Twin Prime Conjeture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the twin prime conjecture.

We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

If a primes pair (p_m, p_{m+1}) is a twin prime, then it can be written as (6k-1, 6k+1) for some k.

Start with the twin prime (5, 7), there are two twin primes, (29, 31) and (41, 43) in the range of $(5^2, 7^2)$.

Choose any one of them, for example (29, 31), there are two twin primes, (857, 859) and (881, 883) in the range of $(29^2, 31^2)$.

We can prove that this procedure can be repeated indefinitely, so there are infinite twin primes.

Theorem;

If (p_m, p_{m+1}) is a twin prime, then there are at least two twin primes in the range of (p_m^2, p_{m+1}^2).

By using sieve of Eratosthenes to natural numbers for number 2 and 3, the remaing numbers are following, 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, ...

By pairing up every two numbers except 1, we have a sequence of pairs (A) as following,

(5, 7), (11, 13), (17, 19), (23, 25), (29, 31), ... (A)

Each pair has a form of (6k-1, 6k+1), for k=1, 2, 3, 4,......

All the twin primes are in this pair sequence (A).

If (p_m, p_{m+1}) is a twin prime,

$p_{m+1}^2 - p_m^2 = (p_m + 2)^2 - p_m^2 = 4(p_m + 1) = 24k$, for some k,

There are $\frac{p_{m+1}^2 - p_m^2}{6} = 4$ k pairs in sequence in the pair sequence (A),

with all the numbers involved being within the range of $(p_m^2, p_{m+1}^2]$.

By seiving of the Eratosthenes for all the primes (p_i, i = 3,4,...,m), if any number of a pair is sieved out, we say the pair being sieved out.

The remaining number of the remaining pairs inside the range

of (p_m^2, p_{m+1}^2) is larger than or equal to,

4k $\prod_{2<i\leq m}(1 - \frac{2}{p_i}) = 4k(\frac{p_3-2}{p_m}) \prod_{3<i\leq m}(\frac{p_i-2}{p_{i-1}}) > \frac{12k}{6k-1} > 2.$

So there are at least two twin primes in the range (p_m^2, p_{m+1}^2).