexity of which always eventually outclasses the power of any finitely axiomatic system.

In 2002, Tadaki suggested augmenting $\Omega$ with a multiplication constant $D$, which acts as a decompression term on $\Omega$.

$$\begin{array}{lcl} \text{Chaitin construction} & \rightarrow & \text{Tadaki ensemble} \\ \Omega = \displaystyle\sum_{q \text{ halts}} 2^{-|q|} & \rightarrow & \Omega_D = \displaystyle\sum_{q \text{ halts}} 2^{-D|q|} \end{array} \tag{21}$$

With this change, Tadaki argued that the Gibbs ensemble compares to the Tadaki ensemble as follows:

$$\begin{array}{ll} \text{Gibbs ensemble} & \text{Tadaki ensemble} \\ Z = \displaystyle\sum_{x \in X} e^{-\beta E[x]} & \Omega_D = \displaystyle\sum_{q \text{ halts}} 2^{-D|q|} \end{array} \tag{22}$$

Interpreted as a Gibbs ensemble, the Tadaki construction forms a statistical ensemble where each program corresponds to one of its micro-state. The Tadaki ensemble admits the following quantities; the prefix code of length $|q|$ conjugated with $D$. As a result, it describes the partition function of a system which maximizes the entropy subject to the constraint that the average length of the codes is some constant $\overline{|q|}$;

$$\overline{|q|} = \sum_{q \text{ halts}} |q| 2^{-D|q|} \tag{23}$$

The entropy of the Tadaki ensemble is proportional to the average length of prefix-free codes available to encode programs:

$$S = k_B \left( \ln \Omega + D\overline{|q|} \ln 2 \right) \tag{24}$$

The constant $\ln 2$ comes from the base 2 of the halting probability function instead of base $e$ of the Gibbs ensemble.

John C. Baez and Mike Stay[12] took the analogy further by suggesting a connection between algorithmic information theory and thermodynamics, where the characteristics of the ensemble of programs are equivalent to thermodynamic observables. In algorithmic thermodynamics, one extends $\Omega$ with algorithmic quantities to obtain:

$$\begin{aligned} &\text{Baez-Stay ensemble} \\ &\Omega' = \sum_{q \text{ halts}} 2^{-\beta E[q] - \gamma V[q] - \delta N[q]} \end{aligned} \tag{25}$$

Noting its similarities to the Gibbs ensemble of statistical physics (8), these authors suggest an interpretation where $E[q]$ is the expected value of the logarithm of the program's runtime, $V[q]$ is the expected value of the length of the program, and $N[q]$ is the expected value of the program's output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper);

"

1. $T = 1/\beta$ is the *algorithmic temperature* (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.
2. $p = \gamma/\beta$ is the *algorithmic pressure* (analogous to pressure). This measures the trade-off between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount while holding the number of programs in the ensemble and their mean output fixed.
3. $\mu = -\delta/\beta$ is the *algorithmic potential* (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

"

–John C. Baez and Mike Stay

From equation (25), they derive analogs of Maxwell's relations and consider thermodynamic cycles, such as the Carnot cycle or Stoddard cycle. For this, they introduce the concepts of *algorithmic heat* and *algorithmic work*. Finally, we note that other authors have suggested other alternative mappings in other but related contexts[10, 9].

Comparing the axioms of science (Section 2) to the very similar computer theoretic setup for algorithmic thermodynamics, it is clear that algorithmic thermodynamics will play a significant role. We now have an algorithmic analog to statistical physics, a framework already familiar to physics and capable of producing conservation equations, itself having a similar setup to our axiomatic model of science. What is left to do is to apply said framework to the axioms of science in such a way that the general conservation equations is mathematically the same as a law of physics. Then under Axiom 1 and Assumption 1, as the manifest is the brute description of the state of affairs, then the general conservation law resulting from the framework will be its maximally informative natural model (i.e. natural information is maximized).

Let us now summarize the relevant prior literature as it will help us introduce the solution that we propose.

### 3.3 Hint 1: Seth Lloyd

In 2002, Lloyd[15] calculated the total number of bits available for computation in the universe, as well as the total number of operations that could have occurred since the universe's beginning. Seth Lloyd's paper was published 17 years ago and has received approximately 520 citations. His calculations stand to this day and they are considered to be uncontroversial.

For both quantities (the quantity of bits stored in the universe and the quantity of operations made on those bits), Lloyd obtains the number $\approx 10^{122} k_B[\text{bit}]$. This number is consistent with other approaches; for instance, the Bekenstein-Hawking entropy[16, 17] of the cosmological horizon (also $\approx 10^{122} k_B[\text{bit}]$), and the entropy of the holographic surface at the cosmological horizon suggested by Susskind[18] (also $\approx 10^{122} k_B[\text{bit}]$).

How did Lloyd derive these numbers? First, he calculated the value for these quantities while ignoring the contribution of gravity and he obtained $\approx 10^{90} k_B[\text{bit}]$. It is only by including the degrees of freedom of gravity that the number $\approx 10^{122} k_B[\text{bit}]$ is obtained, which he does in the second part. As we are interested in the totals, we will go directly to the calculations that include the contribution of gravity. We state Lloyd's main result and note that the details of the calculation can be reviewed in his paper. Lloyd obtains a relation between time and number of operations for the universe:

$$\#\text{ops} \approx \frac{\rho_c c^5 t^4}{\hbar} \approx \frac{t^2 c^5}{G\hbar} = \frac{1}{t_p^2} t^2 \tag{26}$$

where $\rho_c$ is the critical density and $t_p$ is the Planck time and $t$ is the age of the universe. With present-day values of $t$, the result is $\approx 10^{122} k_B[\text{bit}]$. Lloyd concludes that his results are consistent with the Bekenstein bound and the holographic principle. He states:

> "Applying the Bekenstein bound and the holographic principle to the universe as a whole implies that the maximum number of bits that could be registered by the universe using matter, energy, and gravity is $\approx \frac{c^2t^2}{l_p^2} = \frac{t^2}{t_p^2}$."

which is also $\approx 10^{122} k_B$[bit]. A particularly interesting consequence of this result is that these relations appear to imply conservation of both information and operations in space-time (the numerical quantity of $10^{122}$ is obtained by summing over all available degrees of freedom in space-time). So with this hint, we are now looking for a fundamental relationship between entropy, information, operations, and space-time.

