Refutation of predicative collapse and arithmetical comprehension

Abstract: The four equations evaluated are *not* tautologous, hence refuting predicative collapsing principles such as arithmetical comprehension. Therefore these form a tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Freund, A. (2019). Predicative collapsing principles. arxiv.org/pdf/1906.07448.pdf

Abstract. We show that arithmetical transfinite recursion is equivalent to a suitable formalization of the following:
For every ordinal α there exists an ordinal β such that 1+β·(β+α) (ordinal arithmetic) admits an almost order preserving collapse into β. (1.1.1)

LET p, q: α , β.

(#p>%q)>(((%p>#p)+q)&(q+p)) ; **F**NTT **F**NTT **F**NTT **F**NTT (1.1.2)

(#p>%q)>(((%p>#p)+q)&(q+p)))>q ;
TCTT TCTT TCTT TCTT (1.2.2)

Arithmetical comprehension is equivalent to a statement of the same form, with β·α at the place of β·(β+α). … (2.1.1)

((%p>#p)+q)&p ; **F**N**F**T **F**N**F**T **F**N**F**T **F**N**F**T (2.1.2)

(((%p>#p)+q)&p)>q ; TCTT TCTT TCTT TCTT (2.2.2)

Remark 2.2.2: We see the logical equivalence of Eqs. 1.2.2 and 2.2.2, meaning ((1+β·(β+α))→β)=((β·α)→β), which is probably not what the author intended.

The four equations evaluated are *not* tautologous, hence refuting predicative collapsing principles such as arithmetical comprehension.