Is there an order in the distribution of prime numbers?

Abstract

By arranging the prime numbers on four columns ten-to-ten (columns of one, three, seven, nine) and establishing a suitable correspondence between the quadruples obtained and the numbers between zero and fifteen, we obtain a synthetic representation of them which allows to establish that the order in the distribution of prime numbers among positive natural numbers is not random.

Silvio Gabbianelli
gasbion@iol.it

Is there an order in the distribution of prime numbers?

We could arrange prime numbers into four columns (we can exclude 2 and 5 for simplicity) leaving empty the spaces were shoud be not prime (fig. 1)

1	3	7	
11	13	17	19
	23		29
31		37	
41	43	47	
	53		59
61		67	
71	73		79
	83		89
		97	
101	103	107	109

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fig. 1

We could change empty cells with 0 and full one with 1 so to have (fig. 2)

1	1	1	0
1	1	1	1
0	1	0	1
1	0	1	0
1	1	1	0
0	1	0	1
1	0	1	0
1	1	0	1
0	1	0	1
0	0	1	0

1	1	1		1

…………………………………

fig. 2

The possible combinations of the two elements 0,1 are 16 and can be represented by the 15 decimal numbers plus 0 expressed in binary notation as showed at fig.3

0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

fig. 3

We have settled a correspondence between 15 first decimal number plus 0 and the combinations of prime numbers as showed at fig. 2 (fig.4)

1	1	1	0	14
1	1	1	1	15
0	1	0	1	5
1	0	1	0	10
1	1	1	0	14
0	1	0	1	5
1	0	1	0	10

1	1	0	1	13
0	1	0	1	5
0	0	1	0	2
1	1	1	1	15

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fig.4

If we transpose in rows of 10 elements the content of the fifth column of the fig. 4 (fig.5)

14	15	5	10	14	5	10	13	5	2
15	4	2	11	1	10	6	5	8	15
0	8	7	5	8	10	5	10	12	4
2	14	0	10	3	5	2	5	5	2
9	1	8	13	5	2	14	1	2	9
5	0	12	0	10	2	5	10	2	5
10	7	0	8	14	5	8	6	4	8
9	1	2	5	4	10	9	4	2	2
1	8	15	1	0	7	4	2	14	0
2	9	1	2	10	4	2	10	4	10

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fig.5

at the rows and columns we will have represented the tens and hundreds beginning from 0. At first row and at first column hundreds 0 and tens 0. The table shows synthetically that, for example, the tens 1 has all the four prime numbers present 11, 13, 17, 19 (value 15= 1111), while, for example, at hundreds 1 and tens 4 corresponds the only prime number 149 (value 1= 0001).

Analysing the synthetic representation as built up in fig. 5, we can note a certain regularity in the values disposition (fig.6)

14	15	5	10	14	5	10	13	5	2

15	4	2	11	1	10	6	5	8	15
0	8	7	5	8	10	5	10	12	4
2	14	0	10	3	5	2	5	5	2
9	1	8	13	5	2	14	1	2	9
5	0	12	0	10	2	5	10	2	5
10	7	0	8	14	5	8	6	4	8
9	1	2	5	4	10	9	4	2	2
1	8	15	1	0	7	4	2	14	0
2	9	1	2	10	4	2	10	4	10

fig.6

One third of the elements (white background) can be constituted by all the 16th values, while, the other two thirds, can be constituted only by 4 specific values respectively: 5, 4, 1, 0 (yellow background) e 10, 8, 2, 0 (sky blue background).

