Confirmation of Perez' definition of the conceivable statement

Abstract: Of the 16 equations evaluated, 15 are *not* tautologous. Elsewhere we correctly proved and confirmed most of the conjectures of the author such as refutations of ZFC, Cantor, and Gödel. The definition of a conceivable statement is confirmed as a theorem, with the other equations forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Perez, P.A. (2010). Addressing mathematical inconsistency: Cantor and Gödel refuted
arxiv.org/ftp/arxiv/papers/1002/1002.4433.pdf jap717@juanperezmaths.com

Abstract. This article undertakes a critical reappraisal of arguments in support of Cantor's theory of transfinite numbers. The following results are reported:

- Cantor's proofs of nondenumerability are refuted by analyzing the logical inconsistencies in implementation of the *reductio* method of proof and by identifying errors. Particular attention is given to the diagonalization argument and to the interpretation of the axiom of infinity.
- Three constructive proofs have been designed that support the denumerability of the power set of the natural numbers, *P* (N), thus implying the denumerability of the set of the real numbers R. These results lead to a Theorem of the Continuum that supersedes Cantor's Continuum Hypothesis and establishes the countable nature of the real number line, suggesting that all infinite sets are denumerable.

Some immediate implications of denumerability are discussed:

- Valid proofs should not include inconceivable statements, defined as statements that can be found to be false and always lead to contradiction. This is formalized in a Principle of Conceivable Proof.
- Substantial simplification of the axiomatic principles of set theory can be achieved by excluding transfinite numbers. To facilitate the comparison of sets, infinite as well as finite, the concept of relative cardinality is introduced.
- Proofs of incompleteness that use diagonal arguments (e.g. those used in Gödel's Theorems) are refuted. A constructive proof, based on the denumerability of *P* (N), is presented to demonstrate the existence of a theory of first-order arithmetic that is consistent, sound, negation-complete, decidable and (assumed p.r. adequate) able to prove its own consistency. Such a result reinstates Hilbert's Programme and brings arithmetic completeness to the forefront of mathematics.

3. Refutations of Cantor's proofs of nondenumerability

3.1.2 Proofs by external (or conventional) contradiction

(3.2.1) $\neg P \Rightarrow Q_1 \Rightarrow Q_2 \Rightarrow . . . \Rightarrow Q_n \Rightarrow (R \wedge \neg R)$.

(~p>(q>s))>(r&~r) ; FFFF FFFF FFFF FFFF (3.2.2)

(3.3.1) $\neg P \Rightarrow (R \wedge \neg R)$.

~p>(r&~r) ; FTFT FTFT FTFT FTFT (3.3.2)

(3.4.1) $\neg(R \wedge \neg R) \Rightarrow \neg(\neg P) \Rightarrow P$

Remark 3.4.1: Eq. 3.4.1 is a trivial tautology.

3.1.3 Proofs by internal (or self-referential) contradiction

(3.5.1) $\neg P \Rightarrow Q_1 \Rightarrow Q_2 \Rightarrow . . . \Rightarrow Q_n \Rightarrow P$.

(~p>(q>s))>p ; FTTT FTTT FTFT FTFT (3.5.2)

(3.6.1) $\neg P \Rightarrow (P \wedge \neg P)$.

~p>(p&~p) ; FTFT FTFT FTFT FTFT (3.6.2)

Remark 3.6.2: Eq. 3.6.2 is not equal to 3.7.2 or 3.10.2.

(3.7.1) $\neg(P \wedge \neg P) \Rightarrow \neg(\neg P) \Rightarrow P$

Remark 3.7.1: Eq. 3.7.1 is a trivial tautology and not equal to 3.6.1 or 3.10.1.

(3.8.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Leftrightarrow . . . \Leftrightarrow Q_{i-1} \Leftrightarrow Q_i \Rightarrow Q_{i+}1 \Rightarrow . . . \Rightarrow Q_n \Rightarrow P$.

((~p=(q=r))>s)>p ; TTFT FTTT FTFT FTFT (3.8.2)

(3.9.1) $Q_i \Rightarrow \neg P \wedge Q_i \Rightarrow P$

(q>(~p&q))>p ; FTFT FTFT FTFT FTFT (3.9.2)

(3.10.1) $Q_i \Rightarrow (P \wedge \neg P)$.

q>(p&~p) ; TTFF TTFF TTFF TTFF (3.10.2)

Remark 3.10.2: Eq. 3.10.2 is not equal to 3.6.2 or 3.7.2.

