

Refutation of Bob Boyer's paradox

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Abstract: Bob Boyer's paradox is as follows. "A question: It is generally granted that 'p implies p', which is to say, 'if p, then p'. So what about this claim: 'If any number is prime, then any number is prime'?" We find the sentences are unrelated. We then rewrite the second sentence as: "If at least one number is prime, then at least one number is prime"; or as "If at least one number is prime, then possibly all numbers are prime". Both are trivial tautologies, meaning neither is a contraction or paradox, and therefore forming a tautologous fragment of the universal logic $\forall\exists 4$.

We assume the method and apparatus of Meth8/ $\forall\exists 4$ with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee , \cup , \sqcup ; - Not Or; & And, \wedge , \cap , \square , $;$; \ Not And;
 $>$ Imply, greater than, \rightarrow , \Rightarrow , \mapsto , $>$, \supset , \gg ; $<$ Not Imply, less than, \in , $<$, \subset , \prec , $\#$, \ll , \lesssim ;
 $=$ Equivalent, \equiv , $:=$, \Leftrightarrow , \leftrightarrow , $\hat{=}$, \approx , \simeq ; @ Not Equivalent, \neq ;
 $\%$ possibility, for one or some, \exists , \diamond , **M**; # necessity, for every or all, \forall , \square , **L**;
 $(z=z)$ **T** as tautology, \top , ordinal 3; $(z@z)$ **F** as contradiction, \emptyset , Null, \perp , zero;
 $(\%z\#z)$ **N** as non-contingency, Δ , ordinal 1; $(\%z\<\#z)$ **C** as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$), ($x \sqsubseteq y$); $(A=B)$ $(A\sim B)$.
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: cs.utexas.edu/~boyer/

A question

It is generally granted that 'p implies p', which is to say, 'if p, then p'. (1.1)

Remark 1.2: Eq. 1.1 is a trivial tautology as $p \gg p$. (1.2)

So what about this claim: 'If any number is prime, then any number is prime'?

LET p, q : number, prime.

Remark 2.1: We take 1.1 and 2.1 as unrelated, despite the conjunction in 2.1 of "So". Because "any number" is in the singular, we take it as "any one" number, that is, "at least one number as in the singular (but not all numbers as in the plural)". Hence, 'If at least one number is prime, then at least one number is prime'. (2.1)

Remark 2.2: Eq. 2.1 as $(\%p \gg q) \gg (\%p \gg q)$ is also a trivial tautology. (2.2)

So what about this claim: 'If at least one number is prime, then possibly all numbers are prime'? (3.1)

Remark 3.2: Eq. 3.1 as $(\%p \gg q) \gg (\#p \gg q)$ is also a trivial tautology. (3.2)

The three equations tested are tautologous, to mean there is no contradiction or paradox.