### 3.4 Hints 2: Entropy and space-time

A relation between entropy and space-time has been anticipated (or at least hinted at) since probably the better part of four decades. The first hints were provided by the work of Bekenstein[19, 20, 21] regarding the similarities between black holes and thermodynamics, culminating in the four laws of black hole thermodynamics. The temperature, originally introduced by analogy, was soon augmented to a real notion by Hawking[16] with the discovery of the Hawking temperature derivable from quantum field theory on curved space-time. We note the discovery of the Bekenstein-Hawking entropy, connecting the area of the surface of a horizon to be proportional to one fourth the number of elements with Planck area that can be fitted on the surface: $S = k_B c^3/(4\hbar G)A$.

We mention Ted Jacobson[22] and his derivation of the Einstein field equation as an equation of state of a suitable thermodynamic system. To justify the emergence of general relativity from entropy, Jacobson first postulated that the energy flowing out of horizons becomes hidden from observers. Next, he attributed the role of heat to this energy for the same reason that heat is energy that is inaccessible for work. In this case, its effects are felt, not as "warmth", but as gravity originating from the horizon. Finally, with the assumption that the heat is proportional to the area $A$ of the system under some proportionality constant $\eta$, and some legwork, the Einstein field equations are eventually recovered.

Recently, Erik Verlinde[23] proposed an entropic derivation of the classical law of inertia and of classical gravity. He compared the emergence of such laws to that of an entropic force, such as a polymer in a warm bath. Each law is emergent from the equation $TdS = Fdx$, under the appropriate temperature and a posited entropy relation. His proposal has encouraged a plurality of attempts to reformulate known laws of physics using the framework of statistical physics. Visser[24] provides, in the introduction to his paper, a good summary of the literature on the subject. The ideas of Verlinde have been applied to loop quantum gravity ([25]), the Coulomb force ([26]), Yang-Mills gauge fields

([27]), and cosmology ([28], [29] and [30]). Some criticism has, however, been voiced[31, 32, 33, 34, 35], including by Visser[24].

Even more recently, a connection between entanglement entropy and general relativity has been supported by multiple publications[36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53].

Finally, we mention the body of work of George Ellis regarding the evolving block universe hypothesis detailed in [54, 55, 56] and the connection between space-time events, general relativity and quantum mechanics.

We are now ready to attempt a solution.

### 3.5 Attempt 1: Finding a specific system of statistical physics with suitable properties

Our first series of attempts could be grouped under a simple concept: In each case, we attempted to construct a specific system of statistical physics having a double interpretation; one, as a system of algorithmic thermodynamics admitting a conservation relation involving bits and operations, and second, that said conservation relation be interpretable as a physical system of space-time (i.e. as a system described by general relativity).

Finding a specific system of statistical physics means attributing a definition to the functions of the thermodynamic observables that are used in the partition function. This is the approach used by Ted Jacobson and Erik Verlinde in the context of connecting general relativity and classical relativity, respectively, to entropy. In each of their paper, the degrees of freedom of space are assumed to be quadratic (i.e. they grow as an area law). Consequently, the thermodynamic observables $\mathcal{O}$ are defined by a quadratic function $\mathcal{O} = x^2$. Attempting to expand upon these ideas, we have produced over a dozen draft papers regarding the emergence of many physical laws, including a toy model of a cosmology emergent from quadratic degrees of freedom. However, in the end, we felt that there was a general problem with this approach.

The problem with this approach is that any results that we would obtain would specific to the artificially constructed partition function. Even if it had the desired properties, one would still have to justify why this specific partition function and not another happens to be the one which describes the World. Specifically, we were unable to justify by natural argument why the degrees of freedom of space would be quadratic. The model, constructed as a specific system of statistical physics would thus remain artificial.

Furthermore, we were missing out on the full potential of statistical physics. Indeed, statistical physics is able to produce conservation equations on the broadest of scales. As a typical example, we refer to the fundamental relation of thermodynamics involving the conservation of energy over a change in thermodynamic observables:

$$d\overline{E} = TdS - pd\overline{V} \tag{27}$$

This relation applies to all systems of statistical physics (provided they admit $V$ and $E$ as thermodynamic observables and are at equilibrium) regardless of the specific system under investigation.

To capture this generality into our natural model, our final solution was not to search for a specific system of statistical physics, but instead to increase the generality of thermodynamics; in the present case, with a non-commutative algebra applied to the thermodynamic observables. In this generalization, which we call *geometric thermodynamics*, the extremely general conservation relation above becomes a special case of an even more general conservation relation that, surprisingly, has the suitable properties!

Applied to axiomatic science, geometric (or generalized/non-commutative) thermodynamics, gains the suitable properties: First, the protocol, originally referring to the classical operations of the Turing machine, now refers to a series of quantum operations. Second, what used to be a bit now becomes a generalized, or geometric, qubit (which will be referred to as geometric event in the results section). Finally, the structure emergent as the bulk/macroscopic state of the system is simply the structure of arbitrarily curved space-times, from which the Einstein Field equations are easily derived by the principle of stationary action.

We will now introduce geometric thermodynamic.

# 4 Geometric Thermodynamics

We identified the potential to generalize statistical physics with a non-commutative algebra as we attempted to create thermodynamic cycles that are consistent with the symmetries of space-time. By doing so, we realized that such cycles could be produced if the relevant thermodynamic observables obeyed a non-commutative algebra. With this insight, we have "reverse engineered" the type of partition function along with a suitable microscopic object of study which would eventually produce allows cycles with suitable properties.

To understand in more detail, let us investigate a hypothetical cycle involving a number of thermodynamic observables. Lets name them $\overline{X}$, $\overline{Y}$ and $\overline{Z}$. Such quantities would be extensive, have the meter as their unit, and would be conjugated to a Lagrange multiplier $\tilde{k}$ having the inverse units ($m^{-1}$). The equation of state of such a system would be:

$$\tilde{k}^{-1}dS = d\overline{X} + d\overline{Y} + d\overline{Z} \tag{28}$$

For a change over the quantities $\overline{X}$, $\overline{Y}$ and $\overline{Z}$ to be consistent with the symmetries of Euclidean space, one would expect that the change in entropy along two paths of equal distance, say a path going in a straight line from $(0, 0, 0)$ to $(0, 5, 0)$ and a path going in a straight line from $(0, 0, 0)$ to $(3, 4, 0)$, to be equal. Indeed, the Euclidean distance along either path is the same: in this case, 5 meters. Since the paths are related to one another via rotation of the frame of reference, the entropic cost of the transformation should only depend on the Euclidean length of the path, and not on the orientation of the frame of reference.

