Who needs Yukawa's wave equation?

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Abstract

One can think of a wave equation for the nucleus based on the Yukawa potential. It is a natural thing to do from a mathematical point of view. This paper is a didactic exploration of the physical rationale for such wave equation. We relate it to earlier discussions on an oscillator model for the nucleus.

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Introduction

In our previous paper[1], we re-wrote Yukawa's potential function for the nuclear force – a hypothetical force that is supposed to hold nucleons together – introducing a new permittivity factor υ_0, about which we will soon say more:

$$\mathrm{U_N}(r) = -\frac{\mathrm{g_N^2}}{4\pi\upsilon_0}\cdot\frac{1}{r}$$

$$\upsilon_0 = e^{-\frac{r}{a_\mathrm{N}}}\cdot\frac{\mathrm{Y}^2}{\mathrm{N\cdot m^2}}$$

The a_N factor in υ_0 is Yukawa's parameter for the *range* of the strong force, and you can just substitute υ_0 in the U_N formula to get Yukawa's formula in the format that will be more familiar to you: $U_N(r) = -g_N2 \cdot e^{-r/a}/4\pi r$. The reason why we re-write Yukawa's formula is because we want to think through the physical dimensions here. Why do we want to do that? Because we want to think through the physics. We will be analyzing various aspects but let us, indeed, start with a reflection on physical dimensions.

Before we do so, we should probably remind you of the *interpretation* of Yukawa's formula. It is really just the same formula as the one we know for the Coulomb force:

$$\mathrm{V}(r) = -\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r} = -\mathrm{e}^2\frac{1}{r}$$

We just have a *new* charge – g_N instead of q_e – and, of course, we also have the $e^{-r/a}$ factor, so we no longer have that easy (inverse) proportionality between the potential and the distance:

$$\mathrm{V(r)} \propto 1/r$$

That is why Yukawa inserted that $e^{-r/a}$ function. To conclude this quick intuitive explanation, we may quickly want to think about the minus sign of the potential. Do we have positive and negative *nucleon* charges here? No. We do not. The minus sign is there because of the convention that the force *attracts* and so it is like the potential of a gravitational field: two masses will *attract* each other. So that's another reason why this g_N is different from the electric charge. It is just some *positive* real number: no plus or minus. And it's a charge that's common to both protons and neutrons. To make sure you understand this correct, we'll quote Aitchison and Hey on this: "The U(r) potential is the (mutual) potential energy of one point-like nucleon of 'strong charge' g_N due to the presence of another point-like nucleon of *equal* charge g_N at the origin, a distance r away."[2]

[1] Jean Louis Van Belle, *The Nature of Yukawa's Nuclear Force and Charge*, 19 June 2019 (http://vixra.org/abs/1906.0311).
[2] Aitchison and Hey, *Gauge Theories in Particle Physics*, 2013, Vol. I, p. 16.

OK. Let us now think about the physical dimensions in Yukawa's formula.

Physical dimensions

Our new permittivity factor (υ_0), which we will refer to as the *nuclear* permittivity, differs from the *electric* permittivity (ε_0) in two ways:

1. As a physical proportionality constant, it needs to ensure the physical dimension left and right comes out alright. We referred to the unit of nuclear charge (g_N) as the *Yukawa* (Y). Why do we need it? You may think of it as some kind of placeholder name for the time being but the idea is quite fundamental: if you introduce a new force – which is what Yukawa did in his seminal 1935 paper – then you also need to introduce a new charge, and so that's what we're doing here. The numerical value of υ_0 is $e^{-r/a}$ but its physical dimension is $Y^2/N{\cdot}m^2$. We just replace the C^2 factor from the $[\varepsilon_0] = C^2/N{\cdot}m^2$ equation.

2. The nuclear permittivity varies with the distance. That's weird but it is the price we have to pay for the idea of the Yukawa force. I will come back to this. As for now, you just have to swallow this. You should note, of course, that υ_0 is equal to 1 if $r = a_N$, but what's a_N? Some range parameter, which we can determine *empirically*, but what's the theory here? Again, just hold this for a while and we will soon come back to this.

Now that we're here, we should probably say a few words about the redefinition of SI units that came into force just recently—on 20 May 2019, to be precise! Why? Because it involved the electric constant – and some others.

The 2019 redefinition of SI units and the *Zitterbewegung* model

The 2019 redefinition of SI units involves an exact *definition* of the electron charge: q_e is now defined as being equal to $q_e = 1.602176634{\times}10^{-19}$ C, *exactly*. It's not being *measured* anymore: we *define* it as the mentioned fraction ($1.602176634{\times}10^{-19}$) of the *coulomb* charge. Nothing more. Nothing less.[3]

What's measured in labs is the *magnetic moment* of an electron. Labs do this in a one-electron cyclotron—Penning trap[4], which combines magnetic and electric fields to store one single charged particle, so that's one electron in this case. We know the magnetic moment of an electron – of any *pointlike* charged particle, really – is slightly off its theoretical value, and the anomaly is measured in terms of the fine-structure constant (α). Of course, physicists also have a *theory* for the anomaly: quantum field theory. To be precise, most of the difference (about 99.85%) is given by Schwinger's $\alpha/2\pi$ factor. Schwinger's is a first-order correction which he gets from calculating "the one loop electron vertex function in an external magnetic field using his renormalized QED."[5]

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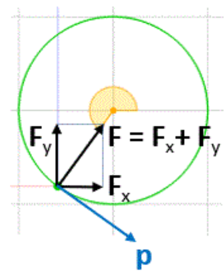
[3] If you are a philosopher, you may say: what's the point? We just kicked the can down the road, didn't we? We have an exact fraction now but what *is* a coulomb? The answer is: a coulomb is a coulomb. It is the unit of electric charge. This answer is good enough for physicists – for whom these discussions actually matter – so it should be good enough for everyone else.

[4] The Wikipedia article on it (https://en.wikipedia.org/wiki/Penning_trap) offers a good and easy read on this.

[5] We are quoting from Ivan Todorov's excellent overview of the matter here. See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/abs/1804.09553v2).

We have a simpler geometric explanation: we think of the fine-structure constant as the *radius* of the naked charge in the *Zitterbewegung* model of the electron.[6] Because we will use a similar model for the nucleon, it is good to briefly recall the basics of the model. Figure 1 illustrates the idea. We have a centripetal force (F) holding a naked charge – something with *zero* rest mass – in a circular orbit around some center.

Figure 1: The *Zitterbewegung* model of an electron



Because the naked charge goes around at the speed of light (or *almost* the speed of light, as we will argue later), it acquires some mass which we'll denote as m_γ. We use the γ subscript here because the *zbw* charge does behave like a photon here: it acquires relativistic mass because of its extreme velocity. The only thing is that our *zbw* charge also has electric charge (all of the charge of the electron, in fact), which a photon doesn't have, of course! Its relativistic mass also gives it some non-zero momentum $p = m_\gamma v = \gamma m_0 v = \gamma m_0 c$, even if m_0 (i.e. the rest mass of the naked charge) is zero.

