Refutation of unification nets (canonical proof net quantifiers)

Abstract: Using the drinker's paradox, as rendered in two equations, we evaluate the unification net, in two equations, as *not* tautologous. To extend the unification net to additives is similarly defective, forming a *non* tautologous fragment of the universal logic VŁ4. We also supply analysis of Smullyan's drinking principle.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Hughes, D.J.D. (2018). Unification nets: canonical proof net quantifiers. arxiv.org/pdf/1802.03224.pdf

1.3 Towards combinatorial proofs for classical first-order logic
A first-order combinatorial proof of Smullyan's drinker paradox [is shown]

$\exists x(Px \Rightarrow \forall y Py)$. (1.3.0.1)

LET p, q, r, s: P, D, x, y.

%r&((p&r)>(#s&(p&s))) ; `CCCC T`**`F`**`T`**`F`**` CCCC TNTN` (1.3.0.2)

Remark 1.3.0.2: We rewrite Eq. 1.3.0.1 for clarity by distributing the respective quantifiers. (1.3.0.1.1)

(p&%r)>(p&#s) ; `TNTN T`**`F`**`T`**`F`**` TNTN TNTN` (1.3.0.1.2)

Eqs. 1.3.0.2 and 1.3.0.1.2 are *not* tautologous and *not* equivalent; also, Smullyan's drinker paradox is stated differently elsewhere*.

By using a semi-combinatorial presentation style ... the unification net becomes more apparent.

$\exists x(\overline{P}x \vee \forall y\, Py)$ (1.3.4.1)

%r&((~p&r)+(#s&(p&s))) ; **`FFFF`** `T`**`F`**`T`**`F`** **`FFFF`** `T`**`F`**`TN` (1.3.4.2)

Remark 1.3.4.2: We rewrite Eq. 1.3.4.1 for clarity by distributing the respective quantifiers. (1.3.4.1.1)

(~p&%r)+(p&#s) ; C**F**C**F** T**F**T**F** CNCN TNTN (1.3.4.1.2)

Eqs. 1.3.4.2 and 1.3.4.1.2 are *not* tautologous and *not* equivalent, refuting unification nets. To extend the unification net to additives is similarly defective.

*From: en.wikipedia.org/wiki/Drinnker_paradox [sic]

[Raymond Smullyan’s drinking principle is known as the drinker’s paradox.]

"There is someone in the pub such that, if he is drinking, then everyone in the pub is drinking." (2.0.1)

The formal statement of the theorem is, where D is an arbitrary predicate and P is an arbitrary nonempty set

$$\exists x \in P.[(D(x) \Rightarrow \forall y \in P.D(y)]. \quad (2.0.1.1)$$

Remark 2.0.1: We disagree that Eq. 2.0.1 maps to 2.0.1.1 (it is *no*t tautologous). Instead we map 2.0.1 in words as:

“If one is in the bar, then if that one in the bar is drinking, then all in the bar are drinking.” (2.0.2.1)

LET p, q, r, s: P pub, D drinking, x one, y all.

(%r<p)>(((%r<p)&q)>((#s<p)&q)) ; TTNT TT**F**T TTNT TTNT (2.0.2.2)

Remark 2.0.2.2: Eq. 2.0.2.2 is also equivalent to (%r<p)>((%r&q)>((#s<p)&q)), excluding the repetitive second “%r<p” for “%r”.

Eq. 2.0.2.2 as rendered is not tautologous, refuting the drinker’s paradox as a paradox, and forming another *non* tautologous fragment of the universal logic VŁ4.