An Oscillator Model for Nuclear Mass

Jean Louis Van Belle, *Drs, MAEc, BAEc, BPhil*

15 June 2019

Abstract

In this paper we use the oscillator or *Zitterbewegung* model of an electron to offer an equally elegant explanation of the mass of nucleons—equally elegant as the explanation we offered for the electron mass, that is. It is based on the same ideas: a nucleon charge with zero rest mass in orbital motion. The difference is the charge. The nucleon charge is a different charge—different from the electric charge. A different charge implies a different force. A different force implies a different amplitude of the oscillation – and we, therefore, find a different *Compton* radius for the nucleon.

Our interpretation of Wheeler's idea of mass without mass – based on equating the $E = m \cdot a^2 \cdot \omega^2 = m \cdot c^2$ using the tangential velocity formula $c = a \cdot \omega$ – remains valid. As an added bonus, we get an equally simple and elegant formula for the coupling constant for the strong force.

Contents

An Oscillator Model for Nuclear Mass

Jean Louis Van Belle, *Drs, MAEc, BAEc, BPhil*

15 June 2019

Introduction

The strong force must be strong: something must hold those positive charges in the nucleus together, and it must be stronger than the electrostatic force—because that force would push them apart. The strong force must also have a short range: otherwise it would be pulling all positive charges in the Universe together, and it does not do that. It is a strange force because we also think of this force as preventing electrons from sitting right on top of protons.[1]

This paper explores the strong force in very classical terms: no quantum field theory, no nuclear force quantum, no talk about field degrees of freedom or other hocus-pocus. Because this paper started with some classical explanation of the Yukawa potential, this first draft of an article will follow the same logic. Let us have a look at it.

The Yukawa potential

Most of what happens in nuclear physics – and in quantum mechanics in general – is discussed in terms of some *potential*. To be precise, physicists found the introduction of the so-called Yukawa potential – that was back in 1935 – extremely useful. So let us have a look at it. The Yukawa potential has the following shape:

$$\mathrm{U}(r) = -\frac{\mathrm{g_N^2}}{4\pi}\frac{e^{-r/a}}{r}$$

That looks weird but when you compare this to the potential we used to calculate electron orbitals, it's actually not as weird as it looks first. The potential we used to calculate electron orbitals was just the electrostatic potential:

$$\mathrm{V}(r) = -\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r} = -\mathrm{e}^2\frac{1}{r}$$

We may now make the following comparisons and remarks:

1. The g_N factor is analogous to the electric charge. The $1/4\pi\varepsilon_0$ factor in the $e^2 = q_e^2/4\pi\varepsilon_0$ expression is a *physical* proportionality factor which also ensures the units come out alright. So we have a 4π factor but do we have something like the ε_0 factor for the Yukawa potential as well?

No. Why not? We don't need such factor because we have some freedom here to define the *unit* for our nuclear charge g_N: there is or was nothing around when Yukawa first jotted this down—back in 1935,

[1] Of course, you probably know that an electron might actually be captured by a proton from time to time, but that's a phenomenon that has nothing to do with the strong force. To be precise, we think of electron capture as a *decay* of a proton. Proton decay – it decays into a neutron, obviously – involves the so-called weak force, which we imagine as an even weirder animal than the strong force so we won't talk about it here.

that is. In contrast, the *coulomb* was established in classical physics as the unit of charge *before* physicists thought of the proton as some kind of *elementary* charge.[2] That's why we have the ε_0 in Gauss' Law in electrostatics and, because of $\boldsymbol{E} = -\nabla V$, we have it in Poisson's equation for electrostatics too:

$$\nabla\cdot\boldsymbol{E} = \rho/\varepsilon_0 \Leftrightarrow \nabla^2 V = -\rho/\varepsilon_0$$

The ε_0 clearly emerges as a *physical* proportionality factor here: it fixes the units. The $\nabla\cdot\boldsymbol{E}$ is the divergence of the electric field $\boldsymbol{E}$, so that's the (outward) flux out of some *volume* around the point we're considering. A small digression to remind ourselves of the physics of the situation might be useful here.

We've written Gauss' Law in *differential form*—as opposed to its integral form. So we're really thinking of some infinitesimal volume here. At the same time, when calculating flux, we will integrate field strength (E) – which is expressed in newton per coulomb (N/C) – over some *surface* of some volume, so we get the $(\mathrm{N/C})\cdot\mathrm{m}^2$ dimension. However, when we write everything in differential form, we will want to express flux in terms of the *unit* volume, which is m^3. Hence, $\nabla\cdot\boldsymbol{E}$ is expressed in $(\mathrm{N/C})\cdot\mathrm{m}^2/\mathrm{m}^3 = \mathrm{N/m\cdot C}$. The ε_0 constant is expressed in $\mathrm{C}^2/\mathrm{N\cdot m}^2$ units, so $\varepsilon_0\cdot\nabla\cdot\boldsymbol{E}$ will, effectively, give us some charge *density*, so that's a charge *per unit volume*: $[\rho] = (\mathrm{C}^2/\mathrm{N\cdot m}^2)\cdot(\mathrm{N/m\cdot C}) = \mathrm{C/m}^3$.

