Cousin Prime Conjeture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the cousin prime conjecture.

We use $p_i$ for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

If a primes pair $(p_m, p_{m+1})$ is a cousin prime, then it can be written as $(6k+1, 6k+5)$ for some k.

Start with the cousin prime $(7,11)$, there are five cousin primes, $(67, 71), (79, 83), (97, 101), (103, 107)$ and $(109, 113)$ in the range of $(7^2, 11^2)$.

Choose any one of them, for example $(67, 71)$, there are more than five cousin primes, $(4513, 4517), (4519, 4523), (4639, 4643), (4729, 4733), (4783, 4787), (4789, 4793), (4933, 4937), (4969, 4973)$ and $(4999, 5003)$ in the range of $(67^2, 71^2)$.

We can prove that this procedure can be repeated indefinitely, so there are infinite cousin primes.

Theorem;

If $(p_m, p_{m+1})$ is a cousin prime, then there are at least five cousin primes in the range of $(p_m^2, p_{m+1}^2)$.

By using sieve of Eratosthenes to natural numbers for number 2 and 3, the remaing numbers are following,

By pairing up every two numbers except 1 and 5, we have a sequence of pairs (A) as following,

(7, 11), (13, 17), (19, 23), (25, 29), (31, 35), (37, 41), (43, 47), (49, 53), ........... (A)

Each pair has a form of (6k+1, 6k+5), for k=1, 2, 3, 4,.......

All the cousin primes are in this pair sequence (A).

If \((p_m, p_{m+1})\) is a cousin prime,

\[p_{m+1}^2 - p_m^2 = (p_m + 4)^2 - p_m^2 = 8(p_m + 2) = 24(2k + 1),\] for some k,

There are \(\frac{p_{m+1}^2 - p_m^2}{6} = 4(2k+1)\) pairs in sequence in the pair sequence (A),

with all the numbers involved being within the range of \([p_m^2, p_{m+1}^2]\).

By seiving of the Eratosthenes for all the primes \(p_i, i = 3, 4, ..., m\), if any number of a pair is sieved out, we say the pair being sieved out.

The remaining number of the remaining pairs inside the range of \([p_m^2, p_{m+1}^2]\) is larger than,

\[4(2k+1) \prod_{2 < i \leq m} \left(1 - \frac{2}{p_i}\right) = 4(2k + 1) \left(\frac{p_{m-2}}{p_m}\right) \prod_{3 < i \leq m} \left(\frac{p_{i-2}}{p_i-1}\right) > \frac{12(2k+1)}{6k+1} > 4.\]

So there are at least five cousin primes in the range \([p_m^2, p_{m+1}^2]\).