One can enforce this property by demanding that the thermodynamic observables obey a suitable non-commutative algebra. Let's see with an example. As the first step, we add generators of the Clifford algebra $(e_1, e_2, e_3)$ to each quantity. We get:

$$\tilde{k}^{-1} dS = d\overline{X} e_1 + d\overline{Y} e_2 + d\overline{Z} e_3 \tag{29}$$

The second step is to verify that the entropy conforms to the Euclidean distance. We can investigate if this case by squaring the equation of state. We obtain:

$$\tilde{k}^{-2}(dS)^2 = e_1^2(d\overline{X})^2 + e_2^2(d\overline{Y})^2 + e_3^2(d\overline{Z})^2 \tag{30}$$

$$+ (e_1e_2 + e_2e_1)d\overline{X}d\overline{Y} + (e_1e_3 + e_3e_1)d\overline{X}d\overline{Z} + (e_2e_3 + e_3e_2)d\overline{Y}d\overline{Z} \tag{31}$$

In the case were $e_1$, $e_2$ and $e_3$ are commutative, the cross terms $e_1e_2 + e_2e_1$, $e_1e_3 + e_3e_1$ and $e_2e_3 + e_3e_2$ do not cancel, but if they are, say matrices[1], with the following properties:

$$e_1^2 = 1 \tag{32}$$

$$e_2^2 = 1 \tag{33}$$

$$e_3^2 = 1 \tag{34}$$

$$e_1e_2 + e_2e_1 = 0 \tag{35}$$

$$e_1e_3 + e_3e_1 = 0 \tag{36}$$

$$e_2e_3 + e_3e_2 = 0 \tag{37}$$

Then, the cross terms cancel and we obtain:

$$\tilde{k}^{-2}(dS)^2 = (d\overline{X})^2 + (d\overline{Y})^2 + (d\overline{Z})^2 \tag{38}$$

The resulting equation of state has the mathematical form of the Euclidean distance $d^2 := \tilde{k}^2(dS)^2$. The entropy, as demanded, is invariant under rotation of the frame of reference. As we will see, if one uses the flexibility of geometric algebra, one can generalize this argument to space-times of any dimensions, any signature, and even including arbitrarily curved space-times.

Geometric thermodynamics is quite easy to construct, yet it is incredibly powerful. One begins by defining an arbitrary non-commutative basis as follows: $\forall \mu \in \{0, ..., n-1\}$ and $\forall \nu \in \{0, ..., n-1\}$, then

[1] We are reminded of the insight of Paul Dirac regarding the elimination the quadratic form of the relativistic energy using matrices with suitable non-commutative properties: the Dirac matrices.

$$g_{\mu\nu} := \frac{1}{2}(e_\mu e_\nu + e_\nu e_\mu) \tag{39}$$

We define the matrix $g$ as:

$$g = \begin{array}{c} \begin{array}{cccccc} {\scriptstyle \mu=0} & {\scriptstyle \mu=1} & {\scriptstyle \mu=2} & {\scriptstyle \mu=3} & \cdots & {\scriptstyle \mu=n-1} \end{array} \\ \begin{pmatrix} g_{00} & g_{10} & g_{20} & g_{30} & \cdots & g_{(n-1)0} \\ g_{01} & g_{11} & g_{21} & g_{31} & \cdots & g_{(n-1)1} \\ g_{02} & g_{12} & g_{22} & g_{32} & \cdots & g_{(n-1)2} \\ g_{03} & g_{13} & g_{23} & g_{33} & \cdots & g_{(n-1)3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{0(n-1)} & g_{1(n-1)} & g_{2(n-1)} & g_{3(n-1)} & \cdots & g_{(n-1)(n-1)} \end{pmatrix} \end{array} \begin{array}{c} {\scriptstyle \nu=0} \\ {\scriptstyle \nu=1} \\ {\scriptstyle \nu=2} \\ {\scriptstyle \nu=3} \\ \vdots \\ {\scriptstyle \nu=n-1} \end{array} \tag{40}$$

One then defines $n$ thermodynamic observables using this geometric basis. The priors defined in equation (10), for example $\overline{E} = \sum_{q\in\mathbb{Q}} E[q]\rho[q]$ are now simply multiplied with a generator $e_i$ of the geometric algebra, yielding $n$ equations. $\forall i \in \{0, ..., n-1\}$:

$$\overline{X_i}e_i = \sum_{q\in\mathbb{Q}} (X_i[q]e_i)\,\rho[q] \tag{41}$$

Then, by maximizing the entropy with these priors as the constraints and by using the method of the Lagrange multipliers, one will obtain a generalized non-commutative thermodynamics conservation relation, in lieu of equation (18).

$$dS = \kappa d\overline{X}_0 e_0 + \cdots + \kappa d\overline{X}_{n-1} e_{n-1} \tag{42}$$

We note that had we instead selected a geometric algebra such that the generators are commutative, then one would recover, as a special case, the traditional conservation relation of energy found in statistical physics. Explicitly, posing the properties of the generators $e_0, ..., e_{n-1}$ to be commutative:

$$e_i^2 = 1 \tag{43}$$

$$e_i e_j = e_j e_i \tag{44}$$

one obtains the relation $dS = \kappa d\overline{X}_0 + ... + \kappa \overline{X}_{n-1}$, which is of the same mathematical form as equation (18). Therefore, geometric thermodynamic is indeed a generalization of statistical physics.

### 4.1 Outline

Our goal will be to derive the geometric partition function $\mathbf{Z}$ (Theorem 1), the geometric probability measure $\boldsymbol{\rho}[q]$ (Theorem 2) the geometric entropy $\mathbf{S}$ (Theorem 3) and the geometric equation of state $d\mathbf{S}$ (Theorem 4). Then, as the main result, to find a scalar expression for $d\mathbf{S} \to dS$ and show that it equates the curvature of space-time (Theorem 5).

### 4.2 Results

To apply geometric thermodynamics to the axioms of science, we will pose our final assumption:

**Assumption 4** (The fundamental assumption of 'substance')**.** *We will equip an experiment $p$ with the observables of geometric thermodynamics. We will call such an experiment a geometric event* $\mathbf{q}$.

We have opted to use the boldface (for instance $\boldsymbol{\rho}$, $\mathbf{q}$) as our notation to designate that the quantity is a vector of the geometric algebra. We will prefix the name of such quantities with the term *geometric*. Geometric quantities contain basis elements within their expression so as to enforce the suitable non-commutative relation between the quantities of the expression.