What about the 4π factor? That's the 4π factor in the formulas for the surface *area* and the *volume* of a sphere, which is equal to $4\pi\cdot r^2$ and $4\pi\cdot r^3$ respectively.[3] We can also re-write Coulomb's Law in the following rather funny way:

$$\varepsilon_0 \mathrm{F} = \frac{\mathrm{q}_1\mathrm{q}_2}{4\pi r^2}$$

This tells us the electrostatic force – repulsive or attractive – is proportional to (1) the magnitude of the charges and (2) the surface area of the volume that's defined by the distance between the two charges. I am just kidding, of course! And then I am not. 😉

The point is: the electromagnetic force is *not* linear. It falls off with the *square* of the distance. In contrast, the potential and the distance do have this easy (inverse) proportionality:

$$V(r) \propto 1/r$$

That is why Yukawa inserted that $e^{-r/a}$ function, which we will discuss now.

2. The $e^{-r/a}$ function introduces the non-proportionality (or non-linearity, as I often call it[4]): it is 1 for $r = 0$ and goes to zero as $1/e^x$ ($x = r/a$) as r goes to infinity. The parameter $\boldsymbol{a}$ functions, therefore, as an effective *range* parameter. This *range* of the strong force is expressed in femtometer or *fermi* (1 fm = 1

[2] We may note here that the q_e is, effectively, the proton charge. The electron charge is $-q_e$ and explains why $V(r)$ is negative: we think of bringing an electron from some far-away place (free space – zero potential) closer to our one-proton nucleus and its potential energy will, therefore, be negative. This is a trivial point but it is useful to remind ourselves of the physics of the situation. For the same reasons, we will want to digress here and there on the physical dimensions of our equations.

[3] In case you'd want to look at how this works exactly, we recommend Feynman's shortcut to these results (see: Feynman's *Lectures*, Vol. II, Chapter 4, section 7 (http://www.feynmanlectures.caltech.edu/II_04.html).

[4] See: https://readingeinstein.blog/the-theory-of-everything/.

fermi = 1×10^{-15} m). We use the same scale when calculating the classical electron radius which we wrote as a fraction of the Compton radius of a proton:

$$r_{\mathrm{e}} = \alpha \cdot r_{\mathrm{C}} \approx \frac{1}{137} \cdot \frac{\hbar}{\mathrm{m_e} \cdot c} \approx \frac{386 \times 10^{-15}\ \mathrm{m}}{137} \approx 2.8 \times 10^{-15}\ \mathrm{m}$$

Now that we're talking about the radius of an electron, could we associate – just for fun – some *Compton* radius with a proton? Of course, we can. There is no *a priori* reason why we would find any meaningful result – as opposed to the Compton radius for an electron and a photon, which we can associate with an effective area of interference based on theoretical grounds[5] – but we can try, right? Let us see what we get:

$$a_{\mathrm{p}} = \frac{\hbar}{\mathrm{m_p} \cdot c} = \frac{\hbar}{\mathrm{E_p}/c} = \frac{(6.582 \times 10^{-16}\ \mathrm{eV} \cdot s) \cdot (3 \times 10^{8}\ m/s)}{938 \times 10^{6}\ \mathrm{eV}} \approx 0.21 \times 10^{-15}\ \mathrm{m}$$

That's the same order of magnitude as the radius of a proton we get out of scattering experiments, which is some value between 0.84 and 0.9 fm. The difference is about 1/4. Can we narrow this down? Not for the time being. A more precise calculation of the $a_{\mathrm{p}} = \hbar/\mathrm{m}c$ – using all of the significant digits for $\hbar$, m and c – yields something like 0.21008452488130184321492656765172, but – in light of the imprecision of the *measured* radius, it looks like this 0.21 fm value is an approximation that's good enough. Indeed, the various experiments – depending on whether one uses normal hydrogen, muonic hydrogen or deuterium – yield a radius between 0.842 0.842 ± 0.001 fm and 0.887 ± 0.012 fm.[6] Hence, the ratio of our theoretical $a_{\mathrm{p}} = \hbar/\mathrm{m}c$ value and the measured value is, effectively, *about* 0.25. Interesting—but what could it mean?

