Twin Prime Conjecture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the twin prime conjecture.

We use $p_i$ for all the primes, 2,3,5,7,11,13,......, i=1,2,3,......,

If a primes pair $(p_m, p_{m+1})$ is a twin prime, then it can be written as $(6k-1, 6k+1)$ for some k.

Start with the twin prime (5, 7), there are two twin primes, (29, 31) and (41, 43) in the range of $(5^2, 7^2)$.

Choose any one of them, for example (29, 31), there are two twin primes, (857, 859) and (881, 883) in the range of $(29^2, 31^2)$.

We can prove that this procedure can be repeated indefinitely, so there are infinite twin primes.

Theorem;

If $(p_m, p_{m+1})$ is a twin prime, then there are at least two twin primes in the range of $(p_m^2, p_{m+1}^2)$.

By using sieve of Eratosthenes to natural numbers for number 2 and 3, the remaining numbers are following,
1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, .......

By pairing up every two numbers except 1, we have a sequence of pairs (A) as following,

(5, 7), (11, 13), (17, 19), (23, 25), (29, 31), ......... (A)

Each pair has a form of (6k-1, 6k+1), for k=1, 2, 3, 4,......

All the twin primes are in this pair sequence (A).

If \((p_m, p_{m+1})\) is a twin prime,

\[ p_{m+1}^2 - p_m^2 = (p_m + 2)^2 - p_m^2 = 4(p_m + 1) = 24k, \]

for some k,

There are \(\frac{p_{m+1}^2 - p_m^2}{6} = 4k\) pairs in sequence in the pair sequence (A),

with all the numbers involved being within the range of \((p_m^2, p_{m+1}^2)\).

By seieving of the Eratosthenes for all the primes \((p_i, i = 3,4,...,m)\), if any number of a pair is sieved out, we say the pair being sieved out.

The remaining number of the remaining pairs inside the range of \((p_m^2, p_{m+1}^2)\) is larger than or equal to,

\[ 4k \prod_{2<i\leq m} (1 - \frac{2}{p_i}) = 4k \left(\frac{p_m - 2}{p_m}\right) \prod_{3<i\leq m} \left(\frac{p_i - 2}{p_i - 1}\right) > \frac{12k}{6k-1} > 2. \]

So there are at least two twin primes in the range \((p_m^2, p_{m+1}^2)\).