What's the nature of the centripetal force? It is electromagnetic: think of a perpetual current in a superconductor. We cannot dwell on this here. The point is: this geometry explains a magnetic moment which we can calculate as being equal to:

$$\mu = \mathrm{I} \cdot \pi r_C{}^2 = \frac{q_e}{2m}\hbar$$

This model also allows us to calculate the angular momentum using a classical (but relativistically correct) formula:

$$\mathrm{L} = I \cdot \omega = m_\gamma \cdot a^2 \cdot \omega = \frac{m_e}{2} \cdot a^2 \cdot \omega = \frac{m_e}{2} \cdot \frac{\hbar^2}{m_e^2 \cdot c^2} \frac{\mathrm{E}}{\hbar} = \frac{\hbar}{2}$$

This, then, gives us the theoretical gyromagnetic ratio of an electron which, as you know, we express in terms of the Bohr magneton $q_e/2m$:

$$g_e = \frac{\mu}{\mathrm{L}} = \frac{q_e \hbar}{2m_e} \frac{2}{\hbar} = 2 \cdot \frac{q_e}{2m_e}$$

Hence, the *theoretical* g-ratio of an electron in free space is equal to two: *two* units of $q_e/2m$, that is. We call an electron in free space (no potential) a spin-only electron so as to distinguish it from an electron in

[6] See: Jean Louis Van Belle, *The Anomalous Magnetic Moment: Classical Calculations*, 11 June 2019 (http://vixra.org/abs/1906.0007). *Zitter* is German for shaking or trembling. It refers to a presumed local oscillatory motion which Erwin Schrödinger stumbled upon when he was exploring solutions to Dirac's wave equation for free electrons. We are not shy about it: we believe this motion to be real. Why? Because it explains a lot—an *awful* lot! And it does so without *hocus-pocus*! No black-box theory. No inexplicable rules. No weird theorems. Just a simple *form factor*.

an atomic orbital or an electron that is in some orbit because of an applied electromagnetic field. As for the latter, you should note we have such field in a Penning trap. The formulas for an orbital electron are similar but incorporate the orbital number *n*:

$$\mu_n = \mathrm{I} \cdot \pi r_n^{\ 2} = \frac{\mathrm{q_e}}{2\mathrm{m_e}} n\hbar$$

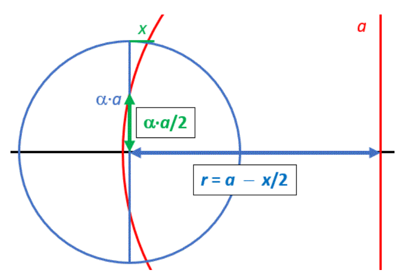
$$\mathrm{L}_n = I \cdot \omega_n = n\hbar$$

$$\mathrm{g}_n = \frac{\mu}{\mathrm{L}} = \frac{\mathrm{q_e}}{2\mathrm{m_e}} \frac{n\hbar}{n\hbar} = 1 \cdot \frac{\mathrm{m}}{2\mathrm{q_e}}$$

Hence, for an orbital electron, we find a g-ratio that is equal to one: *one* units of $q_e/2m$, that is.[7] However, let us get back to the matter at hand—literally. The point is: the empirical or experimental value differs from this theoretical value, and (most of) the difference is given by Schwinger's $\alpha/2\pi$ factor. To be precise, Schwinger's is a first-order correction, which explains about 99.85% of the difference. Higher-order corrections are supposed to explain the rest. As mentioned above, Schwinger gets his $\alpha/2\pi$ factor by calculating "the one loop electron vertex function in an external magnetic field using his renormalized QED."

We get an $\alpha/8$ factor from a *very* approximative geometric argument and we think the missing $4/\pi$ may be explained by precession.[8] In short, we think our geometric argument makes an awful lot of sense. It is based on the idea that the pointlike charge has some radius itself, and that this radius is a fraction of the radius of the electron, which is nothing but the Compton radius $a_e = \hbar/mc$. Figure 2 illustrates the idea: if we think of the *Zitterbewegung* (zbw) charge as a tiny sphere, then the radius of its orbital or oscillatory motion – think of it as the *effective* Compton radius of the electron – will have to be slightly smaller than what it would be if the *zbw* charge was really nothing but a point with zero radius. If not, we'd have a proportion of charge that is *larger* than 1/2 going *faster* than the speed of light.[9]

Figure 2: Geometry of *zbw* charge and electron



Why do we think the *zbw* charge would have some size? There are two reasons for that. The first reason is philosophical—or logical, we'd say. We think a pointlike object does not make any sense: if it has some property – even if it is just some *charge* without any rest mass – then it will have some dimension. An object whose dimension is zero is just plain nothingness: it is a *mathematical* point only.

[7] For the detail of the calculations, and the *rationale* of the model, see: Jean Louis Van Belle, *The Electron as a Harmonic Electromagnetic Oscillator,* 31 May 2019 (http://vixra.org/abs/1905.0521).
[8] Jean Louis Van Belle, *The Anomalous Magnetic Moment: Classical Calculations*, 11 June 2019 (http://vixra.org/abs/1906.0007).
[9] We effectively think of the *zbw* charge here as a tiny sphere of charge, and we assume the charge *density* is the same everywhere.

The second reason is empirical: in elastic scattering experiments (Thomson scattering), the (low-energy) photons seem to bounce off some hard *core* – and we think that's the pointlike charge.[10] The explanation is consistent with experiment, because the Thomson radius is measured as a fraction – the fine-structure constant – of the Compton radius:

$$r_{\mathrm{e}} = \frac{\mathrm{e}^2}{\mathrm{m}c^2} = \alpha \cdot a = \alpha \frac{\hbar}{\mathrm{m}c} \approx 2.818 \ldots \times 10^{-15}\ \mathrm{m}$$

In any case, we don't want to spend too much time and space on this.[11] The point here is that there is, effectively, some easy geometric explanation (a *physical* interpretation, that is) for the quantum-mechanical formula that you have seen many times but probably never quite *understood*:

$$\alpha = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0 \hbar c} = \frac{\mathrm{e}^2}{\hbar c}$$

You should just note that – since the 2019 redefinition of SI units – we think of the electric charge (and the speed of light) as *exact* numbers: $q_e = 1.602176634\times10^{-19}$ C, *exactly*, and c = 299,792,458 m/s, and so that's equally *exactly*. Now, Maxwell's equations tell us that the magnetic and electric constant are related through c^2:

$$c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

In case you haven't seen this expression before, just take it as a fact of the world and check the dimensions: ε_0 is expressed in $\mathrm{C^2/N{\cdot}m^2}$ while μ_0 is expressed in $\mathrm{N/A^2 = N{\cdot}s^2/C^2}$, so $[\varepsilon_0\mu_0] = \mathrm{s^2/m^2}$. It's one of those relations that can be proved easily mathematically but have a profound physical meaning that can*not* be explained very easily. But it's not impossible. Let's first establish *equivalent* time and distance units so the *numerical* value of the speed of light is equal to 1. We can do this in various ways, but one intuitive one is to measure distance in light-seconds: a light-second is the distance that a photon or, if you don't want to talk about photons because you're not sure what they are, plain light[12] would cover in one second: one light-second is, obviously, 299,792,458 m, *exactly*. Why? Because the speed of light is an *exact* physical constant: think of light – as an electromagnetic oscillation – *defining* the time and distance units.[13] So what happens if we replace the *m* in the $\varepsilon_0 = 8.8541878128(13)\times10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$ formula one light-second? One *meter* obviously corresponds to $1/c$ light-seconds. Hence, the numerical value of ε_0 will have to change too if we're going to use light-second as the distance unit. To make a long story short, we will write as:

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[10] We do *not* have that when there is inelastic scattering: Compton scattering. Compton scattering occurs when we use high-energy photons (X or gamma rays): the photon is briefly absorbed, and a new photon is emitted – with a different energy (a longer wavelength). The difference in energy gives the electron some *kinetic* energy: it will accelerate and change direction.

