Primality test with Fibonacci numbers

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Every prime number without exception it´s represented as a factor of one of these two Fibonacci numbers, the previous and the following whose indexes are the previous and the posterior of that prime number.

Example: the prime number 41 is factor obligatorily of the nuumbers of Fibonacci(40) or Fibonacci(42)

Only on of this numbers can have the factor 41

With these ideas, our primality test ends because if we divide Fibonacci(40)/41 o Fibonacci(42)/41 and in either of the two cases it gives us remainder 0, we can conclude that 41 is a prime number.

You can also see it as if:

m.c.d.(Fibonacci(40),41) ó m.c.d (Fibonacci(42)/41) give us the prime 41

Or like if (Fibonacci(40) mod 41)=0 ó
(Fibonacci(42) mod 41)=0

In any of these equivalent situations we can conclude that 41 is a prime number, it´s a sufficient condition to be a prime number.

Now we go with the factors of the Fibonacci numbers that seem to follow the pattern of 2n+1 or 2n-1 being n a natural number in relation to the index of the Fibonacci number.
Only for prime numbers biggers than 5

Example:

Fibonacci(69) = 117669030460994 i´ts factors are:

2 x 137 x 829 x 18077 x 28657

We see that
138/69 = 2
828/69=12
18078/69 = 262

And finally

28656 y 28658 that don´t works and that although it has come out so fast is partly an exception, since according to my observations I believe that at least 80% follow this rule. What I consider a good approximation to publish it in this article.

It should be noted that both in the primality test and in the pattern that mainly Fibonacci numbers factors follow there is a certain random component, because we don´t know exactly if the prime number (x) i´ts in fibonacci(x+1) or fibonacci(x-1) and in the pattern of the factors we do not know a priori either if we have to add one or subtract one from the prime number to divide the prime between the correspondent fibonacci number and obtain a remainder of zero.

But there's more we know that if we have a prime number x divides either fibonacci(x+1) or Fibonacci(x-1)
but if divide fibonacci(x+1) also is factor of fibonacci(x-1)-1
and if divide fibonacci(x-1) also is factor of fibonacci(x+1)-1

This has the important consequence that prime number x also divides to

divide to m.c.d. (fibonacci(x+1),fibonacci(x-1)-1
or
divide to m.c.d. (fibonacci(x-1),fibonacci(x+1)-1