We don't think this is a coincidence, and so let us explore this. We will first want to remind ourselves of the basics of the electron oscillator model, which explains the Compton radius for an electron in a classical and, therefore, very *physical* way.

The oscillator model for an electron

In our classical interpretation of what an electron and a photon might actually *be*[7], we equate the Compton radius of an electron to the radius of Schrödinger's *Zitterbewegung*: the electron is some naked charge – something pointlike with zero rest mass[8] - moving about some center at the speed of light. It can do so because its rest mass is zero. The rest mass of the electron itself is nothing but the *equivalent* mass of the energy in this oscillatory motion: Wheeler's idea of mass without mass.

This concept led us to just take Einstein's mass-energy equivalence relation ($\mathrm{E} = \mathrm{m} \cdot c^2$) and, interpreting c as the tangential velocity of the naked charge, to substitute c for $a \cdot \omega$ (a tangential velocity will always

---

[5] See: *The Electron as a Harmonic Electromagnetic Oscillator* (http://vixra.org/abs/1905.0521) and *A Classical Quantum Theory of Light* (http://vixra.org/abs/1906.0200).
[6] See the Wikipedia article on the proton radius: https://en.wikipedia.org/wiki/Proton_radius_puzzle. For the larger value, see Ingo Sick's January 2018 publication (https://arxiv.org/abs/1801.01746).
[7] See the above-mentioned papers.
[8] Pointlike does not imply it has no dimension whatsoever. We think of the *classical* electron radius as the radius of the zero-mass *Zitterbewegung* charge.

equal the radius times the angular frequency). We then used the Planck-Einstein relation (ω = E/ħ = m·c^2/ħ) to find the Compton radius:

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{\mathrm{m} \cdot c^2} = \frac{\hbar}{\mathrm{m} \cdot c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The novel idea here is that one rotation – one *cycle* of the electron in its *Zitterbewegung* – does not only pack the electron's energy (E = m·c^2): it also packs Planck's quantum of action (S = *h*). The idea of an oscillation packing some amount of physical action may not be very familiar but it is quite simple: physical action is the product of (1) a force (the force that keeps our *zbw* charge in its circular orbit), (2) some distance (the circular loop) and (3) some time (the cycle time). For an electron, we got a cycle time that was equal to:

$$\mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{8.187 \times 10^{-14}\ \mathrm{J}} \approx 0.8 \times 10^{-20}\ \mathrm{s}$$

This is quite phenomenal, because it gives us a household-level current at the sub-atomic scale:

$$\mathrm{I} = \mathrm{q_e} f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}\ (ampere)$$

However, this model gives us consistent values for (1) the magnetic moment (the current times the area of the loop), (2) the angular momentum of an electron (*ħ*/2)[9] and, therefore, (3) the gyromagnetic ratio (aka *g*-factor) for the pure spin moment of an electron:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow g = 2$$

Can we repeat the trick for protons and neutrons? Our first intuitive response should be negative: the *Zitterbewegung* charge can spin around at the speed of light because its *rest* mass is zero. That is why Alexander Burinskii refers to it as the electron photon, or a toroidal photon.[10] We don't have a *Zitterbewegung* charge here, don't we?

We don't. That's the point. We have some other charge here, and a different kind of force.

An oscillator model for nucleons

The $\mathrm{g_N}$ factor is analogous to the electric charge. What does it mean, *exactly*? It means we can, perhaps, imagine, once again, some object that has no other mechanical properties but its charge and – perhaps – some tiny dimension. Its charge must be different than the *electric* charge, of course – because the force is different: we're talking a nuclear force here—*the* nuclear force, for the time being: the strong force. So we have some force holding some charge – with zero rest mass – in some orbit around some

[9] We assume the energy and, hence, the equivalent mass, is spread uniformly over the whole disk.
[10] Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008, https://arxiv.org/abs/hep-th/0507109.

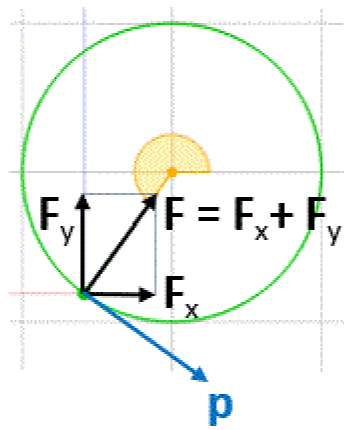
center, as shown below (Figure 1). It's a centripetal force – obviously – and its horizontal and vertical component can be written as the following functions of (1) the *magnitude* of that centripetal force (F) and (2*a*) the sine or cosine of the phase $\theta = \omega \cdot t$ or, alternatively, (2*b*) the *x* and *y* coordinates and the radius of the oscillation a_p:

- $F_x = F \cdot \cos(\theta - \pi) = -F \cdot \cos(\theta) = -F \cdot x/a_p$
- $F_y = F \cdot \sin(\theta - \pi) = -F \cdot \sin(\theta) = -F \cdot y/a_p$

We thus get the following formula for the force:

$$\mathbf{F} = \mathbf{F_x} + \mathbf{F_y} = -F \cdot \mathbf{cos}(\theta) - \boldsymbol{i} \cdot F \cdot \sin(\theta)$$

Figure 1: An oscillator model for nucleons?