[11] For a more elaborate *exposé*, see: Jean Louis Van Belle, *The Nature of Yukawa's Charge and Force*, 19 June 2019 http://vixra.org/abs/1906.0311.

[12] The Philosopher would say: "If we don't know what photons are, *exactly*, then we don't really know what light is." Both concepts are as precise or as imprecise as our definition of it. The idea of light traveling at the speed of light assumes some light *particle* that we can track. We can't measure the speed of a continuous beam. We may not know much about photons but, for all practical purposes, it is the light particle.

[13] This sounds brutal – or even weird – but it's not as brutal or weird as you might think at first.

$$\varepsilon_0 = [8.8541878128(13)\times 299{,}792{,}458^2]\times 10^{-12}\ \mathrm{C^2/N{\cdot}s^2} \approx 0.796\times 10^6\ \mathrm{C^2/N{\cdot}s^2}$$

We can now rephrase or recalculate the relation between the electric and magnetic constant as:

$$c^2 = 1^2 = \frac{1}{\varepsilon_0\mu_0} = \frac{1}{\left(0.796\ldots\times 10^6\,\dfrac{\mathrm{C}^2}{\mathrm{N\cdot s^2}}\right)\left(1.256\ldots\times 10^{-6}\,\dfrac{\mathrm{N\cdot s^2}}{\mathrm{C}^2}\right)} = 1$$

The manipulations above show that – when using natural time and distance units – ε_0 and μ_0 are just each other's reciprocal (1/1.256… = 0.796…). Any case, that's *not* what we want to discuss here, even if we do want you to think about it. What do we want to discuss here? We wanted to link the α, μ_0 and ε_0 constants.

You should check on (1) the CODATA values for α, μ_0 and ε_0, (2) their (relative) standard uncertainty (think of it as the standard deviation of the measurements) and (3) the correlation coefficient for these three physical constants.[14] You will find the results below:

Figure 3: CODATA values and correlation of fine-structure, electric and magnetic constants

	Empirical value	**Relative uncertainty**	**Correlation coefficient**
Fine-structure constant (α)	$7.2973525693(11)\times 10^{-3}$	1.5×10^{-15}	1 (with ε_0 and μ_0)
Electric constant (ε_0)	$8.8541878128(13)\times 10^{-12}$ C²/N·m²	1.5×10^{-15}	1 (with α and μ_0)
Magnetic constant (μ_0)	$1.25663706212(19)\times 10^{-6}$ N/A²	1.5×10^{-15}	1 (with α and ε_0)

Note that the electric and magnetic constant are also referred to as the (electric) *permittivity* and (magnetic) *permeability* of the vacuum (i.e. free space). That makes sense if we think of them as some kind of *force* per unit of flux, in space or in time. For ε_0 (for $1/\varepsilon_0$, we should say), we have the newton per $\mathrm{C^2/m^2}$ unit: the denominator is (electric) charge per meter, squared. For μ_0, we have newton per $\mathrm{C^2/s^2}$: charge per second, squared. The ratio of $1/\varepsilon_0$ and μ_0 gives us the speed of light, squared: $1/\varepsilon_0\mu_0 = c^2$. It is tough to understand this *intuitively*, but not impossible.

The point here is the following: fine-structure constant, magnetic constant and electric constant are all related, and we think the relation is given by the sheer geometry of the situation. Now that we are here, we should probably add a remark here. We said that the charge of an electron (q_e) has been defined as $q_e = 1.602176634\times 10^{-19}$ C, *exactly*. You probably also know that Planck's constant has also been defined as being equal to $h = 6.62607015\times 10^{-34}$ J·s, *exactly*.[15] Hence, you may think that we should think of $\alpha = e^2/\hbar c$ as some *exact* value now—because of the following formula:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0\hbar c} = \frac{e^2}{\hbar c}$$

[14] CODATA is the Committee on Data of the International Science Council and publishes this data (http://www.codata.org). However, I find the US NIST site (https://physics.nist.gov/cuu/Constants/index.html) more user-friendly. Note that the electric and magnetic constant are referred to

[15] The NIST defines the unit as J·Hz^{-1}, which confirms our interpretation of Planck's constant as a fundamental cycle. Note that the *reduced* Planck constant ($\hbar = h/2\pi = 1.054\ 571\ 817\ldots\times 10^{-34}$ J·s) has an infinite number of digits but zero uncertainty. That is because π is an irrational number.

But, no! The e^2 and q_e^2 are related through ε_0 and, hence, the uncertainty about ε_0 will be reflected in an uncertainty about e^2 and, therefore, in the fine-structure constant α.

Let us show one more thing before we move on. We can use the $1/c^2 = \mu_0\varepsilon_0$ to write α as a function of μ_0:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0\hbar c} = \frac{q_e^2\mu_0 c^2}{4\pi\hbar c} = \frac{q_e^2\mu_0 c}{4\pi\hbar}$$

This just shows – once again – how the three constants and, therefore, their uncertainties, are related.

This leads us to another discussion: if α is a fraction of that Compton radius, then what *is* that Compton radius?

The Compton radius of an electron

In our classical interpretation of what an electron (and a photon) might actually *be*[16], we equated the Compton radius of an electron to the radius of Schrödinger's *Zitterbewegung*. Our assumption – the electron as a naked charge – something pointlike with zero rest mass[17] - moving about some center at the speed of light. It can do so because its rest mass is zero. The rest mass of the electron itself is nothing but the *equivalent* mass of the energy in this oscillatory motion: Wheeler's idea of mass without mass. This led us to interpret the speed of light – the c in Einstein's mass-energy equivalence relation ($E = m\cdot c^2$) – as a tangential velocity: $c = a\cdot\omega$.[18] We then used the Planck-Einstein relation ($\omega = E/\hbar = m\cdot c^2/\hbar$) to find the Compton radius:

$$a = \frac{c}{\omega} = \frac{c\cdot\hbar}{m\cdot c^2} = \frac{\hbar}{m\cdot c} = \frac{\lambda_C}{2\pi} \approx 0.386\times10^{-12}\ \mathrm{m}$$

