Cousin Prime Conjeture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the cousin prime conjecture.

We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

If a primes pair (p_m, p_{m+1}) is a cousin prime, then it can be written as (6k+1, 6k+5) for some k.

Start with the cousin prime (7,11), there are five cousin primes, (67, 71), (79, 83), (97, 101), (103, 107) and (109, 113) in the range of $(7^2, 11^2)$.

Choose any one of them, for example (67, 71), there are more than five cousin primes, (4513, 4517), (4519, 4523), (4639, 4643), (4729, 4733), (4783, 4787), (4789,4793), (4933, 4937), (4969, 4973) and (4999, 5003) in the range of $(67^2, 71^2)$.

We can prove that this procedure can be repeated indefinitely, so there are infinite cousin primes.

Theorem;

If (p_m, p_{m+1}) is a cousin prime, then there are at least five cousin primes in the range of (p_m^2, p_{m+1}^2).

By using sieve of Eratosthenes to natural numbers for number 2 and 3, the remaing numbers are following, 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 47, 49, 53, 55, 59, 61, 65, 67, 71, ...

By pairing up every two numbers except 1 and 5, we have a sequence of pairs (A) as following,

(7, 11), (13, 17), (19, 23), (25, 29), (31, 35), (37, 41), (43, 47), (49, 53), ... (A)

Each pair has a form of (6k+1, 6k+5), for k=1, 2, 3, 4,......

All the cousin primes are in this pair sequence (A).

If (p_m, p_{m+1}) is a cousin prime,

$p_{m+1}^2 - p_m^2 = (p_m + 4)^2 - p_m^2 = 8(p_m + 2) = 24(2k + 1)$, for some k,

There are $\frac{p_{m+1}^2 - p_m^2}{6}$ = 4 (2k+1) pairs in sequence in the pair sequence (A),

with all the numbers involved being within the range of $(p_m^2, p_{m+1}^2]$.

By seiving of the Eratosthenes for all the primes (p_i, i = 3,4,...,m), if any number of a pair is sieved out, we say the pair being sieved out.

The remaining number of the remaining pairs inside the range

of (p_m^2, p_{m+1}^2) is larger than,

4(2k+1) $\prod_{2<i\leq m}(1-\frac{2}{p_i}) = 4(2k+1)(\frac{p_3-2}{p_m})\prod_{3<i\leq m}(\frac{p_i-2}{p_{i-1}}) > \frac{12(2k+1)}{6k+1} > 4.$

So there are at least five cousin primes in the range (p_m^2, p_{m+1}^2).