Refutation of $\Pi_1^-$ of self-verifying axioms for grounding functions and relation predicates

© Copyright 2019 by Colin James III   All rights reserved.

Abstract: We evaluate $\Pi_1^-$ self-verifying axioms defining the grounding functions and the relation predicates as not tautologous. This refutes the approach for weak axiom systems to use subtraction and division primitives, rather than addition and multiplication, to encode formally theorems of arithmetic. Therefore the conjectures form a non tautologous fragment of the universal logic $V\mathcal{L}4$.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, $\mathcal{F}$ as contradiction, $\mathcal{N}$ as truthity (non-contingency), and $\mathcal{C}$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Table I: List of $\Pi_1^-$ axioms defining the grounding functions and the relation predicates

3.1 $\forall x \forall y \forall z \{ x = y \land y = z \} \supset x = z$

$$((\#p=\#q)\&(\#q=\#r))>(\#p=\#r) ;$$

TNTN NTNT TNTN TNTN (3.2)

6.1 $\forall x \forall y \forall a \forall b \{ x - y = a - b \land y = b \} \supset x = a$

$$(((\#p-\#q)=(\#r-\#s))\&(\#q=\#s))>(\#p=\#r) ;$$

TTTT TTTT TTTC TTCT (6.2)

10.1 $\forall x \neg x < x$

$$\neg p < p ;$$

TFTF TFTF TFTF TFTF (10.2)

16.1 $\forall x x - 0 = x$

$$(\#p-(p@p))=p ;$$

FCFC FCFC FCFC FCFC (16.2)

18.1 $\forall x \forall y x < y \supset x/y = 0$

$$((\#p\#q>((\#p\#q)=(p@p))) ;$$

TCTT TCTT TCTT TCTT (18.2)

From: Willard, D.E. (2001). Self-verifying axiom systems, the incompleteness theorem and related reflection principles. pdfs.semanticscholar.org/c278/147b7a68385836a90939a175a9959cabbf0b.pdf

Abstract: We will study several weak axiom systems that use the Subtraction and Division primitives (rather than Addition and Multiplication) to formally encode the theorems of Arithmetic ...
19.1 $\forall x \frac{x}{x/0} = x/1 = x$

\[
((\#p(p@p)) = (\#p(\%p>\#p))) = (p@p) \text{ ; F F F F F F F F F F F F F F F F (19.2)}
\]

20.1 $\forall x \forall y x \geq y \geq 1 \supset [x/y > 0 \land x/y - 1 = x-y/y]$

\[
\sim((-(%p>\#p)>\#q)>\#p) > (((\#p\#q)>(p@p)) \& ((\#p\#q) = (%p>\#q)) = ((\#p\#q)\#q)) \text{ ; C T T T C T T T C T T T (20.2)}
\]

Seven axioms are not tautologous. This refutes $\Pi_1^{-}$ self-verifying axioms defining the grounding functions and the relation predicates.