The novel idea here is that one rotation – one *cycle* of the electron in its *Zitterbewegung* – does not only pack the electron's energy ($E = m\cdot c^2$): it also packs Planck's quantum of action ($S = h$). The idea of an oscillation packing some amount of physical action may not be very familiar but it is quite simple: physical action is the product of (1) a force (the force that keeps our *zbw* charge in its circular orbit), (2) some distance (the circular loop) and (3) some time (the cycle time). For an electron, we got a cycle time that was equal to:

$$T = \frac{h}{E} \approx \frac{6.626\times10^{-34}\ \mathrm{J\cdot s}}{8.187\times10^{-14}\ \mathrm{J}} \approx 0.8\times10^{-20}\ \mathrm{s}$$

We can also calculate the electric current:

$$I = q_e f = q_e\frac{E}{h} \approx (1.6\times10^{-19}\ \mathrm{C})\frac{8.187\times10^{-14}\ \mathrm{J}}{6.626\times10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}$$

This looks phenomenal – a household-level current (almost 2 *ampere*) at the sub-atomic scale but, as mentioned above, the model gives us consistent values for the magnetic moment, the angular

[16] See the above-mentioned papers.

[17] Pointlike does not imply it has no dimension whatsoever. We think of the *classical* electron radius as the radius of the zero-mass *Zitterbewegung* charge. Hence, the fine-structure constant ($r_e = \alpha\cdot r_C = \alpha\hbar/mc$) relates the two radii. As mentioned above, this explains the small $\alpha/2\pi$ anomaly in the magnetic moment.

[18] A tangential velocity will always equal the radius times the angular frequency of the rotational motion.

momentum of an electron and the g-ratio, so we see no reason why we wouldn't try to roll with this. The obvious question here is the following: what keeps this photon-like charge in its orbit? What is the *nature* of the centripetal force?[19] We think we have answered this question in our papers – it is the electromagnetic force itself (again, think of a current ring in a superconducting material[20]) – and so we won't dwell on it here. We just wanted to recap the basics of our oscillator model for an electron in order to now apply it to the nucleon.

The Compton radius of protons and neutrons

We have a new charge here – the Yukawa charge – and there is no reason why we wouldn't treat it as a pointlike charge with zero rest mass. In other words, we can apply the same reasoning as the one we used to calculate the Compton radius of an electron. Because the model may not be familiar, we repeat the logic once more:

1. We think of the Yukawa charge as a pointlike charge with zero rest mass. Pointlike does *not* mean it's dimensionless. In fact, we will want to think of it as some sphere of charge, and we will probably also want to develop some theory for the radius of the charge—something like our explanation of the anomalous magnetic moment of the electron in terms of the radius of the naked charge.

2. Because its rest mass is zero, the Yukawa charge will move at the speed of light. We think this motion is some *Zitterbewegung*—something like the *Zitterbewegung* of an electron. We just have a different charge here and, therefore, we also have a different mass and a different radius. However, the $E = m \cdot c^2$ and tangential velocity formula can be combined and give us the $E = m_N \cdot a_N^2 \cdot c^2$ equation. The m_N and a_N are the nucleon mass and the radius or *amplitude* of the oscillation.

3. We think one *cycle* of this nucleon (or nucle*ar*?) oscillation packs (1) the energy of the nucleon and (2) one unit of physical action. Yes. We are talking Planck's quantum of action here. Why? The reason is simple: none of the theoretical developments so far, and none of the experiments so far, give us any reason to try to use some other quantum in the Planck-Einstein relation. Hence, we think the $\omega = E/\hbar = m \cdot c^2/\hbar$ is valid in the nucleus too.

This gives us the Compton radius for an electron, for a nucleon, or for any particle we might associate with some new charge—electric, nuclear or whatever one might come up with (colors, perhaps?):

[19] Alexander Burinskii, a Russian physicist who specializes in *physical* electron models, wrote me the following when I first contacted him back in December 2018: "I know many people who considered the electron as a toroidal photon and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: "Microgeons with spin". Editor E. Lifschitz prohibited me then to write there about Zitterbewegung [because of ideological reasons], but there is a remnant on this notion. There was also this key problem: what keeps [the pointlike charge] in its circular orbit?"

[20] We do not have a *material* ring to guide the electron here – no little wire – but we believe the scale and current are such that all is kept in place. We refer to Burinskii's and Hestenes research. Burinskii integrates gravity in his Kerr-Newman geometries. As for David Hestenes, his calculations revived the *Zitterbewegung* interpretation of an electron in the 1980s and early 1990s and he will, therefore, forever be associated with the so-called *Zitterbewegung* interpretation of quantum mechanics.

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{\mathrm{m} \cdot c^2} = \frac{\hbar}{\mathrm{m} \cdot c}$$

If we try the mass of a proton (or a neutron—almost the same), we get this:

$$a_{\mathrm{p}} = \frac{\hbar}{\mathrm{m_p} \cdot c} = \frac{\hbar}{\mathrm{E_p}/c} = \frac{(6.582 \times 10^{-16}\ \mathrm{eV} \cdot s) \cdot (3 \times 10^{8}\ m/s)}{938 \times 10^{6}\ \mathrm{eV}} \approx 0.21 \times 10^{-15}\ \mathrm{m}$$

The order of magnitude is right, because we get a radius between 0.84 and 0.9 fm out of proton scattering experiments.[21] But a factor of the order of 1/4 is, perhaps, a bit much. To put things into perspective, we may remind ourselves that the order of magnitude of the *Compton* radius of the proton that we have calculated here is the same as the order of magnitude of the *Thomson* radius of an electron, i.e. the radius of the *Zitterbewegung* charge: $r_e = \alpha \cdot r_C \approx 2.8$ fm. However, while the order of magnitude is the same, we should also note our theoretical proton radius – the Compton radius above: 0.21 fm – is about 13.5 times *smaller* than the classical electron radius.

Why are we mentioning this? We might be tempted to think of a proton as a neutron with an added positron (the anti-matter counterpart of an electron) or, vice versa, of a neutron as a proton with an added electron[22], but the calculation above shows that doesn't make much sense: our proton, or our neutron, should be much larger then! Even if we accept the empirical value (0.84 and 0.9 fm) – which we should accept, of course, because that's what we measure ! – then the two radii (proton or neutron radius versus the classical electron radius) differ by a factor that is close to π. Indeed, $2.8/\pi \approx 0.89$. It must be a coincidence, right?

The Compton radius of quarks

What if we try the mass of quarks? Which ones? We have six: *u*, *d*, *c*, *s*, *t* and *b* quarks. We will probably want to limit ourselves to the first generation, because the second and third might be explained by some resonance or higher (non-equilibrium?) energy states. The *u* and *d* quarks[23] have an equivalent mass of 2.3 ± 0.7 ± 0.5 and 4.8 ± 0.5 ± 0.3 MeV/c^2 respectively. The weird $x \pm \sigma_1 \pm \sigma_2$ expression accounts for statistical versus systematic uncertainty respectively.[24] These are very imprecise numbers—which is one of the many reasons why I don't like quark theory but that is entirely subjective,

[21] We refer to Wikipedia once again for a very readable account of these experiments and their results (https://en.wikipedia.org/wiki/Proton_radius_puzzle).

