Refutation of Π_1^- of self-verifying axioms for grounding functions and relation predicates

Abstract: We evaluate Π_1^- self-verifying axioms defining the grounding functions and the relation predicates as *not* tautologous. This refutes the approach for weak axiom systems to use subtraction and division primitives, rather than addition and multiplication, to encode formally theorems of arithmetic. Therefore the conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Willard, D.E. (2001). Self-verifying axiom systems, the incompleteness theorem and related reflection principles. pdfs.semanticscholar.org/c278/147b7a68385836a90939a175a9959cabbf0b.pdf

Abstract We will study several weak axiom systems that use the Subtraction and Division primitives (rather than Addition and Multiplication) to formally encode the theorems of Arithmetic ...

Table I: List of Π_1^- axioms defining the grounding functions and the relation predicates

3.1 ∀x ∀y ∀z { x = y ∧ y = z } ⊃ x = z

((#p=#q)&(#q=#r))>(#p=#r) ; TNTN NTNT TNTN NTNT (3.2)

6.1 ∀x ∀y ∀a ∀b { x − y = a − b ∧ y = b } ⊃ x = a

(((#p-#q)=(#r-#s))&(#q=#s))>(#p=#r) ;
TTTT TTTT TTTC TTCT (6.2)

10.1 ∀x ¬x < x

~p<p ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (10.2)

16.1 ∀x x − 0 = x

(#p-(p@p))=p ; **F**C**F**C **F**C**F**C **F**C**F**C **F**C**F**C (16.2)

18.1 ∀x ∀y x < y ⊃ x/y = 0

(#p<#q)>((#p\#q)=(p@p)) ; TCTT TCTT TCTT TCTT (18.2)

19.1 $\forall x\ x/0 = x/1 = x$

((#p\(p@p))=(#p\(%p>#p)))=(p@p) ;**F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N (19.2)

20.1 $\forall x\ \forall y\ x \geq y \geq 1 \supset [\ x/y > 0 \wedge x/y - 1 = x - y/y]$

~(~((%p>#p)>#q)>#p)>((((#p\#q)>(p@p))&((#p\#q)=(%p>#q)))=((#p-#q)\#q)) ;
CTTT CTTT CTTT CTTT (20.2)

Seven axioms are *not* tautologous. This refutes Π_1^- self-verifying axioms defining the grounding functions and the relation predicates.