When thinking all of this through in depth, we will get some same mathematical absurdities but – as we pointed out when developing our classical electron model – these aren't any more or less absurd than, say, Dirac's delta function: the function makes sense and then it doesn't. In fact, I would like to say a few words about it because it's quite relevant here.

Both mathematicians and physicists have rather heated discussions on what the delta function is and what it isn't. Mathematicians will often say there's nothing weird about it: they'll just *define* the Dirac delta function as the limit of a sequence of zero-centered normal distributions. But what does that mean in terms of the *physics* of the situation? If you think about that, you'll find it's a weird beast. The Wikipedia article on it[11] offers a fairly balanced view of what it is and what it isn't, but I like Feynman's characterization of it – and this characterization should also be sufficient in the context of this paper:

> "Dirac's $\delta(x)$ function has the property that it is 0 everywhere except at $x = 0$ but the $\int\delta(x)dx$ integral is finite: it's equal to one. *We must imagine that the δ(x) function has such a fantastic infinity at one point that the total area comes out equal to one*."

A line integral will give us some surface area, but here we have the product of zero and infinity. As Feynman notes: that infinity is so fantastic that its product with zero gives us some finite value (unity). As you will see, we'll be confronted with the same absurdity here: a product of zero and infinity. Let's go through the development step by step.

[11] See: https://en.wikipedia.org/wiki/Dirac_delta_function.

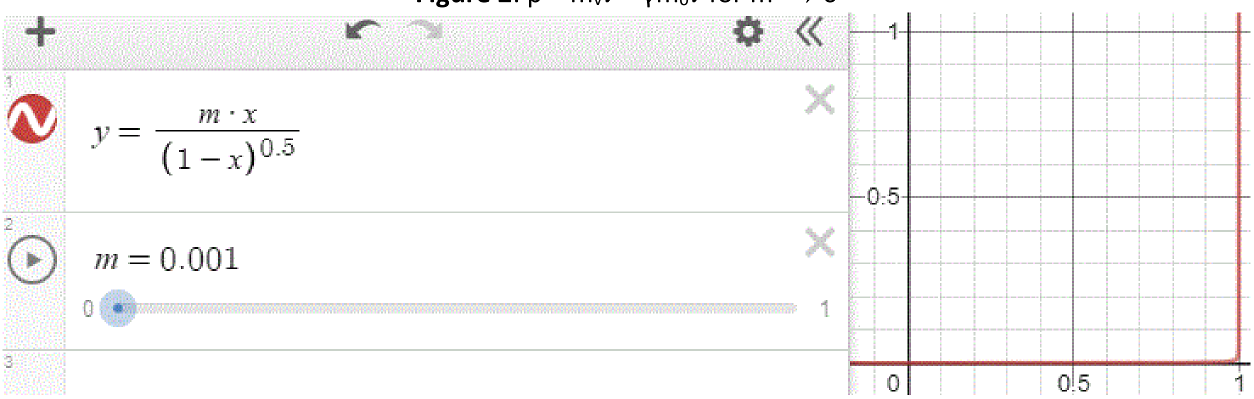
So we have some charge whizzing around at the speed of light. It's not the *electric* charge. It's some other charge. We can't define it for the time being: we can only relate it to the force—the strong force, or the nuclear force, or whatever other placeholder term you want to use for the time being. Let's call it the **nucleon force**, so we have some justification for the N in that g_N factor in Yukawa's potential.[12]

Let us now think about the momentum vector **p** in Figure 1. It should be relativistic momentum of course, so its magnitude is equal to:

$$p = mc = \gamma m_0 c$$

How should we calculate this? The m_0 factor is zero: it's the rest mass of our nuclear charge. The rest mass of g_N. It has to be zero because we think of it as whizzing around at the speed of light. Now, the Lorentz factor goes to infinity as the velocity goes to *c*, and m_0 is equal to zero. So we are multiplying zero by infinity. What do we get? An online graphing tool shows the behavior of the $p = \gamma m_0 v$ function is quite weird. We used desmos.com to produce the graph in Figure 2, which shows what happens with the $p = m_v v = \gamma m_0 v$ for $m = 0.001$ and v/c ranging between 0 and 1.

