Refutation of Kripke frames from incompleteness of BAO's with ◊⊥=⊥

Abstract: Because Kripke frames require ◊⊥=⊥, *not* tautologous, they are refuted. What follows is BAOs so defined are also refuted (which we respectively demonstrate elsewhere), namely: Jónsson-Tarski, Lemmon-Scott; Fine-Thomason, van Benthem, Boolos-Sambin, and Lindenbaum-Tarski. These results also make the Blok dichotomy suspicious. Therefore these conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ▽, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Holliday, W.H.; Litak, T. (2019). Complete additivity and modal incompleteness. arxiv.org/pdf/1809.07542.pdf tadeusz.litak@fau.de wesholliday@berkeley.edu

1 Introduction

The discovery of Kripke incompleteness, the existence of normal modal logics that are not sound and complete with respect to any class of Kripke frames, [is] called one of the two forces that gave rise to the "modern era" of modal logic … Kripke incompleteness was demonstrated with a bimodal logic … , shortly thereafter with complicated unimodal logics … , and later with simple unimodal logics. The significance of these discoveries can be viewed from several angles. From one angle, they show that Kripke frames are too blunt an instrument to characterize normal modal logics in general. More fine-grained semantic structures are needed. …

1.1 The semantic angle

The first angle on Kripke incompleteness—the realization that Kripke frames are not fine-grained enough for the study of normal modal logics in general—renewed interest in the algebraic semantics for normal modal logics based on Boolean algebras with operators (BAOs). A BAO is a Boolean algebra together with one or more unary operators, i.e., unary operation ◊ such that for all elements x, y of the algebra, ◊(x∨y)=◊x∨◊y [a trivial tautology] , and for the bottom element ⊥ of the algebra,

◊⊥=⊥. (1.1.1)

%(p@p)=(p@p) ; `NNNN NNNN NNNN NNNN` (1.1.2)

Remark 1.1.2: Eq. 1.1.2 is *not* tautologous (all `T`) , but at the nearest table result state of truthity (`N` as non-contingency).

> Every normal modal logic is sound and complete with respect to a BAO, namely, the Lindenbaum-Tarski algebra of the logic, according to a straightforward definition of when a modal formula is valid over a BAO. Kripke incompleteness can be better understood in light of the fact that Kripke frames correspond to BAOs that are complete (C), atomic (A), and completely additive (V), or CAV-BAOs.

Because Kripke frames require $\Diamond\bot=\bot$, *not* tautologous, they are refuted. What follows is BAOs so defined are also refuted (which we respectively demonstrate elsewhere), namely: Jónsson-Tarski, Lemmon-Scott; Fine-Thomason, van Benthem, Boolos-Sambin, and Lindenbaum-Tarski. These results also make the Blok dichotomy suspicious.