[22] The mass of a neutron is about 939,565,413 eV/c^2 and about 938,272,081 eV/c^2 for the proton. Hence, the energy *difference* is almost 1.3 MeV. That's quite considerable when thinking photons or other low-energy phenomena. Everything is relative, of course! We need to do some thinking about that difference. It is about the same as the upper limit of the energy of a gluon, which is thought of as ranging between 0 and 1.3 MeV/c^2. Also note that the neutron mass is the larger one. Perhaps it absorbed an electron? The electron mass is about 0.511 MeV/c^2. So one might add some oscillation energy perhaps to explain the remaining difference? These thoughts are, obviously, very speculative and, as mentioned, we get into trouble when thinking of the *size* of an electron.

[23] I love the idea of quarks and the analogy with *flavors* and – for gluons – *colors*, but their names: quarks: *up*, *down*, *strange*, *charm*, *bottom*, and *top*? Up/down and bottom/top sounds to similar and so that's quite confusing. As for the distinction between strange and charm, *you* tell *me*!

[24] For the difference between the two: https://en.wikipedia.org/wiki/Systematic_error. We also took the mentioned mass estimates for the *u* and *d* quark from the Wikipedia article on quarks. It may be mentioned that these values have not yet made it into the NIST tables of fundamental physical constants – unlike, say, the mass of neutrons, protons, alpha particles, and the most common leptons (electron, tau, etcetera). You can check for yourself: https://physics.nist.gov/cuu/Constants/Table/allascii.txt. It's a long list, so I might have made a mistake!

of course. Let's get back to the point here. If these $\pm\,\sigma_1 \pm \sigma_2$ numbers are, effectively, standard deviations from the mean, then we may say the mass of quarks varies between 0 and 7.2 MeV/c^2.[25] However, let us take the 2.3 and 4.8 values to get some idea of the order of magnitude:

$$a_u = \frac{\hbar}{\mathrm{m_p} \cdot c} = \frac{\hbar}{\mathrm{E_p}/c} = \frac{(6.582 \times 10^{-16}\ \mathrm{eV} \cdot s) \cdot (3 \times 10^{8}\ m/s)}{2.3 \times 10^{6}\ \mathrm{eV}} \approx 86 \times 10^{-15}\ \mathrm{m}$$

$$a_d = \frac{\hbar}{\mathrm{m_p} \cdot c} = \frac{\hbar}{\mathrm{E_p}/c} = \frac{(6.582 \times 10^{-16}\ \mathrm{eV} \cdot s) \cdot (3 \times 10^{8}\ m/s)}{4.8 \times 10^{6}\ \mathrm{eV}} \approx 41 \times 10^{-15}\ \mathrm{m}$$

The result is unsurprising: the Compton radius is *inversely* proportional to the mass and, hence, a mass that's 200 or 400 times *smaller* will yield a Compton radius that is 200 to 400 times *larger*. The radius of an atom ranges from The diameter of a nucleus ranges from the mentioned 0.8 fm – the hydrogen nucleus, or a single proton – to about 5.86 fm for the heaviest atoms, like uranium. The 41-86 fm scale is 50 or 100 times the proton size, so the Compton radius of a quark doesn't make much sense.

Should we, therefore, think of quarks as some kind of mathematical abstraction rather than something real? No. Quark theory does make it easier to explain the *particle zoo* and – much more importantly – the existence of quarks has been confirmed experimentally, right? Hence, our oscillator model – the *Zitterbewegung* – might not apply to quarks: perhaps they are *pure* charges. No *Zitter*.

We do to do some research here but, yes, the properties of quarks are being measured in experiments. These experiments are interesting. Many were done in a particle accelerator in Hamburg, HERA[26], in which electrons or positrons were collided with protons at a center of mass energy of 318 GeV. That's *huge*! HERA was closed down in 2007 but measured many things, including the *size* of quarks, for which they found an upper limit that was equal to $0.43{\times}10^{-18}$ m.[27] So that's about 2,000 times smaller than the mentioned empirical value we got for protons and neutrons (0.84 and 0.9 fm). Yes: *two thousand* times.

We will have to come back to this. As for now, we should note that the quark model implies that most of the mass of the proton or the neutron must come from some *oscillation* or interaction between the quarks. So what could that oscillation or interaction possibly *be*?

We will let the matter rest for the time being – literally – and explore some other topic: excitations.

Excited states

We will start, once again, with the theory for an electron, and then examine if we can apply the same concepts to nucleons.

[25] If the distribution is normal, then the three-sigma (3σ) rule tells us this will capture 99.73% of all observations.
[26] HERA stands for Hadron-Elektron-Ringanlage (in German) or, in English, Hadron-Electron Ring Accelerator. See the Wikipedia article on this installation: https://en.wikipedia.org/wiki/HERA_(particle_accelerator).
[27] See: *Limits on the effective quark radius from inclusive e-p scattering at HERA*, ZEUS Collaboration, March 2016 (https://arxiv.org/pdf/1604.01280.pdf).

The energy states of the electron

The idea of an *excited* state is interesting. It reminds us of our results for an electron in some Bohr orbital, which we compare with the results we obtained for an electron in free space below.[28]

Table 1: Intrinsic spin versus orbital angular momentum

Spin-only electron (*Zitterbewegung*)	**Orbital electron (Bohr orbitals)**
$\mathrm{S} = \mathrm{h}$	$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \dots$
$\mathrm{E} = \mathrm{m}c^2$	$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$	$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\omega = \frac{v}{r} = c \cdot \frac{\mathrm{m}c}{\hbar} = \frac{\mathrm{E}}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L} = I \cdot \omega = \frac{1}{2} \cdot \mathrm{m} \cdot a^2 \cdot \omega = \frac{\mathrm{m}}{2} \cdot \frac{\hbar^2}{\mathrm{m}^2c^2}\frac{\mathrm{E}}{\hbar} = \frac{\hbar}{2}$	$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu = \mathrm{I} \cdot \pi {r_\mathrm{C}}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$	$\mu_n = \mathrm{I} \cdot \pi {r_n}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}n\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$	$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

Could we excite an electron? We assume an electron gets excited – temporarily – when it absorbs a high-energy photon in Compton scattering experiments. It is good to briefly recall the logic here.

Compton scattering involves electron-photon interference: a high-energy photon (the light is X- or gamma-rays) will hit an electron and is briefly absorbed before the electron comes back to its equilibrium situation. It does so by emitting another photon, whose wavelength will be longer. The photon has, therefore, less energy, and the difference in the energy of the incoming and the outgoing photon gives the electron some linear momentum. We refer to Compton scattering as inelastic because of this interference effect.

In contrast, low-energy photons scatter elastically. Elastic scattering experiments yield the smaller radius of the electron: the Thomson radius, which is given by the fine-structure constant: $r_\mathrm{e} = \alpha \cdot r_\mathrm{C}$. Thomson scattering radius is referred to as *elastic* because the photon seems to bounce off some hard *core*: there is no interference. This picture is fully consistent with the *Zitterbewegung* model of an electron: we think of the hard core as the pointlike charge itself, but pointlike does not mean its size is zero! It is, in fact, this picture that inspires the tangential velocity formula ($c = a \cdot \omega$) that we are using.