Figure 2: $p = m_v v = \gamma m_0 v$ for $m \to 0$



It is quite enlightening: p is (very close to) zero for *v/c* going from 0 to 1 but then becomes infinity at *v/c* = 1 itself. What can we say about this? Perhaps we should say that the momentum of an object with zero rest mass is a nonsensical concept. Let us avoid this for the time being. Let us just think of the momentum vector p as some kind of Dirac function: some weird beast, but we'll assume we can use it without having to worry. Let us now get back to our analysis of the force.

We will want to distinguish between (1) **p** as a *vector* with some magnitude p (and some direction that goes round and round, and (2) its horizontal and vertical component $\mathbf{p_x}$ and $\mathbf{p_y}$, whose directions do *not* change, but whose magnitudes p_x and p_y change all of the time. How *exactly* do they change? They change with the horizontal and vertical velocity, obviously: $p_x = mv_x = \gamma_x m_0 v_x$ and $p_y = mv_y = \gamma_x m_0 v_y$ respectively. *Huh?* What's γ_x and γ_y? If we distinguish directions and velocities, we will also want to distinguish the associated Lorentz factors.

Note that we wrote the force formula as $\mathbf{F} = \mathbf{F_x} + \mathbf{F_y} = -F\cdot\mathbf{cos}(\theta) - \boldsymbol{i}\cdot F\cdot\sin(\theta)$. It's a bit of a special notation: we use boldface for **cos**(θ) and for the imaginary unit ***i*** here so as to ensure we think of them as vector quantities: they have a magnitude, but they also have a direction and – importantly – some origin. An

[12] Of course, the N in g_N could also refer to nucle*ar* instead of nucle*on*, but perhaps we will want to reserve the term nucle*ar* to refer to something else later.

origin? Yes. We need to think about the reference frame. We know that we can represent the position vector ***r*** using the elementary wavefunction:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\theta) + i \cdot a \cdot \sin(\theta) = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

Hence, we might be tempted to write the force vector as $\mathbf{F} = -\mathrm{F} \cdot e^{-i\theta}$ but we shouldn't be doing this: the origin of the force vectors is not the same: the origin moves with the position vector. To be precise, origin is a term that is usually reserved to denote the origin of the reference frame. Vectors have an *initial* and a *terminal* point, and what we are saying here is that the initial point of our velocity, force and acceleration vector is *not* the origin. However, we will want to do that. We will, therefore, assume some quantization of space: we will want to think in terms of the force grabbing onto some linear space—not just one single point. That linear space is given by the radius of the oscillation a_p. I know this sounds outrageous but it's got to do with our interpretation of Planck's quantum as representing an elementary cycle—the cycle of an electron, a photon, an electron orbital and, in this new development here, the cycle of a nucleon. Physicists (and mathematicians) will probably want to see some lengthy philosophical argument here but we will spare our reader (and ourselves) from that for the time being.

The point is: we should think of the real and imaginary part of our wavefunction varying as the function of the position of our pointlike nucleon charge (g_N).[13] We can now calculate the centripetal acceleration: it's equal to $a_c = v_t^2/a = a \cdot \omega^2$. This formula is relativistically correct. In fact, it is useful to remind ourselves how we get this formula—again, just to make sure we understand the *physicality* of what we're writing here. The radius vector ***a*** has a horizontal and a vertical component: $x = a \cdot \cos(\omega t)$ and $y = a \cdot \sin(\omega t)$. We can, therefore, calculate the two components of the (tangential) velocity vector $\boldsymbol{v} = \mathrm{d}\boldsymbol{r}/\mathrm{dt}$ as $v_x = -a \cdot \omega \cdot \sin(\omega t)$ and $v_x\, y = -a \cdot\ \omega \cdot \cos(\omega t)$ and, in the next step, the components of the (centripetal) acceleration vector $\boldsymbol{a_c}$: $a_x = -a \cdot \omega^2 \cdot \cos(\omega t)$ and $a_y = -a \cdot \omega^2 \cdot \sin(\omega t)$. The magnitude of this vector is calculated as follows:

$$a_c^2 = a_x^2 + a_y^2 = a^2 \cdot \omega^4 \cdot \cos^2(\omega t) + a^2 \cdot \omega^4 \cdot \sin^2(\omega t) = a^2 \cdot \omega^4 \Leftrightarrow a_c = a \cdot \omega^2 = v_t^2/a$$