3.2. **Cantor's diagonalization argument**

3.2.1 *Logical objection to the proof* .

(3.11.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Rightarrow Q_3 \Leftrightarrow P$

((~p=(q=r))>s)=p ; TTFF FFTT FTFT FTFT (3.11.2)

3.3. **Cantor's Theorem: the power set.**
3.3.2. *First proof for higher powers* .

(3.23.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Rightarrow Q_3 \Leftrightarrow P$

((~p=(q=r))>s)=p ; TT**FF** **FF**TT **F**T**F**T **F**T**F**T (3.23.2)

(3.24.1) $\neg P \Leftrightarrow Q \Leftrightarrow C$

~p=(q=r) ; T**FF**T **F**TT**F** T**FF**T **F**TT**F** (3.24.1.2)

Consequently, (3.24[.1]) equates to writing $\neg P \Leftrightarrow C$, and this is taken as a sufficient argument supporting the truth of the theorem. (3.24.2.1)

(~p=(q=r))=(~p=r) ; **FF**TT **FF**TT **FF**TT **FF**TT (3.24.2.2)

3.4. **The Axiom of Infinity.**

(3.25.1) $\exists y(\emptyset \in y \wedge \forall x(x \in y \Rightarrow x \cup \{x\} \in y))$.

LET p, q: x, y

((s@s)<%q)&((#p<%q)>(#p+(#p<%q))) ;
FFFF FFFF FFFF FFFF (3.25.2)

3.5. **Cantor's first proof of the nondenumerability of** R **.**

(3.32.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Rightarrow Q_3 \Leftrightarrow P$

(~p=(q=r))>(s=p) ; TTT**F** T**F**TT **F**TTT TT**F**T (3.32.2)

5. Implications for proofs by reductio (ad absurdum)
The first requirement, a method to identify incorrect mathematical statements, is addressed by the following definition.

Definition 5.1. A mathematical statement Q is said to be inconceivable when there is another statement P such that

(5.1.i.1) $(Q \Rightarrow P) \wedge (Q \Rightarrow \neg P)$, or

(q>p)&(q>~p) ; TT**FF** TT**FF** TT**FF** TT**FF** (5.1.i.2)

(5.1.ii.1) $Q \Rightarrow ((P \Rightarrow \neg P) \vee (\neg P \Rightarrow P))$.

Otherwise, the statement Q is considered conceivable. (5.1.1)

(5.1.i.1) or (5.1.ii.1) ((q>p)&(q>~p))+(q>((p>~p)+(~p>p))) ;
TTTT TTTT TTTT TTTT (5.1.2)

Remark 5.1.2: Eq. 5.1.2 is tautologous to confirm the definition of a conceivable statement.

The mathematical statement Q can itself define a given mathematical object or entity. In such a case, this object is also considered inconceivable or conceivable, in line with the statement which defines it.

Principle 5.2 (of Conceivable Proof). No mathematical proof can be judged valid if its construction includes an inconceivable statement; the exception is if the purpose of the proof is to demonstrate the falsehood of an inconceivable statement, provided that the resulting contradiction is not conceptually linked to the initial assumption of the proof.

Conjecture 5.3 (of Logical Imperfection). Any sound and/or consistent system of mathematics is capable of generating inconceivable statements.

(5.1.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Leftrightarrow Q_3 \Rightarrow Q_4 \Leftrightarrow Q_5 \Leftrightarrow Q_6 \Rightarrow C$

(~p=(q=r))>((s=t)>u) ;

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FTTF TFFT TTTT TTTT( 1)
TTTT TTTT FTTF TFFT( 1)
TTTT TTTT TTTT TTTT( 2)
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(5.1.2)

Of the 16 equations evaluated, 15 are *not* tautologous. Elsewhere we correctly proved and confirmed most of the conjectures of the author such as refutations of ZFC, Cantor, and Gödel. The author's definition of a conceivable statement is confirmed as a theorem.