Now we can arrange, for example, the “yellow one” series to ten columns (fig. 7)

5	5	5	4	1	5	0	5	5	4
0	5	5	1	5	1	5	0	5	5
0	5	4	1	4	4	1	1	4	0
1	4	4	5	1	0	5	0	4	4
5	1	5	1	0	4	1	5	0	5
4	5	4	5	0	0	1	4	0	0
4	0	1	4	1	5	4	1	5	1
1	4	0	4	4	4	1	5	0	5
4	1	0	0	5	1	1	4	4	5
1	4	1	1	4	1	5	1	5	1
4	0	5	1	0	0	5	0	0	1

5	1	1	4	1	0	0	5	0	4
4	1	0	1	0	1	4	4	4	0
5	0	1	5	1	5	0	5	0	0
1	5	5	0	1	4	1	0	4	4

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fig.7

Each row will synthesize 3 rows of the table at fig. 6, the first value will indicate the tens 2 and specifically the two prime numbers 23 and 29 (value 5=0101), the second one is 3 tens from the first one and represents the prime numbers 53 and 59 (value 5) and so on 3 tens for 3 tens.

Analyzing the synthetic representation as built up in fig. 7, we can note a certain regularity in the values disposition (fig.8)

5	5	5	4	1	5	0	5	5	4
0	5	5	1	5	1	5	0	5	5
0	5	4	1	4	4	1	1	4	0
1	4	4	5	1	0	5	0	4	4
5	1	5	1	0	4	1	5	0	5
4	5	4	5	0	0	1	4	0	0
4	0	1	4	1	5	4	1	5	1
1	4	0	4	4	4	1	5	0	5
4	1	0	0	5	1	1	4	4	5
1	4	1	1	4	1	5	1	5	1
4	0	5	1	0	0	5	0	0	1

5	1	1	4	1	0	0	5	0	4
4	1	0	1	0	1	4	4	4	0
5	0	1	5	1	5	0	5	0	0
1	5	5	0	1	4	1	0	4	4
4	0	4	0	5	5	4	4	0	5
0	0	1	1	4	4	5	1	0	1
0	4	1	0	0	5	5	4	0	5
0	0	4	1	0	4	0	1	1	4
0	0	4	4	5	1	4	1	0	0
1	4	1	4	4	4	4	0	5	1
5	5	1	0	1	4	0	0	5	1
0	5	1	1	0	1	4	4	5	1
0	1	4	5	4	1	5	0	0	4
1	4	4	0	1	0	0	4	0	1
5	1	5	0	5	4	4	0	0	4
5	4	4	1	1	0	1	1	1	4
4	0	0	1	4	4	0	0	5	0

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fig.8

The emphasized diagonals are constituted, respectively only by: 4 and 0 the strong yellow , 1 and 0 the light yellow, it means that the first value emphasized represents the prime number 1523 (value 4=0100) while the second element of the diagonal represents the fact that none of the numbers 1851, 1853, 1857, 1859 is prime, and so on.

Now we can arrange the strong yellow series to ten columns (fig. 9)

4	0	0	0	4	0	0	4	0	4
0	4	4	4	4	4	4	0	0	4
4	0	4	0	0	0	4	4	0	4
4	0	0	4	4	0	0	0	0	0
4	4	4	0	0	0	4	4	0	0
0	4	0	4	0	0	0	4	4	0
4	0	0	0	4	4	0	4	0	0
4	4	0	0	4	0	0	0	0	0
0	0	0	0	4	4	0	0	0	4
4	0	4	4	0	0	4	4	4	0
0	0	0	0	0	0	0	4	0	4
0	4	0	0	0	0	0	0	0	0
0	4	0	0	4	4	4	0	0	0
0	4	4	4	0	4	0	0	0	0
0	0	4	0	4	0	0	4	4	0
0	0	4	0	0	4	4	0	0	4
4	0	4	4	0	0	0	4	4	0
0	0	0	4	4	4	0	0	0	0
4	4	4	0	0	0	4	0	0	0
4	4	0	4	4	0	0	4	4	0

....

fig. 9

Analysing the table (that synthesize the numbers ending for 3 from 1523 and growing with ratio 330 up to 551633) so built up, we could not observe positive and regular particularity or repetitions, the particularity of these numbers corresponding to these diagonal values 4, 1, 8, 2, are to could be prime (and in this case the only representative in its tens) was confirmed for the first million of prime numbers.