[28] See: Jean Louis Van Belle, *The Emperor Has No Clothes: A Classical Interpretation of Quantum Mechanics*, 21 April 2019 (http://vixra.org/abs/1901.0105).

The point is: Compton scattering does involve the idea of an excited electron. How should we think of its energy states? The energy states that we associate with Bohr orbitals do *not* include the rest energy of the electron. Hence, we should not be too obsessed by them.

Let us see what happens if we apply the $S_n = n \cdot h$ formula (n = 1, 2, 3, …) to the spin-only electron. We no longer have a unique energy and, therefore, we no longer have a unique frequency. Different frequencies imply different cycle times $T_n = \lambda_n/v_n$. Hence, we have different radii $a_n = \lambda_n/2\pi$ and different tangential velocities v_n. Or perhaps not. If our charge has no rest mass, then its tangential velocity should remain what it is—the speed of light, right? Yes.

That sounds like an idea. Let's apply straight away to our hypothetical nucleon charge.

The energy states of the nucleon

Let us solve this intuitively. If our equilibrium state – the non-excited state – is written and defined by $E_1 \cdot T_1 = h$, then our $S_n = n \cdot h$ formula[29] (or, if you prefer the angular momentum expression, $S_n = n \cdot h$ implies that $E_2 \cdot T_2 = 2 \cdot h$, $E_3 \cdot T_3 = 3 \cdot h$, …, or – more generally - $E_n \cdot T_n = n \cdot h$. If we take the ratio, then we get:

$$\frac{E_n \cdot T_n}{E_1 \cdot T_1} = n$$

Now, the cycle time is equal to the distance over the loop divided by the velocity: $T_n = \lambda_n/v_n$. We can, therefore, write the E_nT_n/E_1T_1 ratio as:

$$\frac{E_n \cdot \lambda_n/v_n}{E_1 \cdot \lambda_1/v_n} = \frac{E_n}{E_1} \cdot \frac{\lambda_n}{\lambda_1} \cdot \frac{v_1}{v_n} = n$$

If v_1 and v_n have to be equal, and equal to the speed of light ($v_1 = v_n = c$), then this might work if $E_n = n^2 \cdot E_1$ and if $\lambda_n = \lambda_n/n$:

$$\frac{E_n}{E_1} \cdot \frac{\lambda_n}{\lambda_1} \cdot \frac{v_1}{v_n} = \frac{n^2 E_1}{E_1} \cdot \frac{\lambda_1}{n \cdot \lambda_1} \cdot \frac{c}{c} = n$$

A *shorter* loop means a *higher* frequency. We can calculate the frequency as $f_n = 1/T_n = v_n/\lambda_n = c/\lambda_n = n \cdot f_1$. That makes sense: the energy of an oscillation is proportional to the *square* of its frequency. Let's put our formulas in a little table (Table 2).

Table 2: Non-excited and excites states of a nucleon: key values

$S_n = n \cdot \text{h for } n = 1, 2, \ldots$
$E_n = n^2 \cdot E_1 = n^2 \cdot m_N \cdot c^2$
$a_n = \frac{1}{n} \cdot a_1 = \frac{1}{n} \cdot \frac{\hbar}{m_N c}$
$v_n = c$
$\omega_n = \frac{c}{a_n} = n \cdot \frac{m_N c^2}{\hbar} = n \cdot \frac{E_1}{\hbar} = n \cdot \omega_1$

[29] We could rephrase this in terms of angular momentum – an expression you know from quantum mechanics – but we find this more convenient and, more importantly, more powerful as an *explanation* of what we think is the case.

These formulas feel somewhat counter-intuitive because we are used to see the solution of the black-body radiation, where we assumed energy states were defined as $E_1 = h \cdot f$, $E_2 = 2 \cdot h \cdot f$, $E_3 = 3 \cdot h \cdot f$,…, $E_n = n \cdot h \cdot f$. These energy states were all separated by the same amount of energy: $E_n - E_{n-1} = \hbar \cdot \omega = h \cdot f$. It was a classical application of the energy equipartition theorem. Here we do *not* have energy equipartition: the orbitals are separated *not* by equal energies $h \cdot f$ but by *equal amounts of physical action*: Planck's quantum of action (not $E = h \cdot f$) is the fundamental unit here and we, therefore, do *not* have one single frequency: the frequency depends on the orbital, as shown in Table 2.

We get the result that the 1st, 2nd, nth orbital now packs an amount of energy that is equal to $E_n = n \cdot h \cdot f_n = n^2 \cdot h \cdot f_1 = n^2 \cdot E_1$ The energy *difference* between two orbitals – or two *excitation states*, we should say – can be calculated as:

$$\Delta E = E_n - E_{n-1} = n^2 \cdot E_1 - (n-1)^2 \cdot E_1 = [n^2 - (n-1)^2] \cdot E_1 = (2n - 1) \cdot E_1$$

ΔE is no longer constant: it is now a linear function of n, as shown in the table below (Table 3)

Table 3: Energy differences: $\Delta E = (2n - 1) \cdot E_1$

n	**1**	**2**	**3**	**4**	**5**	**…**
$\Delta E = (2n - 1) \cdot E_1$	$1 \cdot E_1$	$3 \cdot E_1$	$5 \cdot E_1$	$7 \cdot E_1$	$9 \cdot E_1$	…
$E_2 - E_1$	$2 \cdot E_1$					
$E_3 - E_1$	$4 \cdot E_1$					
$E_4 - E_1$	$6 \cdot E_1$					
$E_5 - E_1$	$8 \cdot E_1$					
…	…					
$E_n - E_1$	$2 \cdot (n - 1) \cdot E_1 = (2n - 2) \cdot E_1$					

The idea of a nuclear force quantum

Standard quantum field theory assumes the exchange of nuclear force quanta. What does our model say about them? We obviously have an *energy* quantum here, but it's the energy of the nucleon itself:

$$E_1 = m_N \cdot c^2$$

Can we *imagine* some exchange of a *photon-like* particle carrying this energy between one nucleon and another? Of course, we can. In fact, we can see skipping one or more of the higher states would allow to take this particle to have an energy that's equal to any multiple of $E_1 = m_N \cdot c^2$.

The point is: this particle cannot be photon-like because it is *at least* as massive as the nucleon. We are talking the mass of protons and neutrons here. To be precise, that mass is about 939,565,413 eV/c^2 for the neutron and about 938,272,081 eV/c^2 for the proton. Why should we assume the nucleons stick together by exchanging particles that are at least as heavy as themselves?

To be fair, the idea of a nuclear force quantum was, obviously, very different from the idea of the *gluon* that Murray Gell-Mann advanced in the early 1960s, as part of his quark model: we no longer think of protons and neutrons as elementary particles nowadays. Still, gluons are imagined as being quite heavy:

anything between 0 and 1.3 MeV/c^2. That upper limit[30] – 1.3 MeV/c^2 – happens to coincide with the mass *difference* between protons and neutrons. That brings us to the next question: what might explain that mass difference? We will come back to that. Let us first provide some more elements for the matter at hand here: is there a nuclear force quantum?

