Refutation of the HOL/Isabelle rejection of E.J. Lowe's modal ontological argument

Abstract: Of 20 equations evaluated, 16 are *not* tautologous. This effectively refutes Lowe's proof, as rendered by the authors. This also invalidates the authors' rejection of Lowe's proof due to incompleteness (six of Lowe's conclusions are dismissed without evaluation) and due to an interactive, trial by error approach to reconstruct Lowe. Therefore an ideal showcase for the computer-assisted interpretive method using HOL/Isabelle failed. These results form another *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,· ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⇸ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≤ ;
= Equivalent, ≡, :=, ⇔, ↔, ≙, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Fuenmayor, D.; Benzmüller, C. (2019, nee 2017).
Computer-assisted reconstruction and assessment of E. J. Lowe's modal ontological argument.
isa-afp.org/browser_info/current/AFP/Lowe_Ontological_Argument/outline.pdf

Abstract: Computers may help us to understand –not just verify– philosophical arguments. By utilizing modern proof assistants in an iterative interpretive process, we can reconstruct and assess an argument by fully formal means. Through the mechanization of a variant of St. Anselm's ontological argument by E. J. Lowe, which is a paradigmatic example of a natural-language argument with strong ties to metaphysics and religion, we offer an ideal showcase for our computer-assisted interpretive method [tool named HOL/Isabelle].

2 E. J. Lowe's Modal Ontological Argument
2.1 Introduction
E. J. Lowe … "A modal version of the ontological argument"... features eight premises from which new inferences are drawn until arriving at a final conclusion: the necessary existence of God (which in this case amounts to the existence of some "necessary concrete being").

(P1.1) God is, by definition, a necessary concrete being.

LET p, q, r, s, t, u, v, w, x, y, z:
being, dependent, explanation, space, time, abstract, concrete, world, x, y, z.

Remark 1: The verb depend is taken to mean the imply operator, whereas the adjectives dependent (not independent) are taken as variables. While the verb explain can be taken to mean the imply operator, the noun explanation is taken as a variable standing on its own.

God: #(v&p)=(z=z) ; **FFFF FFFF FFFF FFFF**(4)
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N(4) (P1.2)

(P2.1) Some necessary abstract beings exist.

%#(u&p)=(z=z) ; **FFFF FFFF FFFF FFFF**(2)
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N(2) (P2.2)

(P3.1) All abstract beings are dependent beings.

(q&p)>#(u&p) ; TTTF TTTF TTTF TTFT(2)
TTTN TTTN TTTN TTTN(2) (P3.2)

(P4.1) All dependent beings depend for their existence on independent beings.

~(q&p)>#(q&p) ; **FFF**T **FFF**T **FFF**T **FFF**T (P4.2)

(P5.1) No contingent being can explain the existence of a necessary being.

(~(%z<#z)&p)>%#p ; TTTT TTTT TTTT TTTT (P5.2)

(P6.1) The existence of any dependent being needs to be explained.

%#(q&p)>r ; TTTC TTTT TTT<u>C</u> TTTT (P6.2)

(P7.1) Dependent beings of any kind cannot explain their own existence.

~(#(q&p)>(r>%#(q&p)))=(z=z) ;
FFFF FFFF FFFF FFFF (P7.2)

(P8.1) The existence of dependent beings can only be explained by beings on which they depend for their existence.

p>(#r>%(q&p)) ; TTTT T<u>C</u>TT TTTT TCTT (P8.2)

We will consider in our treatment only a representative subset of the [ten] conclusions, as presented in Lowe's article.

Remark 2 The authors summarily dismiss four of the ten conclusions (C2.1, C3.1, C4.1, and C6.1), suggesting an incomplete approach.

(C1.1) All abstract beings depend for their existence on concrete beings. (Follows from P3.1 and P4.1 together with D3.1 and D4.1.)

((((q&p)>#(u&p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&
((x>(u&p))=((s&t)>~%x))))>((v&p)>((v&p)>%#p)) ;
TTTT TTTT TTTT TTTT (C1.2)

(C5.1) In every possible world there exist concrete beings. (Follows from C1.1 and P2.1.)

((((((q&p)>#(u&p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&
((x>(u&p))=((s&t)>~%x))))>((v&p)>((v&p)>%#p)))&(%#(u&p)))>(#%w>%(v&p)) ;

```
                          TTTT TTTT TTTT TTTT(10)
                          TFTF TFTF TFTF TFTF( 2)
                          TTTT TTTT TTTT TTTT( 4)
```
(C5.1)

(C7.1) The existence of necessary abstract beings needs to be explained. (Follows from P2.1, P3.1 and P6.1.)

((%#(u&p))&(((q&p)>#(u&p))&(%#(q&p)>r)))>(%#(u&p)>r) ;

```
                          TTTT TTTT TTTT TTTT( 2)
                          TCTT TTTT TCTT TTTT( 2)
```
(C7.2)

(C8.1) The existence of necessary abstract beings can only be explained by concrete beings. (Follows from C1.1, P3.1, P7.1 and P8.1.)

(((((((q&p)>#(u&p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&
((x>(u&p))=((s&t)>~%x))))>((v&p)>((v&p)>%#p)))&((q&p)>#(u&p)))&
((~(#(q&p)>(r>%#(q&p))))&(p>(#r>%(q&p)))))>(%#(u&p) >(r>(v&p))) ;

```
                          TTTT TTTT TTTT TTTT
```
(C8.2)

(C9.1) The existence of necessary abstract beings is explained by one or more necessary concrete beings. (Follows from C7.1, C8.1 and P5.1.)

