Refutation of the axiomatic translation principle for modal logic via resolution decision procedure

Abstract: The basis of the axiomatic translation principle is that "no resolution decision procedures are known for decidable first-order logic fragments relevant to modal logics that include the formula $\forall xyz(R(x,y)\wedge R(x,z)\rightarrow R(y,z))$ to express the Euclideanness of a relation R". We prove a decision procedure for that equation, and hence refute the axiomatic translation principle on which the conjecture is based. Therefore, the axiomatic translation principle is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Schmidt, R.A.; Hustadt, U. (2007). The axiomatic translation principle for modal logic. ACM transactions on computational logic. 8:4.19.
citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.153.7750&rep=rep1&type=pdf

1. Introduction: [N]o resolution decision procedures are known for decidable first-order logic fragments relevant to modal logics that include the formula $\forall xyz(R(x,y)\wedge R(x,z)\rightarrow R(y,z))$ expressing the Euclideanness of a relation R. (1.1)

LET p, q, r, s: x, y, R, z.

((r&(#p&#q))&(r&(#p&#s)))>(r&(#q&#s)) ;
TTTT TTTT TTTT TTTT (1.2)

Eq. 1.2 as rendered is tautologous, thereby refuting the conjecture of Eq. 1.1 and hence the axiomatic translation principle on which it is based.