First, let us define a space-time event:

**Definition 6** (Space-time event)**.** *A space-time event $q$ is an $(m+n)$-tuple.*

$$q := (\underbrace{X_0, ...X_{n-1}}_{\text{time terms}}, \underbrace{X_n, ...X_{n+m-1}}_{\text{space terms}}) \tag{45}$$

*The quantities $X_i$ are elements of $\mathbb{R}$. We pose immediately that the first $n$ terms denote the 'time dimensions' and the next $m$ terms denote the 'space dimensions'. A space-time event can be represented algebraically using geometric algebra:*

$$\mathbf{q} := \underbrace{X_0 e_0 + ... + X_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{X_n e_n + ... + X_{n+m-1} e_{n+m-1}}_{\text{space terms}} \tag{46}$$

Now, let us define a geometric event:

**Definition 7** (Geometric event)**.** *A geometric event $\mathbf{q_g}$ is a generalization of a space-time event. A geometric event is a multi-vector of the geometric algebra. For example,*

$$\mathbf{q_g} := \mathbf{q} + \underbrace{A + Be_0 e_1 + Ce_0 e_2 + ... + De_0 e_1 e_2 + ...}_{\text{geometric terms}} \tag{47}$$

Without loss of generality and to keep the steps of the proof to manageable lengths, we will derive the results using space-time events. The geometric terms can be trivially added to all the results that follow.

**Definition 8** (Set of events)**.** *We define a set of events $\mathbb{Q}$ as:*

$$\left(\mathbb{Q} \subset \{q : q \in \mathbb{R}^{n+m}\}\right) \wedge |\mathbb{Q}| < 2^{\aleph_1} \tag{48}$$

*The quantity $n+m$ denotes the number of dimensions of the events. The notation $|\mathbb{Q}| < 2^{\aleph_1}$ indicates that we will be dealing with countable sets, and thus the sum notation of the entropy will be used instead of the differential form.*

**Definition 9** (Interval)**.** *Let $\mathbf{q}_1$ and $\mathbf{q}_2$ be events written using the geometric algebra:*

$$\mathbf{q}_1 := t_1 e_t + x_1 e_x + y_1 e_y + z_1 e_z \tag{49}$$
$$\mathbf{q}_2 := t_2 e_t + x_2 e_x + y_2 e_y + z_2 e_z \tag{50}$$

*Then, the interval between these events is defined in reference to the familiar metric tensor $g$:*

$$(\mathbf{q}_1 - \mathbf{q}_2)^2 = ((t_1 - t_2)e_t + (x_1 - x_2)e_x + (y_1 - y_2)e_y + (z_1 - z_2)e_z)^2 \tag{51}$$
$$= ((\Delta t)e_t + (\Delta x)e_x + (\Delta y)e_y + (\Delta z)e_z)^2 \tag{52}$$
$$\begin{aligned}=&(\Delta t)e_t(\Delta t)e_t + (\Delta t)e_t(\Delta x)e_x + (\Delta t)e_t(\Delta y)e_y + (\Delta t)e_t(\Delta z)e_z\\ &+ (\Delta x)e_x(\Delta t)e_t + (\Delta x)e_x(\Delta x)e_x + (\Delta x)e_x(\Delta y)e_y + (\Delta x)e_x(\Delta z)e_z\\ &+ (\Delta y)e_y(\Delta t)e_t + (\Delta y)e_y(\Delta x)e_x + (\Delta y)e_y(\Delta y)e_y + (\Delta y)e_y(\Delta z)e_z\\ &+ (\Delta z)e_z(\Delta t)e_t + (\Delta z)e_z(\Delta x)e_x + (\Delta z)e_z(\Delta y)e_y + (\Delta z)e_z(\Delta z)e_z\end{aligned} \tag{53}$$
$$\begin{aligned}=&(\Delta t)^2 e_t^2 + (\Delta x)^2 e_x^2 + (\Delta y)^2 e_y^2 + (\Delta z)^2 e_z^2\\ &+ \Delta t \Delta x(e_t e_x + e_t e_x) + \Delta t \Delta y(e_t e_y + e_y e_t) + \Delta t \Delta z(e_t e_z + e_z e_t)\\ &+ \Delta x \Delta y(e_x e_y + e_y e_x) + \Delta x \Delta z(e_x e_z + e_z e_x)\\ &+ \Delta y \Delta z(e_y e_z + e_z e_y)\end{aligned} \tag{54}$$
$$= \sum_{\mu\nu} g_{\mu\nu} \Delta X_\mu \Delta X_\nu \tag{55}$$

*where* $\forall \mu \in \{0, 1, 2, 3\} \forall \nu \in \{0, 1, 2, 3\} \left[g_{\mu\nu} = \frac{1}{2}(e_\mu e_\nu + e_\nu e_\mu)\right]$.

We will derive our ensemble of events using the following definition for the entropy:

**Definition 10** (Geometric entropy)**.**

$$\mathbf{S} = -\sum_{q \in \mathbb{Q}} \boldsymbol{\rho}[q] \ln \boldsymbol{\rho}[q] \tag{56}$$

| **Symbol** | **Name** | **Units** | **Type** |
|---|---|---|---|
| $x(q)$ | space | [meter] | extensive |
| $\tilde{k}$ | entropic repetency | [1/meter] | intensive |
| $\overline{x}$ | thermal space | [meter] | macroscopic |
| $t(q)$ | time | [second] | extensive |
| $f$ | entropic frequency | [1/second] | intensive |
| $\overline{t}$ | thermal time | [second] | macroscopic |
| $c := f/\tilde{k}$ | entropic speed | [meter/second] | intensive |

Table 2: The physical quantities of the ensemble

**Definition 11** (Physical quantities of the partition function)**.** *As we derive the partition function for an ensemble of events, two physical quantities will be introduced as Lagrange multipliers. They are: 1) the entropic repetency $\tilde{k}$ (normally the repetency is represented by the symbol $\tilde{\nu}$, but we already use the symbol $\nu$ extensively for the indices of our basis; therefore, we opt to use $\tilde{k}$ here to avoid ambiguity); and 2) the entropic frequency $f$. Specifically, $\tilde{k} = k/2\pi = 1/\lambda$, where $k$ is the wave-number and $\lambda$ is the wavelength.*

These quantities are the conjugated variables to a distance $x$ and time $t$, respectively. By convention, we prefix the Lagrange multipliers with the word "entropic", and its averaged conjugated quantity will be prefixed with the word "thermal". $\tilde{k}$ and $f$ are both intensive properties, whereas $x$ and $t$ are extensive. Indeed, a process taking 1 min followed by a process taking 2 min takes a total of 3 min (extensive). For the $x$ quantity; walking 1 meter followed by walking 2 meters implies one has walked a total of 3 meters (extensive). Adding or removing clocks from a group of clocks ticking at a frequency $f$ (say once per second) has no impact on the frequency of the other elements of the group (intensive). The same argument applies to the entropic repetency (intensive). The units of $\tilde{k}$ are $m^{-1}$, the units of $x$ are the meters, the units of $t$ are the seconds, and the units of $f$ are $s^{-1}$. Finally, we define the quantity $c := f/\tilde{k}$. These quantities are summarized in Table 2.

