Goldbach Conjeture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove that any large even numbers can always be expressed as sums of two prime numbers.

N is a large even number.
We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

Let p_M be the largest prime which satisfies $p_M < N$. Let $p_m < \sqrt{N}$, and $p_{m+1} > \sqrt{N}$.

By using method of sieve of Eratosthenes up to the first m primes, p_i, i=1,...m, there are total $\kappa \geq [\prod_{i=1,...,m}(1-1/p_i)N]$ numbers of the remaining numbers less than N . Here $[a]$ means the greatest integer less than or equal to a.

Obviously $\kappa = M - m$,

These remaining numbers are also prime numbers.

let $(p_{m+i}, i = 1, 2, .., \kappa)$ be these primes less than N,

Now we use a modified Eratosthenes mothod, every time we sieve an element, x, less than N, by p_i for $i \leq m$, we check both for x=0 (mod p_i), and for x = N (mod p_i),

The remaining number is larger than $[\frac{N}{2}\prod_{1<i\leq m}(1-\frac{2}{p_i})] > [\frac{\sqrt{N}}{2}]$.

These remaining numbers are prime numbers, $(p_{m+i}, i \leq \kappa)$, except 1 is a remaining number too.

So there can be more than 3 remaining number when N is large enough, and there is at least one prime numbers p_{m+l} which is not 1, or N-1.

Since $p_{m+l} \neq N$ (mod p_i for all i=1,2,...m), the number N-p_{m+l} is also a remaining number, and a prime number, p.

We have, $N = p + p_{m+l}$.