(((((%#(u&p))&(((q&p)>#(u&p))&(%#(q&p)>r)))>(%#(u&p)>r))&(((((((q&p)>
#(u&p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&((x>(u&p))=((s&t)>~
%x))))>((v&p)>((v&p)>%#p)))&((q&p)>#(u&p)))&((~(#(q&p)>(r>%#(q&p))))&
(p>(#r>%(q&p)))))>(%#(u&p) >(r>(v&p)))))&((~(%z<#z)&p)>%#p))>(%#(u&p)>(r>
%#(v&p))) ;

```
                          TTTT TTTT TTTT TTTT( 2)
                          TTTT TCTC TTTT TCTC( 2)
                          TTTT TTTT TTTT TTTT( 4)
```
(C9.2)

(C10.1) A necessary concrete being exists. (Follows from C9.1.)

(((((((%#(u&p))&(((q&p)>#(u&p))&(%#(q&p)>r)))>(%#(u&p)>r))&(((((((q&p)>
#(u& p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&((x>(u&p))=((s&t)>~
%x))))>((v&p)>((v&p)>%#p)))&((q&p)>#(u&p)))&((~(#(q&p)>(r>%#(q&p))))&
(p>(#r>%(q&p))))) >(%#(u&p) >(r>(v&p)))))&((~(%z<#z)&p)>#p))>(%#(u&p)>(r>
%#(v&p))))>(%#(v&p)) ;

```
                          FFFF FFFF FFFF FFFF( 2)
                          FFFF FNFN FFFF FNFN( 2)
                          FNFN FNFN FNFN FNFN( 4)
```
(C10.2)

Lowe also introduces some informal definitions which should help the reader understand the meaning of the concepts involved in his argument (necessity, concreteness, ontological dependence, metaphysical explanation, etc.). In the following discussion, we will see that most of these definitions do not bear the significance Lowe claims

Remark 3: The definitions in fact bear significance on their face. Examples are the injections of time to define omnipresence and space to define omnipotence (akin to the reasons in Popper's obscure footnote proof E(Gx)).

(D1.1) x is a necessary being := x exists in every possible world.

LET s, t, w, x, y: space, time , world, x, y.

(x>#p)=(#%w>%x) ;

```
TTTT TTTT TTTT TTTT( 8)
CCCC CCCC CCCC CCCC( 8)
FNFN FNFN FNFN FNFN(16)
```
(D1.2)

(D2.1) x is a contingent being := x exists in some but not every possible world.

(x>(%z<#z))=((%~#%w>%x) ;

```
TTTT TTTT TTTT TTTT( 8)
CCCC CCCC CCCC CCCC(24)
```
(D2.2)

(D3.1) x is a concrete being := x exists in space and time, or at least in time.

(x>(v&p))=(((%s&t)+t)>%x) ;

```
TTTT TTTT TTTT TTTT( 1)}x8
CCCC CCCC CCCC CCCC( 1)}
FFFF FFFF FFFF FFFF( 4)}x2
FTFT FTFT FTFT FTFT( 4)}
```
(D3.2)

(D4.1) x is an abstract being := x does not exist in space or time.

(x>(u&p))=((s&t)>~%x) ;

```
TTTT TTTT TTTT TTTT( 1)}x8
TTTT TTTT NNNN NNNN( 1)}
FFFF FFFF FFFF FFFF( 1)}x4
FFFF FFFF TTTT TTTT( 1)}
FTFT FTFT FTFT FTFT( 1)}
FTFT FTFT TFTF TFTF( 1)}
```
(D4.2)

(D5.1) x depends for its existence on y := necessarily, x exists only if y exists.

(%y>x)=#(%y>%x) ;

```
TTTT TTTT TTTT TTTT(16)
NNNN NNNN NNNN NNNN(16)
```
(D5.2)

(D6.1) (For any predicates F and G) F depend for their existence on G := necessarily, Fs exist only if Gs exist.

LET p, q: F, G.

#(p&q)>((%q>p)=#(%q>%p)) ;

```
TTTT TTTT TTTT TTTT
```
(D6.2)

We will work iteratively on Lowe's argument by temporarily fixing truth values and inferential relationships among its sentences, and then, after choosing a logic for formalization, working back and forth on the formalization of its axioms and theorems by making gradual adjustments while getting automatic real-time feedback about the suitability of our changes, vis-a-vis the argument's validity. In this fashion, by engaging in an iterative process of trial and error, we work our way towards a proper understanding of the concepts involved in the argument, far beyond of what a mere natural-language based discussion would allow.

Remark 4: The iterative process of back and forth formalization of axioms for adjustments

based on trial and error is not an exact approach because it suggests an *a priori* goal, such as consistently to refute proofs of the existence of God using the HOL/Isabelle tool.

Of 20 equations evaluated, 11 are *not* tautologous. This effectively refutes Lowe's proof, as rendered by the authors. This also invalidates the authors' rejection of Lowe's proof due to incompleteness (six of Lowe's conclusions are dismissed to avoid evaluation) and due to an interactive, trial by error approach to reconstruct Lowe. Therefore, the HOL/Isabelle tool failed as a showcase.