To derive the ensemble of events, we will assume that we are allowed to do as follows: Instead of creating an ensemble of $M$ (a manifest) selected over $\mathbb{W}$ (the set of all manifests), we create $n$ ensembles of $p$ (an experiment) selected over $\mathbb{D}$ (the domain of science). In this case, the ensemble $M \in \mathbb{W}$ is the grand canonical ensemble to $n$ canonical ensembles $p \in \mathbb{D}$. At any point, should we prefer to work with $M \in \mathbb{W}$, rather than with $n$ systems of $p \in \mathbb{D}$, we can redress to a grand-canonical ensemble simply by introducing $\mu N(M)$ as a thermodynamic observable in the grand-canonical ensemble and summing $M \in \mathbb{W}$ instead of $p \in \mathbb{D}$. Specifically, the assumption is that $\mu N(M)$ is a valid thermodynamic observable of a manifest.

As this assumption is about experiments, and geometric events are experiments equipped with additional structure, then we will also inherit this assumption for geometric events. With geometric events, we will now create a canonical ensemble by summing over $q \in \mathbb{Q}$.

**Definition 12** (Ensemble of events)**.** *The probability measure of an ensemble of events maximizes the entropy under the constraints of the macroscopic priors defined in Table 2 expressed in the geometric algebra of events. Specifically,* $\forall i \in \{0, ..., n+m-1\}$ *the priors to the ensemble are:*

$$\overline{X}_i e_i = \sum_{q \in \mathbb{Q}} (X_i[q] e_i)\, \boldsymbol{\rho}[q] \tag{57}$$

*The functions* $X_i[q]$ *are maps* $X_i : \mathbb{Q} \to \mathbb{R}$ *where* $X_i[q]$ *returns the value of the* $i^{\text{th}}$ *element of the* $(m+n)$*-tuple of* $q$*. There are* $m+n$ *priors. The terms* $\overline{X}_i e_i$ *are averages.*

Note: we will prove Theorem 1 and 2 together.

**Theorem 1** (Geometric partition function)**.** *The partition function of the ensemble of events is:*

$$\mathbf{Z} = \sum_{q \in \mathbb{Q}} \exp\left( -f \underbrace{(X_0[q]e_0 + ... + X_{n-1}[q]e_{n-1})}_{\text{time terms}} - \tilde{k} \underbrace{(X_n[q]e_n + ... + X_{n+m-1}[q]e_{n+m-1})}_{\text{space terms}} \right) \tag{58}$$

*where* $\tilde{k}$ *and* $f$ *are Lagrange multipliers.*

**Theorem 2** (Geometric probability measure)**.** *The probability measure* $\boldsymbol{\rho}(q)$ *is:*

$$\boldsymbol{\rho}(q) = \mathbf{Z}^{-1} \exp\left( -f \underbrace{(X_0[q]e_0 + ... + X_{n-1}[q]e_{n-1})}_{\text{time terms}} - \tilde{k} \underbrace{(X_n[q]e_n + ... + X_{n+m-1}[q]e_{n+m-1})}_{\text{space terms}} \right) \tag{59}$$

*Proof.* We will now prove Theorem 1 and Theorem 2. One obtains the partition function $\mathbf{Z}$ with the usual method of the Lagrange multipliers.

1. The constraints, are:

$$I = \sum_{q \in \mathbb{Q}} \boldsymbol{\rho}[q] \tag{60}$$

   where $I$ is the identity element, and

$$\forall i \in \{0, ..., n+m-1\} \left[ \overline{X}_i e_i = \sum_{q \in \mathbb{Q}} (X_i[q] e_i) \, \boldsymbol{\rho}[q] \right] \tag{61}$$

2. The function to maximize is:

$$\mathbf{S} = -\sum_{q \in \mathbb{Q}} \boldsymbol{\rho}[q] \ln \boldsymbol{\rho}[q] \tag{62}$$

3. The Lagrange equation is:

$$\boldsymbol{\mathcal{L}} = \left( -\sum_{q \in \mathbb{Q}} \boldsymbol{\rho}[q] \ln \boldsymbol{\rho}[q] \right) - \lambda \left( \sum_{q \in \mathbb{Q}} \boldsymbol{\rho}[q] - I \right) - \sum_{i=0}^{n+m-1} \left( \lambda_i \sum_{q \in \mathbb{Q}} (X_i[q] e_i) \, \boldsymbol{\rho}[q] - \overline{X}_i e_i \right) \tag{63}$$

where $\lambda$ and the set of $\lambda_i$ are Lagrange multipliers.

4. Maximizing $\boldsymbol{\mathcal{L}}$ with respect to $\boldsymbol{\rho}[q]$ is done by taking its derivative and posing it equal to $\mathbf{0}$.

$$\frac{\partial \boldsymbol{\mathcal{L}}}{\partial \boldsymbol{\rho}[q]} = \mathbf{0} \tag{64}$$

where $\mathbf{0}$ is the null vector. Here we will be dealing with a number of vector derivatives and related identities. Specifically, we require scalar-by-vector and vector-by-vector. From *The Matrix Cookbook* by [57], these derivatives behave as we would expect them to from what we know of derivatives involving scalar variables. Then using the appropriate corresponding identities, one easily gets:

$$\frac{\partial \boldsymbol{\mathcal{L}}}{\partial \boldsymbol{\rho}[q]} = -\ln \boldsymbol{\rho}[q] - I - \lambda I - \sum_{i=0}^{n+m-1} (\lambda_i X_i[q] e_i) = \mathbf{0} \tag{65}$$

5. Solving for $\boldsymbol{\rho}(q)$ in (Equation 65) one obtains:

$$\boldsymbol{\rho}(q) = \exp(-I - \lambda I) \exp\left( -\sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i \right) \tag{66}$$

6. From the constraint $I = \sum_{q \in \mathbb{Q}} \boldsymbol{\rho}(q)$, we can find the expression for $\exp(-I - \lambda I)$ as follows:

$$I = \sum_{q \in \mathbb{Q}} \boldsymbol{\rho}(q) \tag{67}$$

$$= \exp(-I - \lambda I) \sum_{q \in \mathbb{Q}} \exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right) \tag{68}$$

$$\implies I \left( \sum_{q \in \mathbb{Q}} \exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right)\right)^{-1} = \exp(-I - \lambda I) \tag{69}$$

