Refutation of inquisitive modal logic via flatness grade

Abstract: We evaluate two seminal definitions for flatness grade with neither tautologous. This refutes inquisitive modal logic and relegates it to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Meißner, S.; Otto, M. (2019). A first-order framework for inquisitive modal logic. arxiv.org/pdf/1906.04981.pdf

3.2 Graded flatness and the standard translation

Definition 3.4 (flatness grade).
The flatness grade ♭ (ϕ) ∈ N of ϕ ∈ InqML is defined by syntactic induction, for all ψ, χ ∈ InqML, according to

♭ (ψ → χ) := ♭ (χ); (3.4.1.1)

LET p, q, r, s: p, ψ, χ, ♭ ; + \V (intuitive disjunction).

(s&(q>r))=(s&r) ; TTTT TTTT **FF**TT TTTT (3.4.1.2)

♭ (ψ \V χ) := ♭ (ψ) + ♭ (χ) + 1. (3.4.2.1)

(s&(q+r))=(((s&q)+(s&r))+(%p>#p)) ;
CCCC CCCC CCTT TTTT (3.4.2.2)

Eqs. 3.4.1.2 and 3.4.2.2 as rendered are *not* tautologous. This refutes flatness grade and hence inquisitive modal logic.