Now, the force law tells us that F is equal to $\mathrm{F} = \mathrm{m} \cdot a_c = \mathrm{m} \cdot a \cdot \omega^2$ but, again, we have this problem of determining what the mass of our pointlike charge actually is. The m_0 in our $\mathrm{m} = \gamma \mathrm{m}_0$ formula is zero! We need to find some other way. You'll say I've done a lot of hocus-pocus already, but this is really the last and final logical step to find some wonderfully elegant result. *Note how the horizontal and vertical force component behave like the restoring force in a linear harmonic oscillation*. This restoring force depends linearly on the (horizontal or vertical) displacement from the center, and the (linear) proportionality constant is usually written as k. In case of a mechanical spring, this constant will be the *stiffness* of the spring. We don't have a spring here so it is tempting to think it models some elasticity of space itself. However, we should probably not engage in such philosophical thought. Let us write down what we have:

---

[13] At this point, you may wonder why we use a g in g_N. N for nucleon, right? But g? Using a small *n* for the nucleon charge – just like we use a small e for the electron charge – would have been more logical, right? Perhaps. The answer is: we don't know. This nucleon charge is always denoted by g_N. It's a convention. Yukawa's formula is associated with quantum field theory and, in his article, he predicted the existence of some *nuclear force quantum*. Think of it as a predecessor of the idea of a *g*luon—if only because then you have some explanation for the *g*! It's got definitely *nothing* to do with a gyromagnetic ratio!

$$F_x = dp_x/dt = -k \cdot x = -k \cdot a \cdot \cos(\omega t) = -F \cdot \cos(\omega t)$$

$$F_y = dp_y/dt = -k \cdot y = -k \cdot a \cdot \sin(\omega t) = -F \cdot \sin(\omega t)$$

It's important to note that, while this doesn't *look* like it's relativistically correct, it actually is relativistically correct: we're doing nothing wrong here![14] Now, it is quite straightforward to show that the constant k can always be written as:

$$k = m \cdot \omega^2$$

We get that from the *solution* we find for ω when solving the differential equations $F_x = dp_x/dt = -k \cdot x$ and $F_y = dp_y/dt = F_y = dp_y/dt = -k \cdot y$ and assuming there is nothing particular about p and m. In other words, we assume there is nothing wrong with the $p = m \cdot v = \gamma m_0 v$ relation. So, again, we just temporarily make abstraction from the weird behavior of that function. In that sense, it's really just like Dirac's delta function: the function may or may not make sense mathematically but we think it's OK to use it as some *limit* and, when we do use it like we'd use any other function, our results come out OK. Let's move on and wrap this up.

So we have the $k = m \cdot \omega^2$ equation, but we know m is *not* the rest mass of our nucleon here. We need to find some innovative way of referring to it. Let's call it the *effective* mass of g_N as it's whizzing around at the speed of light. We need to remember it's a measure of inertia – and we measure that inertia along the horizontal and vertical axis respectively and, hence, we should, perhaps, write something like this: $m = m_\gamma = m_x = m_y$, in line with the distinction we made between p, p_x and p_y. Why m_γ? The notation is just a placeholder: we need to remind ourselves it is a relativistic mass concept and so I used γ (the symbol for the Lorentz factor) to remind ourselves of that.[15] So let us write this:

$$k = m_\gamma \cdot \omega^2$$

From the equations for F_x and F_x, we also know that $k \cdot a = F$, so $k = F/a$. Hence, the following equality must hold:

$$F/a = m_\gamma \cdot \omega^2 \Leftrightarrow F = m_\gamma \cdot a \cdot \omega^2 \Leftrightarrow F/a = m_\gamma \cdot a^2 \cdot \omega^2 = \Leftrightarrow F/a \cdot m_\gamma = a^2 \cdot \omega^2$$

We know the sum of the potential and kinetic energy in a linear oscillator adds up to $E = m \cdot a \cdot \omega^2/2$. We have *two* independent linear oscillations here so we can just add their energies and the ½ factor vanishes. We also know that the total energy in this oscillation must be equal to $E = m \cdot c^2$. The mass factor here is the *rest mass of our electron*, so it's *not* that weird relativistic m_γ concept. *However*, we did equate c to $a \cdot \omega^2$. Hence, we can now write the following:

$$E = m \cdot c^2 = m \cdot a^2 \cdot \omega^2 = m \cdot F/a \cdot m_\gamma$$

The force is, therefore, equal to:

$$F = (m_\gamma/m) \cdot (E/a)$$

[14] We invite the reader to tell us what we'd be doing wrong!

[15] The γ symbol may also remind you of a photon, and that's OK too because it's just like Burinskii's 'electron photon', or the 'toroidal photon' as he also referred to it in his email communications with me.

