Infinite number of Fibonacci and Lucas primes

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If we do the summation between two Fibonacci numbers as follows:

\[
\sum_{n=F(x-1)}^{F(x+1)} \cdot n
\]

We arrive sooner or later to obtain a Lucas prime in the prime factors of that sum.

For example when we do the summation of \((n)\) between \(F(18)\) to \(F(20)\) the largest factor of this sum is the Lucas (19), which is prime.

I will not expand with more examples just to emphasize that I have verified all the numbers that I have been able to calculate by this method. The last Lucas prime that I have found was Lucas (313), which is the biggest prime factor of the result of the summation between Fibonacci (312) to Fibonacci (313)
What I try to prove is that if there are infinite number of Fibonacci and infinitely many prime numbers it is logical to think that when we do the summation with higher Fibonacci numbers in the way described above, a new prime factor greater than the previous prime factor will appear.

And it seems that these new and bigger prime factor than the previous biggest prime is allways a new Lucas prime.

So there would be infinite number of Lucas primes.

The reciprocal is true if we do the same operation with Lucas numbers we get bigger Fibonacci primes.

A mathematical expression is:
If $p$ is the biggest factor of all previous sums the next factor $(p + 1)$ that fulfill the inequation $(p + 1) > p$ is a Lucas prime or a Fibonacci prime if we use numbers of Lucas in the sums.

Definitely this limit tends to Infinite number of Fibonacci and Lucas primes in the factors of these sums.