Refutation of comorphism of sites

Abstract: The complex equation evaluated is *not* tautologous, hence refuting the conjecture of *comorphism of sites* as a functor with a *covering lifting property*. What follows is that the following are also refuted: surjections, inclusions, localic morphisms, hyperconnected morphisms, and equivalences of toposes. This further relegates category theory of Grothendieck to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ▽, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Caramello, O. (2019). Denseness conditions, morphisms and equivalences of toposes. arxiv.org/pdf/1906.08737.pdf olivia.caramello@uninsubria.it

Abstract: We establish a general theorem providing necessary and sufficient explicit conditions for a morphism of sites to induce an equivalence of toposes. This results from a detailed analysis of arrows in Grothendieck toposes and denseness conditions, which yields results of independent interest. We also derive site characterizations of the property of a geometric morphism to be an inclusion (resp. a surjection, hyperconnected, localic), as well as site-level descriptions of the surjection inclusion and hyperconnected-localic factorizations.

6 Geometric morphisms induced by comorphisms of sites

Recall that a *comorphism of sites* (D, K) → (C, J) (where J and K are Grothendieck topologies respectively on C and D) is a functor F : D → C which has the *covering lifting property*, that is the property that for every d ∈ D and any J-covering sieve S on F (d) there is a K-covering sieve R on c such that F (R) ⊆ S. (6.1)

LET	p,	q,	r,	s,	t,	u,	v,	w,	x,	y:
	C,	D,	R,	S,	K,	F,	J,	c,	d,	J-covering sieve

((((v>p)&(t>q))>((x&t)>(p&v)))>(u=(q>w)))>((#(x<q)&%(y>(u&x)))>((r>w)>~(s<(u&r)))) ;

TTTT TTTT TTTT TTTT(16)
TTTT TTTT TTTT TTTT(2)}x4
TTTT TTTT CCTT TTTT(2)} (6.2)

Eq. 6.2 as rendered is not tautologous, hence refuting the conjecture of *comorphism of sites* as a functor with a *covering lifting property*. By extension, also refuted are: surjections, inclusions, localic morphisms, hyperconnected morphisms, and equivalences of toposes; and further the category theory Grothendieck.