Refutation of category theory by lattice identity, and graph-theoretic / set-of-blocks in partitions

Abstract: We evaluate a definition and two models of a method in graph theory to define any Boolean operation. The definition is *not* tautologous, refuting that only partition tautologies using only the lattice operations correspond to general lattice-theoretic identities. Defined models of graph-theoretic and set-of-blocks do not produce a common edge, but rather show the graph-theoretic definition implies the set-of-blocks definition. This refutes the graph-theoretic model as defining *any* Boolean operation on lattice partitions of category theory. What follows is that general lattice theory is also refuted via partitions. Therefore the conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ellerman, D. (2019). "A graph-theoretic method define any Boolean operation on partition". The Art of Discrete and Applied Mathematics 2 (2): 1–9. arxiv.org/pdf/1906.04539.pdf

Abstract: The lattice operations of join and meet were defined for set partitions in the nineteenth century, but no new logical operations on partitions were defined and studied during the twentieth century. Yet there is a simple and natural graph-theoretic method presented here to define any n-ary Boolean operation on partitions. An equivalent closure-theoretic method is also defined. In closing, the question is addressed of why it took so long for all Boolean operations to be defined for partitions.

4 The Implication Operation on Partitions

The real beginning of the logic of partitions, as opposed to the lattice theory of partitions, was the discovery of the set-of-blocks definition of the implication operation $\sigma \Rightarrow \pi$ for partitions. [T]he corresponding relation holds in the partition case: $\sigma \Rightarrow \pi = 1$ iff $\sigma \preccurlyeq \pi$.

A logical formula in the language of join, meet, and implication is a *subset tautology.* All partition tautologies are subset tautologies but not vice-versa. *Modus ponens* $(\sigma \wedge (\sigma \Rightarrow \pi)) \Rightarrow \pi$ is both a subset and partition tautology but Peirce's law, $((\sigma \Rightarrow \pi) \Rightarrow \sigma) \Rightarrow \sigma$, accumulation, $\sigma \Rightarrow (\pi \Rightarrow (\sigma \wedge \pi))$, and distributivity, $((\pi \vee \sigma) \wedge (\pi \vee \tau)) \Rightarrow (\pi \vee (\sigma \wedge \tau))$, are examples of subset tautologies that are not partition tautologies.

Remark 4.5: The equations above are tautologous, and hence must be subset tautologies and partition tautologies in order for partition logic to be bivalent.

The importance of the implication for partition logic is emphasized by the fact that the only partition tautologies using only the lattice operations, e.g., $\pi \vee 1$, correspond to general lattice-theoretic identities, i.e., $\pi \vee 1 = 1$. (4.6.1)

LET p, q, r, s, t, u, v: π, σ, B, C (or c), C' (or c), u (or a), u' (or b) .

(p+(%p>#p))=(%p>#p) ; TNTN TNTN TNTN TNTN (4.6.2)

Remark 4.6.2: Eq. 4.6.2 is *not* tautologous, hence that only partition tautologies using only the lattice operations correspond to general lattice-theoretic identities.

There is a link $u - u'$ in $G(\sigma \Rightarrow \pi)$ in and only in the following situation where $(u, u') \in \text{indit}(\pi)$ and $(u, u') \in \text{dit}(\sigma)$–which is exactly the situation when B is not contained in any block C of σ: Figure 1: Links $u - u'$ in $G(\sigma \Rightarrow \pi)$.

Remark Fig. 1 and 2: The diagrams' graphic images are not redrawn here as Figs. 4.7.1 or 4.8.1.

(((u<(r&s))&(v<(r&t)))>(u-v)) ;

TTTT TTTT TTTT TTTT(6)
FFFF FFFF FFFF TTTT(1)
FFFF TTTT **FFFF** TTTT(1) (4.7.2)

Thus the graph-theoretic and set-of-blocks definitions of the partition implication are equivalent.

Figure 2: Example of graph for partition implication.

Example [2] Let U = {a, b, c, d} so that K(U) = K4 is the complete graph on four points. Let σ = {{a} , {b, c, d}} and π = {{a, b} , {c, d}} so we see immediately from the set-of-blocks definition, that the π-block of {c, d} will be discretized while the π-block of {a, b} will remain whole so the partition implication is σ ⇒ π = {{a, b} , {c} , {d}}. After labelling the links in K (U), we see that only the a−b link has the Fσ⇒π 'truth value' so the graph G (σ ⇒ π) has only that a−b link (thickened in Figure 2). Then the connected components of G (σ ⇒ π) give the same partition implication σ ⇒ π = {{a, b} , {c} , {d}}. (4.8.1)

(((u+(v&(s&t)))>((u&v)+(s&t)))=((u&v)+(s+t)))>(u-v) ;

TTTT TTTT TTTT TTTT(3)
FFFF FFFF FFFF FFFF(5) (4.8.2)

Remark 4.9: For the graph-theoretic and set-of-blocks definitions of the partition implication to be equivalent, Eqs. 4.7.2 and 4.8.2 should be equivalent. (4.9.1)

(((u<(r&s))&(v<(r&t)))>(u-v))>((((u+(v&(s&t)))>((u&v)+(s&t)))=((u&v)+(s+t)))>(u-v)) ;

TTTT TTTT TTTT TTTT(2)
FFFF FFFF TTTT TTTT(1)
TTTT TTTT **FFFF FFFF**(2)
FFFF FFFF FFFF FFFF(1)
TTTT TTTT TTTT **FFFF**(1)
TTTT **FFFF** TTTT **FFFF**(1) (4.9.2)

Remark 4.10: To resuscitate the conjecture of Eq. 4.9.1, we remove the consequent in the models of (u-v) to test for equality of the antecedent models. (4.10.1) ((u<(r&s))&(v<(r&t)))=(((u+(v&(s&t)))>((u&v)+(s&t)))=((u&v)+(s+t))) ;

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TTTT TTTT FFFF FFFF( 1)
FFFF FFFF FFFF FFFF( 1)
FFFF FFFF TTTT TTTT( 1)
TTTT TTTT FFFF FFFF( 2)
FFFF FFFF FFFF FFFF( 1)
TTTT TTTT TTTT FFFF( 1)
TTTT FFFF TTTT FFFF( 1)
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(4.10.2)

Remark 4.11: To further coerce Eq. 4.9.1 from 4.10.1, we weaken the argument in 4.10.1 from an equality to a conditional via the imply connective. (4.11.1)

((u<(r&s))&(v<(r&t)))>(((u+(v&(s&t)))>((u&v)+(s&t)))=((u&v)+(s+t))) ;

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TTTT TTTT TTTT TTTT
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(4.11.2)

While the result of Eq. 4.11.2 is tautologous, it means that the graph-theoretic and set-of-blocks definitions do not produce the common edge of Figs. 1 or 2, but rather that Fig. 1 implies Fig. 2, to mean the graph-theoretic definition implies the set-of-blocks definition. This refutes the graph-theoretic model as defining any Boolean operation on lattice partitions of category theory.