Numbers with the same propriety were found out also beginning from fig. 6 as showed in fig. 10

14	15	5	10	14	5	10	13	5	2
15	4	2	11	1	10	6	5	8	15
0	8	7	5	8	10	5	10	12	4
2	14	0	10	3	5	2	5	5	2
9	1	8	13	5	2	14	1	2	9
5	0	12	0	10	2	5	10	2	5
10	7	0	8	14	5	8	6	4	8
9	1	2	5	4	10	9	4	2	2
1	8	15	1	0	7	4	2	14	0
2	9	1	2	10	4	2	10	4	10
1	5	8	13	1	8	13	0	2	14
5	2	5	0	0	12	4	8	10	4
8	6	5	10	1	1	0	3	5	10
14	1	10	0	0	0	10	4	8	1
1	0	7	5	2	13	0	8	15	5
0	8	4	8	5	5	2	9	4	2
11	5	10	2	0	2	7	0	0	7
1	0	12	4	10	5	0	2	7	0
8	8	4	8	2	0	10	15	1	0

10	4	0	12	1	8	0	5	2	7
4	10	3	1	0	4	5	0	15	1
0	12	1	10	12	4	8	1	0	0
6	4	8	3	4	8	3	4	10	6
1	8	0	5	10	10	0	10	13	5
0	10	4	2	10	1	2	6	0	0
4	0	8	9	5	10	0	1	0	12
1	2	8	4	2	3	4	10	7	5
2	13	1	8	9	4	2	2	1	10
12	1	0	6	4	10	8	1	2	2
5	2	2	1	0	6	5	8	0	1

....

fig.10

beginning from 209 (value 1), 113 (value 4), 97 (value 2) and 211 (value 8), all with ratio 210. Really numbers with this property do exist for every series, beginning from an opportune number, with ratio 3x7x10=210, 3x11x10=330, 3x13x10=390, 3X17x10=510 and so on for each multiple of 30 for every prime number beginning from 7, as showed at fig. 11 for the case of the series of value 2/0

14	15	5	10	14	5	10	13	5	2	2/0=3x7x10
15	4	2	11	1	10	6	5	8	15	2/0=3x11x10
0	8	7	5	8	10	5	10	12	4	2/0=3x13x10
2	14	0	10	3	5	2	5	5	2	2/0=3x17x10
9	1	8	13	5	2	14	1	2	9	2/0=3x19x10

5	0	12	0	10	2	5	10	2	5
10	7	0	8	14	5	8	6	4	8
9	1	2	5	4	10	9	4	2	2
1	8	15	1	0	7	4	2	14	0
2	9	1	2	10	4	2	10	4	10
1	5	8	13	1	8	13	0	2	14
5	2	5	0	0	12	4	8	10	4
8	6	5	10	1	1	0	3	5	10
14	1	10	0	0	0	10	4	8	1
1	0	7	5	2	13	0	8	15	5
0	8	4	8	5	5	2	9	4	2
11	5	10	2	0	2	7	0	0	7
1	0	12	4	10	5	0	2	7	0
8	8	4	8	2	0	10	15	1	0
10	4	0	12	1	8	0	5	2	7
4	10	3	1	0	4	5	0	15	1
0	12	1	10	12	4	8	1	0	0

fig. 11

Analyze now the thirds of the quadruple which have the white background (fig. 12)

15	14	13	15	11	6	15	7	10	12
14	3	5	9	13	14	9	12	2	2
7	14	6	9	5	9	2	15	7	14
9	10	10	1	13	13	14	5	12	10

6	1	3	14	0	10	1	7	13	15
8	5	9	11	2	7	7	12	5	7
8	2	15	10	12	0	7	3	4	15
12	12	1	6	3	3	6	0	10	13
10	10	6	4	9	0	12	8	3	7
13	9	2	12	6	8	2	2	6	0
9	9	1	1	2	7	8	9	15	0
5	6	12	2	4	15	9	3	3	12
6	4	14	9	5	11	6	12	12	9
11	6	0	14	0	0	13	3	7	0

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fig.12

Each row will synthesize 3 rows of the table at fig. 6, the first value will indicate the ten 1 and specifically the four prime numbers 11, 13, 17 and 19 (value 15=1111), the second one is the 4th ten and represents the prime numbers 41, 43 and 47 (value 14) and so on 3 tens for 3 tens.