7. We then define the inverse of the left term as the partition function:

$$\mathbf{Z} := \sum_{q \in \mathbb{Q}} \exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right) \tag{70}$$

8. Finally, we write $\boldsymbol{\rho}(q)$ using $\mathbf{Z}$. We obtain:

$$\boldsymbol{\rho}(q) = \mathbf{Z}^{-1} \exp\left(- \sum_{i=0}^{n+m-1} \lambda_i X_i[q] e_i\right) \tag{71}$$

Out of isotropic considerations, we pose $\forall j \in \{0, ..., n-1\}[\lambda_j = f]$ and $\forall k \in \{n, ..., n+m-1\}[\lambda_k = \tilde{k}]$, and we obtain (Equation 58) and (Equation 59). $\square$

**Theorem 3** (Geometric entropy)**.** *The entropy of the ensemble of events is:*

$$\mathbf{S} = \ln \mathbf{Z} + \underbrace{f\overline{X}_0 e_0 + ... + f\overline{X}_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{\tilde{k}\overline{X}_n e_n + ... + \tilde{k}\overline{X}_{n+m-1} e_{n+m-1}}_{\text{space terms}} \tag{72}$$

*We interpret the entropy as the information one gains by knowing which event $q$ was randomly selected from the set of events $\mathbb{Q}$ under the probability distribution $\boldsymbol{\rho}[q]$.*

*Proof.* Replacing $\boldsymbol{\rho}[q]$ in the definition for the entropy $\mathbf{S}$ (Equation 56) with the probability distribution for the ensemble of events ($\boldsymbol{\rho}[q]$ in Equation 59), one obtains:

$$\mathbf{S} = -\sum_{q\in\mathbb{Q}}\left[\mathbf{Z}^{-1}\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\ln\left(\mathbf{Z}^{-1}\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\right)\right] \tag{73}$$

With a few rearrangements, one obtains:

$$\mathbf{S} = -\mathbf{Z}^{-1}\sum_{q\in\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\left(\ln\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)-\ln\mathbf{Z}\right)\right] \tag{74}$$

The logarithm of the exponential of a matrix is equal to the matrix $\ln\exp\mathbf{A} = \mathbf{A}$. Therefore,

$$\mathbf{S} = -\mathbf{Z}^{-1}\sum_{q\in\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\left(\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)-\ln\mathbf{Z}\right)\right] \tag{75}$$

$$= -\mathbf{Z}^{-1}\sum_{q\in\mathbb{Q}}\left[\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\right]+\mathbf{Z}^{-1}\ln\mathbf{Z}\sum_{q\in\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\right] \tag{76}$$

From definition (58), $\mathbf{Z} = \sum_{q\in\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\right]$. Therefore,

$$\mathbf{S} = -\mathbf{Z}^{-1}\sum_{q\in\mathbb{Q}}\left[\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\exp\left(-\sum_{i=0}^{n+m-1}\lambda_i X_i[q]e_i\right)\right]+\ln\mathbf{Z} \tag{77}$$

From definition (57), the average of a quantity $\overline{X}_i e_i = \sum_{q\in\mathbb{Q}}\left(X_i[q]e_i\right)\boldsymbol{\rho}[q]$. Therefore,

$$\mathbf{S} = \ln\mathbf{Z} + \sum_{i=0}^{n+m-1}\overline{X}_i e_i \tag{78}$$

□

**Theorem 4** (Geometric equation of state)**.** *The equation of state of the ensemble of events is:*

$$d\mathbf{S} = \underbrace{fd\overline{X}_0 e_0 + ... + fd\overline{X}_{n-1}e_{n-1}}_{\text{time terms}} + \underbrace{\tilde{k}d\overline{X}_n e_n + ... + \tilde{k}d\overline{X}_{n+m-1}e_{n+m-1}}_{\text{space terms}} \tag{79}$$

*Proof.* The differential form extends easily from $\mathbf{S}$ to $d\mathbf{S}$ using partial derivatives to recover the applicable laws of thermodynamics, then grouping them in the form of an equation of state as per introductory statistical physics. □

## 4.3 Space-time entropy

We note that geometric entropy $\mathbf{S}$ is not an element of the reals, but a vector of the geometric algebra. This situation is undesirable —one would prefer a definition of entropy that is a real number— and, in fact, it will be by resolving this problem that we will obtain the interesting physics connecting the entropy to the curvature of space-time.

**Theorem 5** (Space-time entropy)**.** *Due to the peculiar non-commutative properties of the algebra of events, the multi-vector* $d\mathbf{S}$ *becomes a real number simply by squaring (taking the geometric product of* $d\mathbf{S}$ *with itself) the equation of state:*

$$(d\mathbf{S})^2 = \left(\underbrace{f d\overline{X}_0 e_0 + ... + f d\overline{X}_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{\tilde{k} d\overline{X}_n e_n + ... + \tilde{k} d\overline{X}_{n+m-1} e_{n+m-1}}_{\text{space terms}}\right)^2 \tag{80}$$

*Squaring* $d\mathbf{S}$ *does not change the meaning of the equation nor the entropy, yet it nonetheless erases base-specific information making* $d\mathbf{S}$ *a scalar as desired. To help us interpret the squared equation, we now divide each side of the above equation by* $\tilde{k}^2$*. Finally, we identify the term* $\tilde{k}^{-2}(d\mathbf{S})^2$ *as the space-time interval* $(ds)^2$*. Then, the equation of state is easily recognized as the interval of general relativity (the metric) in* $m+n$ *space. Let* $(ds)^2 := \tilde{k}^{-2}(d\mathbf{S})^2$*, then:*

$$\underbrace{(ds)^2}_{\text{interval}} = \left(\underbrace{c d\overline{X}_0 e_0 + ... + c d\overline{X}_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{d\overline{X}_n e_n + ... + d\overline{X}_{n+m-1} e_{n+m-1}}_{\text{space terms}}\right)^2 \tag{81}$$

*Proof.* Expanding the power of two to the right-hand side of the equation and rearranging, and by using the definition of the interval (Definition 9) it is straightforward to recover the definition of the metric $g$;

$$\underbrace{\tilde{k}^{-2}(d\mathbf{S})^2}_{\text{entropic distance}} = \underbrace{(ds)^2}_{\text{interval}} = \underbrace{\sum_{\mu\nu} g_{\mu\nu} d\overline{X}_\mu d\overline{X}_\nu}_{\text{metric}} \tag{82}$$

□

## 4.4 Einstein field equations

With access to a generally curved space-time expressed by a metric $g$, deriving the Einstein field equations can be done straightforwardly by appealing to the principle of stationary action. The action $\mathcal{A}$ is defined as (here we use $\mathcal{A}$ for the action instead of $S$ to avoid confusing the symbol with that of the entropy):

$$\mathcal{A} = \int \mathcal{L} d^{(4)} V \tag{83}$$

In curved space-time, the 4-volume element $d^{(4)}V$ is given by:

$$d^{(4)} V = \sqrt{-g} d^4 x \tag{84}$$

where $g$ is the determinant of the metric tensor matrix. We then take the Ricci scalar as the simplest curvature invariant which produces a scalar, and we pose $\mathcal{L} := R$. We then obtain:

$$\mathcal{A} = \int R \sqrt{-g} d^4 x \tag{85}$$

which, up to a multiplication constant, we recognize as the Hilbert-Einstein action.