Now what can we say about the m_γ/m ratio? We know m_γ is sort of undefined—but it shouldn't be zero and it shouldn't be infinity. It is also quite sensible to think m_γ should be smaller than m. It cannot be larger because than the energy of the oscillation would be larger than $E = mc^2$. What could it be? 1/2, 1/2π? Rather than guessing, we may want to remind ourselves that protons and neutrons – or nucleons in general – have angular momentum, and their spin-1/2 particles so their angular momentum is equal to $L = \hbar/2$. We calculated it using the $L = I \cdot \omega$ formula – using an educated guess for the moment of inertia ($I = m \cdot a^2/2$) – in the context of our electron but we also have the $\mathbf{L} = \boldsymbol{r} \times \mathbf{p}$ formula, of course! The lever arm is the radius here, so we can write:

1. $L = \hbar/2 \Leftrightarrow p = L/a = (\hbar/2)/a = (\hbar/2) \cdot mc/\hbar = mc/2$
2. $p = m_\gamma c$

$\Rightarrow m_\gamma c = mc/2 \Leftrightarrow m_\gamma = m/2$

We found the grand result we expected to find: the *effective* mass of the pointlike nucleon charge – as it whizzes around the center of the two-dimensional oscillation that makes up our electron – must be equal to *half* (1/2 times) the rest mass of the nucleon.

We can now calculate the nucleon force using our $F = (m_\gamma/m) \cdot (E/a) = E/2a$ formula:

$$F_N = \frac{E_p}{2a_p} \approx \frac{1.5 \times 10^{-10}\ \mathrm{J}}{2 \cdot 0.21\ \times 10^{-15}\ \mathrm{m}} \approx 358{,}000\ \mathrm{N}$$

This force is equivalent to a force that gives a mass of 358 metric ton (1 g = 10^{-3} kg) an acceleration of 1 m/s per second. Does this make any sense? Probably not, right? But perhaps it does. Let us compare with the results we found for the force holding the electron together:

$$F_e = \frac{E_e}{2a_e} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{2 \cdot 386\ \times 10^{-15}\ \mathrm{m}} \approx 0.106\ \mathrm{N}$$

This force is equivalent to a force that gives a mass of about 106 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second. This was already *huge* at the sub-atomic scale, but a force that's equal to about 358,000 N? Because this is so enormous, we need to think about energy densities and, perhaps, wonder if *general* relativity comes into play. Have we been modeling a black hole? We can do an easy check by calculating the Schwarzschild radius for the proton. If we would pack all of the mass of the proton into a black hole, then the Schwarzschild formula gives us a radius that is equal to:

$$r_s = \frac{2 \cdot G \cdot m_p}{c^2} \approx \frac{2 \cdot 6.674 \times 10^{-11} \dfrac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \cdot 1.673 \times 10^{-27}\ \mathrm{kg}}{8.988 \times 10^{16} \dfrac{\mathrm{m}^2}{\mathrm{s}^2}} \approx 2.48 \times 10^{-54}\ \mathrm{m}$$

What was the Schwarzschild radius of an electron again?

$$r_s = \frac{2 \cdot G \cdot m_e}{c^2} \approx \frac{2 \cdot 6.674 \times 10^{-11} \dfrac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \cdot 9.1 \times 10^{-31}\ \mathrm{kg}}{8.988 \times 10^{16} \dfrac{\mathrm{m}^2}{\mathrm{s}^2}} \approx 1.35 \times 10^{-58}\ \mathrm{m}$$

This shows we should probably not worry all that much. This exceedingly small number has no relation whatsoever with the Compton radius. In fact, its scale has no relation with whatever distance one encounters in physics: it is *much* beyond the Planck scale, which is of the order of 10^{-35} meter and which, for reasons deep down in relativistic quantum mechanics, physicists consider to be the smallest possibly sensible distance scale.

Charges, forces, scales and coupling constants

It is easy to see that the ratio of our nucleon force and electron force – or whatever you'd want to call it – is

$$\frac{F_N}{F_e} = \frac{\frac{E_p}{2a_p}}{\frac{E_e}{2a_e}} = \frac{E_p}{E_e} \cdot \frac{a_e}{a_p} = \frac{m_p}{m_e} \cdot \frac{\frac{\hbar}{m_e \cdot c}}{\frac{\hbar}{m_p \cdot c}} = [\frac{m_p}{m_e}]^2 \approx 1{,}838^2 \approx \frac{358{,}000 \text{ N}}{0.106 \text{ N}}$$