This third of the numbers represents the hard core of prime numbers; in fact, it has roughly the same number of prime numbers as the other two thirds put together! Analysing the synthetic representation as built up in fig. 12, we can't note regularity in the values disposition, if we take the values 7 to 7 that is to say every 210 numbers starting from the first quadruple (11, 13, 17, 19) we find all 15 values represented (fig.13)

15	7	13	14	7	13	3	15	7	10
12	0	9	9	6	7	12	12	6	14
11	12	3	0	9	8	11	5	2	8
10	9	6	3	9	1	6	1	4	1
1	7	11	13	2	14	3	6	13	12
4	5	3	1	12	0	3	10	2	9
13	13	6	9	10	1	7	14	4	4
6	5	11	1	9	10	12	7	0	6

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fig.13

but if we start from the second or third quadruple of fig. 12 (fig. 13 and 14)

14	10	14	6	14	14	14	8	12	12
12	10	0	2	0	8	2	6	12	0
12	10	4	10	10	2	14	10	6	12
4	12	12	10	2	4	6	2	0	14
6	0	14	0	12	10	0	0	0	14
0	4	0	14	4	6	10	8	2	6
10	4	4	0	2	14	14	0	14	6
10	10	8	6	12	10	10	2	10	12
4	4	0	10	12	8	0	12	12	8
2	2	0	10	0	14	8	8	2	0
0	0	2	4	8	12	6	8	10	2
4	2	14	4	8	2	2	8	8	14

……………………………………………………………………………

fig. 13

13	12	9	9	9	5	0	5	5	0
1	13	12	12	9	9	4	4	12	0
12	12	8	5	5	4	8	13	4	4
5	8	12	8	13	0	1	8	5	5
0	9	4	5	9	8	5	9	9	1
0	9	4	0	8	8	13	13	12	0
8	1	5	1	4	13	12	5	0	12
9	4	5	8	1	4	4	1	12	8
9	0	13	1	4	13	12	8	4	0
5	12	8	5	5	13	12	12	0	0
0	4	13	8	8	5	0	13	8	12

4 0 5 8 1 5 1 0 9 0

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fig. 14

we see that only some values are represented (0, 2, 4, 6, 8, 10, 12, 14) fig. 13 and (0, 1, 4, 5, 8, 9,12, 13) fig. 14 respectively with relative lower presence of prime numbers. In fact on a sample of 143 quadruples for each group the percentage of primes present goes from 48.4% of the initial quadruple 11, 13, 17, 19 to 36.2% of the initial quadruple 41, 43 , 47.49 and at 34.8% of the initial quadruple 71.73,77,79.

The conjecture that we can state at the end of this work trying to extend the upper showed property to the whole prime numbers is the following:

– excluding the 2 and the 5 we can represent the prime numbers through the correspondence between the quadruples of numbers, as illustrated at the beginning of this work, and the natural numbers from one to fifteen adding the zero;
– two thirds, can be constituted only by 4 specific values respectively: 5, 4, 1, 0 (yellow background) e 10, 8, 2, 0 (sky blue background). These numbers have not twins nor quadruple of prime numbers;
– through this synthetic representation we have seen that two thirds of these numbers contain roughly the same amount of prime numbers as the last third (white background), we can therefore define it as the hard core of prime numbers;
– within this nucleus (the hard core) there is in turn by appropriate intervals of quadruples, a core that is represented through all the 16 combinations unlike the two remaining thirds representing themselves through half plus the zero of these combinations (0, 2, 4, 6, 8, 10, 12, 14), (0, 1, 4, 5, 8, 9,12, 13);
– the percentages of prime numbers within the various but easily identifiable representations, showed in this work, vary significantly proving that the distribution of prime numbers is not random.