# 5 Discussion

We will now have a discussion about the solutions proposed by this model regarding some of the well-known problems of physics. In this paper, we are only beginning to scratch the surface of axiomatic science, therefore our intention is not to completely resolve all of these problems, but rather to present the case made by axiomatic science in this regard such that it may encourage future research.

First, we will discuss how geometric thermodynamics suggests an entropic model of space-time. The relation $(ds)^2 = \tilde{k}^{-2}(d\mathbf{S})^2$ equates the space-time interval to an entropy. We will call $\tilde{k}^{-2}(d\mathbf{S})^2$ the *entropic distance*. The entropic distance quantifies the informational departure of some hypothesized future or past manifest from the reference manifest. Let us begin by comparing the Euclidean case to the Minkowski case.

1. In Euclidean space, the geometric equation of state is $\tilde{k}^{-2}(d\mathbf{S})^2 = (dx)^2 + (dy)^2 + (dz)^2$. In this case, we note that all events (points) have a non-zero entropic distance from the origin.

2. In Minkowski space-time, the geometric equation of state is $\tilde{k}^{-2}(d\mathbf{S})^2 = (dx)^2 + (dy)^2 + (dz)^2 - (dt)^2$. We note the existence of a nullsurface (light cone) on which all events have zero entropic distance from the origin.

**Conjecture 1** (Present)**.** *We note that since the entropy is constant on the nullsurface, then the entropic distance between all events on the surface is zero.*

*We thus conjecture that all events on the nullsurface are known to the observer because they have an entropic distance of zero from it. Consequently, none of these events are hidden by entropy. The information available in the reference manifest is the information available on the nullsurface.*

**Conjecture 2** (Past/Future)**.** *We can then define a position in time away from the present and quantified by the entropy as:*

$$ds = \pm\sqrt{\tilde{k}^{-2}(d\mathbf{S})^2} \tag{86}$$

Conjecture 2 suggests an evolving block universe[56].

**Conjecture 3** (Evolving block universe)**.** *The relation $ds = \sqrt{\tilde{k}^{-2}(d\mathbf{S})^2}$ quantifies the informational departure of a future event from the present. The role of this entropy is to hide from an observer knowledge of events outside its present. This entropy is eliminated if and when the gap between the observer and the entropically-distant event is reduced to* 0 *(that is when the observer moves forward in time until the event occurs). Thus, the future is hidden from the present by entropy.*

**Conjecture 4** (Events and measurements)**.** *Since the entropy represents the number of microscopic states compatible with the current macroscopic state of the system, an observer pondering about its future will conclude that many alternative futures appear possible but would expect only one to be actual when the entropic distance is eventually reduced to zero as time advances.*

**Conjecture 5** (Agreement)**.** *Since no events are hidden by entropy on the nullsurface, all observers along this surface necessarily agree on the events. A disagreement between observers is impossible as all observers know (or can know) what events all others have measured.*

**Conjecture 6** (Entropic speed of light)**.** *The speed of light, usually taken as an axiom in special relativity, is here emergent as the ratio of the Lagrange multipliers of the partition function $c := f/\tilde{k}$. The speed of light here fulfills a role similar to the role fulfilled by the temperature in standard thermodynamics. Essentially, the speed of light is the "temperature" of space-time. The speed of light is a property emergent from the random selection of events from a larger set under the principle of maximum entropy. The speed of light is constant in an ensemble of space-time events at equilibrium for the same reason that the temperature is constant in a system at thermodynamic equilibrium. Indeed, when $f$ is the inverse of the Planck time, and $\tilde{k}$ the inverse of the Planck length, we get c, the speed of light:*

$$f/\tilde{k} = \left(\sqrt{\frac{c^3}{\hbar G}}\sqrt{\frac{\hbar G}{c^5}}\right)^{-1} = c \tag{87}$$

**Conjecture 7** (Arrow of time)**.** *The gradient of space-time entropy always peaks in the direction of the observer's future. Therefore, a space-time system seeking to statistically increase its entropy is powered to evolve forward in time by entropy. Indeed at thermodynamic equilibrium, the entropic frequency does become an entropic power as* $k_B T f = P$*.*

We will now use this model of time and the axioms of science to investigate an interpretation of quantum mechanics.

Let us first provide a brief recap of the quantum measurement problem. In the case of quantum physics, the unitary evolution of the wave function is deterministic, but the notion breaks down if measurements are occurring in the system (or are performed on the system). In the Von Neumann scheme, a measurement of the second kind, for a quantum object with wave function $|\psi\rangle$ and a quantum apparatus with wave function $|\phi\rangle$, is defined as:

$$|\psi\rangle|\phi\rangle \to \sum_n c_n|\chi_n\rangle|\phi_n\rangle \tag{88}$$

After the measurement the system is in one of eigenstates $|\chi_n\rangle$ with probability $|c_n|^2$. That the otherwise deterministic unitary evolving system adopts the "undeterministic" initiative to collapse itself randomly in one of multiple states after measurement is quite the mystery. In fact, nothing in quantum physics predicts that such a thing would occur. Consequently, the notion of the measurement is introduced into quantum mechanics, formally, as a full-blown axiom not derivable from the Schrodinger equation itself, or from any of the other axioms of quantum mechanics.