Yukawa predicted one: his nuclear quantum is usually denoted as m_U (and is referred to as the U-quantum), and it is based on his model of the nuclear force which – in turn – is based on his model for the forces in a nucleus, which are governed by this special potential he invented: the Yukawa potential. Let's discuss that in more detail. Yukawa thought of some wave equation for the nucleus. Something like Schrödinger's equation. Let us explore that idea.

A wave equation for the nucleus?

The augmented Rutherford-Bohr model

Why do we need a wave equation? We get the energy levels out of our oscillator model, don't we? Yes and no. To be precise, the answer is: no. Let us re-visit the electron oscillator model to show why it would not work for nucleons.

Our formulas for the orbital electron (see Table 1) effectively assume circular orbitals separated by an amount of action (or angular momentum) that is equal to h (or $\hbar$ if we're talking angular momentum):

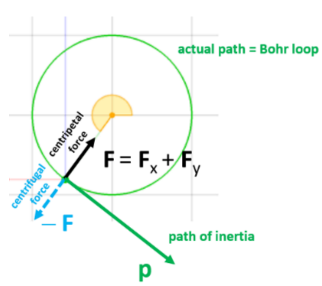
$$S_n = n \cdot h \Leftrightarrow L_n = n \cdot \hbar$$

We got the following energy formula for the energy of the n^{th} orbital:

$$E_n = -\frac{1}{2}\frac{\alpha^2}{n^2}mc^2 = -\frac{1}{n^2}E_R$$

How did we do that? It is just The Bohr model: we have a positively charged nucleus at its center and the electron has an effective rest mass: the radial velocity $v = a \cdot \omega$ of the electron is, therefore, some *fraction* of the speed of light ($v = \alpha \cdot c$). It also has some non-zero momentum $p = m \cdot v$ which we can relate to the electrostatic centripetal force using the simple classical formula $F = p \cdot \omega = mv^2/a$. In contrast, the model of an electron in free space is based on the presumed *Zitterbewegung*, which combines the idea of a very high-frequency circulatory motion with the idea of a pointlike *charge* which – importantly – has *no inertia* and can, therefore, move at the speed of light ($v = c$). Figure 4 illustrates the idea.

Figure 4: The position, force and momentum vector in a Bohr loop



[30] We are yet to do a more detailed analysis. This is general information from the Wikipedia article on gluons and, therefore, needs further verification.

The formulas in the Bohr-Rutherford model are derived from the basic quantum-mechanical rule that angular momentum comes in units of $\hbar = h/2\pi$. We rephrased that rule as: physical action comes in units of h. We also associated Planck's quantum of action with a *cycle*: one rotation will pack some energy over some time (the cycle time) or – what amounts to the same – some momentum over some distance (the circumference of the loop). We can, therefore, write:

$$S = h = \mathrm{E}\cdot\mathrm{T} = \mathrm{p}\cdot\lambda = \mathrm{p}\cdot 2\pi\cdot r_{\mathrm{B}}$$

Using the $v = \alpha\cdot c$ and $r_{\mathrm{C}} = \alpha\cdot r_{\mathrm{B}}$ relations, one can easily verify this is, effectively, the case:

$$S = \mathrm{p}\cdot 2\pi\cdot r_{\mathrm{B}} = \mathrm{m}v\cdot 2\pi\cdot (r_{\mathrm{C}}/\alpha) = \mathrm{m}\alpha c\cdot\frac{2\pi\hbar}{\alpha\mathrm{mc}} = \mathrm{h}$$

We can also calculate S by calculating the force and then multiply the force with the distance and the time. The force is just the (centripetal) electrostatic force between the (negative) electric charge and the (positive) hydrogen nucleus (the proton).

$$\mathrm{F} = \frac{\mathrm{q_e}^2}{4\pi\varepsilon_0 {r_{\mathrm{B}}}^2} = \alpha\cdot\frac{\hbar c}{{r_{\mathrm{B}}}^2}$$

We can then also calculate S as:

$$S = \mathrm{F}\cdot r_{\mathrm{B}}\cdot\mathrm{T} = \alpha\cdot\frac{\hbar c}{{r_{\mathrm{B}}}^2}\cdot r_{\mathrm{B}}\cdot\frac{2\pi r_{\mathrm{B}}}{v} = \alpha\cdot\frac{\mathrm{h}c}{\alpha c} = \mathrm{h}$$

All is consistent. For the energy, we get the following:

$$S = h = \mathrm{E}\cdot\mathrm{T} = \mathrm{E}\cdot\frac{2\pi r_{\mathrm{B}}}{v} = \mathrm{E}\cdot\frac{\frac{h}{\alpha\mathrm{m}c}}{\alpha c} \Leftrightarrow \mathrm{E} = \alpha^2\mathrm{m}c^2$$

This is *twice* the ionization energy of hydrogen ($\mathrm{Ry} = \alpha^2\mathrm{m}c^2/2$) – the *Rydberg* energy – and it is also *twice* the kinetic energy ($\hbar^2/2\mathrm{m}a^2 = \alpha^2\mathrm{m}c^2/2$). That is OK because the oscillator model adds the kinetic and potential energy of *two* oscillators – one perpendicular to the other – and we only need the kinetic energy, so we should add a 1/2 factor.[31]

It is easy to generalize these results for n = 2, 3, etcetera. One can, indeed, show that the energy of the n^{th} orbital is what we wrote above:

$$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$$

We have an augmented Rutherford-Bohr model of an atom here. This 105-year old model[32] was designed to explain the wavelength of a photon that is emitted or absorbed by a hydrogen atom – a

[31] Alternatively, we can show that – because of the circular motion – the *effective* mass will be equal to 1/2 of the total mass.

[32] Around 1911, Rutherford had concluded that the nucleus had to be very small. Hence, Thomson's model – which assumed that electrons were held in place because they were, somehow, embedded in a uniform sphere of positive charge – was summarily dismissed. Bohr immediately used the Rutherford hypothesis to explain the emission spectrum of hydrogen atoms, which further confirmed Rutherford's conjecture, and Niels and Rutherford jointly presented the model in 1913. As Rydberg had published his formula in 1888, we have a gap of about 25 years between experiment and theory here.

one-electron atom, and it does so very well. The idea is that the energy of a photon that is emitted or absorbed is equal to the *difference* in energy between the various orbitals. The energy of these orbitals is usually expressed in terms of the energy of the *first* Bohr orbital, which is now referred to as the *ground state* of (the electron in) the hydrogen atom. The *Rydberg* energy E_R is then the energy of the electron in the first Bohr orbital and, as we have shown above, it can be expressed in terms of the fine-structure constant (α) and the rest energy ($E_0 = mc^2$) of the electron[33]:

$$\mathrm{E}_R = \frac{\alpha^2 \mathrm{m}c^2}{2} = \frac{1}{2}(\frac{\mathrm{q_e}^2}{2\varepsilon_0 \mathrm{h}c})^2 \mathrm{m}c^2 = \frac{\mathrm{q_e}^4 \mathrm{m}}{8\varepsilon_0{}^2 \mathrm{h}^2} \approx 13.6\ \mathrm{eV}$$