The theory of quantum decoherence is a modern take on Von Neumann measurements. De-coherence from the environment $|e\rangle$ is introduced as follows:

$$|\psi\rangle|\phi\rangle|e\rangle \to \sum_n c_n|\chi_n\rangle|\phi_n\rangle|e_n\rangle \tag{89}$$

Under contact with an environment having multiple degrees of freedom, any interference pattern normally observable from $|\psi\rangle$ will be smudged by the environment beyond the ability of instruments to detect it. De-coherence explains why a quantum superposition of eigenstates is unlikely to be observed macroscopically as interaction with the environment very quickly causes the system to evolve towards a classical probability distribution. However, decoherence is ultimately of no help in regards to explaining why one eigenstate out of many is randomly selected for the system to be in post-measurement.

The measurement postulate, as a law, is derived empirically and it is introduced purely so that quantum physics predicts a single macroscopic world (not a superposition of many worlds), consistent with observations. The two primary competing interpretations of this phenomenon are a) the Copenhagen interpretation and b) the Everett many-worlds interpretation, but there exists at least half a dozen others. None of these interpretations are, however, considered satisfactory by mainstream physics and thus the question remains unsettled; in the first case the collapse is simply postulated but no mechanisms are generally accepted for it, and in the second case it is postulated that the observer becomes coupled with a specific result of the measurement causing the appearance of collapse, but no mechanism to account for this coupling is generally accepted either. The interpretational problem is retained in all extensions of quantum theory from the Dirac equation to any valid quantum field theory, etc. As one is generally free to apply any of the compatible interpretations to quantum theory, deciding which one is correct, if any, is often criticized as a non-falsifiable problem.

Axiomatic science, as one should no doubt be expecting at this point, proposes its own interpretation and, interestingly, is incompatible with the others. Axiomatic science flips the foundation responsible for producing the measurement problem. Instead of assuming a deterministic equation describing the time evolution of the probability amplitude (e.g. Schrödinger equation) then introducing a measurement postulate to correct the multiple predictions of this equation to a singular observed world, axiomatic science first postulates the existence of a reference manifest (Axiom 1, a.k.a one world), and then quantifies the informational departure of future or past manifests from the reference manifest using the geometric equation of state (Theorem 4) as the sizing tool. As this entropy quantifies the number of random events which must occur between now and some future or past manifest, one obtains a similar random behavior as the quantum measurement but without its perplexing interpretations.

Specifically, in axiomatic science, the quantum measurement problem, initially paraphrased as 'how does a system in a superposition eventually collapses to a single result, or does it?', is instead converted to a non-problem: 'how strongly does the prediction of the laws of physics diverges from the eventual real result as the system is solved for its future or past state, using only the reference manifest as the initial data'. In this conversion, the measurement is simply a necessary consequence of any natural argument and it accounts for the fact that the future or the past, described by a different manifest, implies a different model. As a future manifest $M$ cannot be logically derived in its entirety from the reference manifest, additional data must be introduced in an amount as quantified by the geometric equation of state in order to connect the reference manifest to the eventual future manifest. Finally, as the argument is natural, no model can supply this missing data, and thus it appears as random data to any possible model.

One can see the details of this even more clearly by investigating the axioms, definitions, and assumptions of axiomatic science within this optic. Specifically, Axiom 1 postulates the existence of the reference manifest making it special. However, to maximize natural information, one must sum over all possible manifests in $\mathbb{W}$, even those that are not granted the status of existence by Axiom 1 (this is the case for all manifests except for $\mathbb{M}$). In the sum, the reference manifest $\mathbb{M}$ does not hold a privileged position, and its statistical weight is given by $\rho(\mathbb{M})$ just as if it were any other $M$. As this sum is required to derive the laws of physics in the framework, we note that it is not the case that the reference manifest by itself is sufficient to derive the laws of physics, as one must sum over all possible manifests to derive them — *even summing on those manifests $M$ that do not exists $\neq \mathbb{M}$.* We often think of the law of physic as those that explain the state of affairs of the world, but in axiomatic science, this is close but not quite the case: the laws of physics accounts for all possible state of affairs. Consequently, reversing the argument; that is, taking the laws of physics as the starting point and solving them for possible manifests recovers a plurality of possible manifests as valid solutions, only one of which is the reference manifest, but which one is it has been erased by the sum. Confusion about various interpretations and the follow-up introduction of non-sensical concepts such as 'collapses' follows from this artificial inversion, and vanishes as a non-problem for the natural construction.

A mathematically explicit connection between the formalism of quantum mechanics and geometric thermodynamics/axiomatic science will be investigated by the author in a future paper. For now, we will provide a sketch of the derivation of the Nambu–Goto action, relevant to string theory, from the geometric equation of state and then we will call it a day.

## 5.1 Sketch 1: Nambu–Goto action

Extending the geometric equation of state from space-time events to geometric events, one obtains:

$$
\begin{aligned}
d\mathbf{S} = & \underbrace{f d\overline{X_0} e_0 + ... + f d\overline{X}_{n-1} e_{n-1}}_{\text{time terms}} \\
& + \underbrace{\tilde{k} d\overline{X}_n e_n + ... + \tilde{k} d\overline{X}_{n+m-1} e_{n+m-1}}_{\text{space terms}} \\
& + \underbrace{\nu dA_{01} e_0 e_1 + \nu dA_{02} e_0 e_2 + ... + p dV_{012} e_0 e_1 e_2 + ...}_{\text{geometric terms}}
\end{aligned}
\tag{90}
$$

This is the geometric interval between two events. Keeping the area terms and crossing out the time, space and higher-dimensional geometric terms, one obtains the area elements as invariants, as follows:

$$
d\mathbf{S}_A = \underbrace{\nu dA_{01} e_0 e_1 + \nu dA_{02} e_0 e_2 + ...}_{\text{invariant area terms}} \tag{91}
$$

One can then construct an action where the area of the geometric event is invariant. Under the relation $dA = d^2\Sigma\sqrt{-g}$, one obtains the action $\mathcal{A}$ of an invariant area:

$$
\mathcal{A} = \frac{T}{c} \int d^2\Sigma\sqrt{-g} \tag{92}
$$

for which the Nambu-Goto action is a special case. In fact, an invariant area is a *world-sheet*.

One can assign the usual parameters $\sigma$ and $\tau$ as the coordinates of the area. Then, one then obtains the induced metric as:

$$g_{\tau\sigma} = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \sigma} \tag{93}$$

Finally, one rewrites (92) as

$$\mathcal{A} = -\frac{T}{c} \int d^2\Sigma \sqrt{\left(\frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \sigma}\right)^2 - \left(\frac{\partial X^\mu}{\partial \tau}\right)^2 \left(\frac{\partial X^\nu}{\partial \sigma}\right)^2} \tag{94}$$

which is the Nambu-Goto action.

# 6 Conclusion

A world which exists brutely and for no logically provable reason has the laws of physics as its only laws.

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