To be precise, the *difference* in energy between the various orbitals should be equal to:

$$\Delta \mathrm{E} = \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right) \cdot \mathrm{E}_R$$

The Rydberg formula then becomes self-evident. The idea of the wavelength of a wave (λ), its velocity of propagation (c) and its frequency (f) are related through the $\lambda = c/f$ relation, and the Planck-Einstein relation ($E = h \cdot f$) tells us the energy and the wavelength of a photon are related through the frequency:

$$\lambda = \frac{c}{f} = \frac{hc}{\mathrm{E}}$$

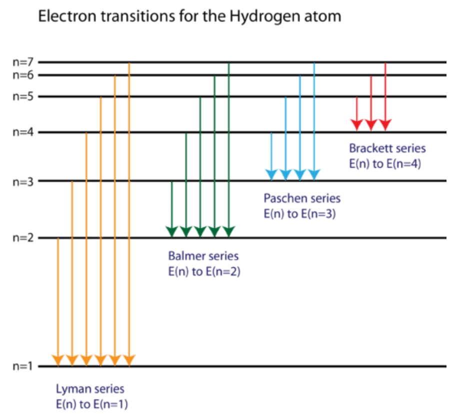
Hence, we can now write the Rydberg formula by combining the above:

$$\frac{1}{\lambda} = \frac{\Delta \mathrm{E}}{\mathrm{h}c} = \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right) \cdot \frac{\mathrm{E}_R}{\mathrm{h}c} = \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right) \cdot \frac{\alpha^2 \mathrm{m}c^2}{2\mathrm{h}c}$$

The Rydberg formula has the fine-structure constant in it but it actually describes the so-called *gross* structure of the hydrogen spectrum only (see Figure 5). Indeed, when the spectral lines are examined at *very* high resolution, the spectral lines are split into finer lines. This is due to the intrinsic spin of the electron. This *intrinsic* spin of the electron is to be distinguished from its orbital motion. It shows we should not be thinking of the electron as a pointlike (infinitesimally small) particle. It has a radius: the Compton radius! Hence, we speak of *spin* angular momentum versus *orbital* angular momentum. There must be some *coupling* between the two motions. Such coupling – and possibly other factors – should explain the fine structure.

[33] We should write m_0 instead of m – *everywhere*. But we are using non-relativistic formulas for the velocity and kinetic energy everywhere. Hence, we dropped the subscript.

Figure 5: The gross structure of the hydrogen spectrum[34]



The point to note here is that the energy in the orbitals is only a very small fraction ($\alpha^2 \approx 0.00005325$) of the rest energy of the electron. That is what we want, right? Can't we develop a similar model for nucleons? Nuclear orbitals separated by relatively small energy differences which then explain some photon-like particle that we can associate with the nuclear force?

We must make two remarks here:

1. This augmented Bohr-Rutherford model is based on the assumption of an immovable proton at the center: it is immovable because its mass is about 1,836 times the mass of the electron. This is why we can imagine these circular orbits. We cannot think of two nucleons attracting each other in the same way: they would have the same mass and, hence, the orbital would not be circular. We should think of the trajectories of binary stars here !
2. We should carefully think of why we'd need 'photon-like particles that we can associate with the nuclear force': we have no use for new theoretical particles !

Having said that, just assuming that the nucleons will just sit on top of each other is rather boring. Hence, the idea of a wave equation might make sense.

The rationale for a wave equation

We interpreted Schrödinger's wave equation as a differential equation whose solutions give us *all* possible electron orbitals, including non-circular orbitals.[35] In fact, we think our *augmented* Rutherford-Bohr model is just a mathematical abstraction of the atom. It only gives us the *principal* quantum number n, which gives us the energy level. Actual electron orbitals are defined by two more numbers:

1. The orbital angular momentum number $l = -n+1, -n+2, \ldots, 0, \ldots, n-2, n-1$
2. The magnetic quantum number $m_l = -l, -l+1, \ldots, 0, \ldots, l-2, l-1$

[34] The illustration was taken from: http://hydrogenatomgirikosa.blogspot.com/2017/03/emission-spectrum-of-hydrogen-atom.html.
[35] See: Jean Louis Van Belle, *A Geometric Interpretation of Schrödinger's Equation*, 12 December 2018 (http://vixra.org/abs/1812.0202).

In addition, the electron may have its spin up or down and there is, therefore, room for *two* electrons in one orbital. Our augmented Rutherford-Bohr model doesn't capture the *l* and *m* numbers and, therefore, it doesn't capture the fine structure of the hydrogen spectrum.

In contrast, Schrödinger's wave equation does the trick. It is, therefore, effectively very tempting to *not* simply assume that nucleons will just sit on top of each other and think of modeling some kind of wave equation for nucleons – using Yukawa's potential function.

Arguments against the idea

While we mentioned why the idea might make sense, we would also like to note why it might *not* make sense.

1. We are modeling something very different: a nucleon – be it a proton, a neutron or a quark – is not electrically neutral (the neutron is but, as mentioned, we think of it having some internal pieces whose electric charges cancel out). A wave equation would, therefore, need to integrate not one but *two* potentials: Yukawa's potential *and* the Coulomb potential. We cannot neglect the Coulomb potential because we argued that the Yukawa and Coulomb forces are equally important at the distance $r = a_N$.[36]

2. Things are complicated because this potential applies to *two* nucleons only. What if we have only one, or if we have three, four or *n* nucleons? With Schrödinger's equation, things get complicated, but they are not impossible. Feynman says the following about this:

> "To get a solution, we would have to solve Schrödinger's equation for Z electrons in a Coulomb field. For helium [two electrons], no one has found an analytic solution, although solutions for the lowest energy states have been obtained by numerical methods. With three, four or five electrons it is hopeless to try to obtain exact solutions, and it is going too far to say that quantum mechanics has given a precise understanding of the periodic table. It is possible, however, even with a sloppy approximation—and some fixing—to understand, at least qualitatively, many chemical properties which show up in the periodic table."[37]

If it is that difficult for more than one electron – and for such simple potential – then it looks like an impossible task to try to model anything real for more than two nucleons.

3. The most important question, however, is still the following: why would we need an equivalent of 'photons' to be associated with the nuclear force? Just because it looks nice? Or to find some other use for complicated quantum field theories?

There is no experimental evidence for these 'nuclear photons' and the whole theory feels, therefore, quite artificial. What is that it tries to explain, *exactly*?

Jean Louis Van Belle, 22 June 2019

[36] See: Jean Louis Van Belle, *The Nature of Yukawa's Nuclear Force and Charge*, 19 June 2019 (http://vixra.org/abs/1906.0311).

[37] Feynman's *Lectures*, Vol. III, Chapter 19 (http://www.feynmanlectures.caltech.edu/III_19.html#Ch19-S6).