Nice. The ratio of the two forces is (1) proportional to the ratio of the two masses and (2) *inversely* proportional to the ratio of their (Compton) radii. We can do an easy check on this formula—a check which also helps to understand what is that we are trying to model here. So we are calculating a ratio between two forces. What does that mean? We can think of it as follows: we have that k-factor in our oscillator model:

$$k = m_\gamma \cdot \omega^2$$

The m_γ was the *effective* mass of the pointlike charge as it speeds around at its lightning speed—literally. It was equal to $m_p/2$ for the proton and $m_e/2$ for the electron. The 1/2 factor is always there in this oscillator model. So we have a k for the electromagnetic force in the *Zitterbewegung* model for an electron, and now we have a k in this oscillator model for a nucleon. Hence, let us take the ratio of both:

$$\frac{k_N}{k_e} = \frac{\frac{m_p \cdot \omega_p}{2}}{\frac{m_e \cdot \omega_e}{2}} = \frac{\frac{m_p \cdot E_p}{2\hbar}}{\frac{m_e \cdot E_e}{2\hbar}} = \frac{\frac{m_p \cdot m_p \cdot c^2}{2\hbar}}{\frac{m_e \cdot m_e \cdot c^2}{2\hbar}} = [\frac{m_p}{m_e}]^2$$

We get the same result.[16] This is a ratio between forces, or between two elasticity coefficients, perhaps. What's the relation with coupling constants? Think of the formula for the electromagnetic coupling constant – the fine-structure constant, that is – which, after some manipulation, we could write as the ratio of (1) the product of the energies and the radii of the Bohr orbitals and (2) the product of the energy and the radius of the photon[17]:

$$\alpha = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma} = \frac{\frac{1}{n^2}\alpha^2 mc^2 \cdot \frac{n^2}{\alpha}\frac{\hbar}{mc}}{E_\gamma \cdot \frac{\hbar \cdot c}{E_\gamma}} = \alpha$$

[16] The k_e factor should not be interpreted as Coulomb's constant here. It's a related but *different* concept. We leave it to the reader to relate the two as an exercise.

[17] See: *A Classical Quantum Theory of Light* (http://vixra.org/abs/1906.0200).

What do we get if we write some similar number here? Let's call it a mass coupling constant. Why? Because this force is what seems to give our electron and our proton their rest mass (or rest energy). We will write it as α_N for the time being (we can also swap to some other symbol later):

$$\alpha_N = \frac{E_p}{E_e} \cdot \frac{a_p}{a_N} = \frac{m_p}{m_e} \cdot \frac{\frac{\hbar}{m_p \cdot c}}{\frac{\hbar}{m_e \cdot c}} = 1$$

A trivial result? I don't think so. It's the coupling constant of the strong force:

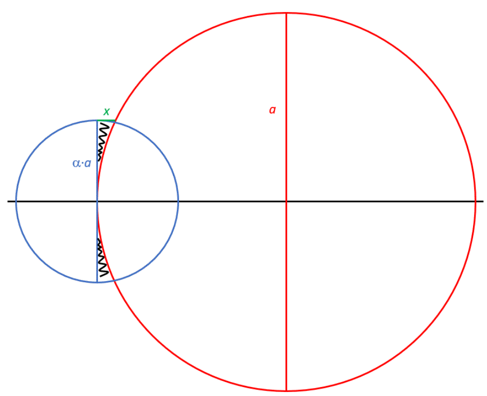
$$\alpha_N = \alpha_S = 1$$

We get the value we hoped we would find, and we get it from a surprisingly simple construction: an oscillator model, Einstein's mass-energy equivalence and the Planck-Einstein relation.

The coupling constant and the anomalous magnetic moment

The discussion above triggers an interesting question. In our classical analysis of the anomalous magnetic moment[18], we thought of the fine-structure constant as the radius of the charge. To be precise, we thought of it as the ratio between the radius of the charge and the Compton radius, as illustrated below (Figure 3).

Figure 3: The radius of the charge as a fraction of the *Compton* radius



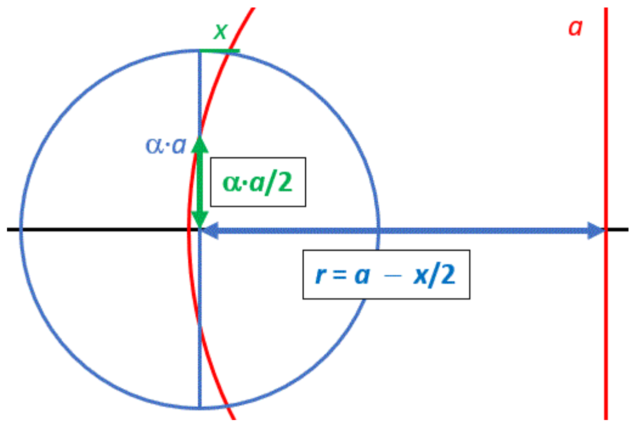
We also noted that the non-zero radius of our charge implied more of the charge was actually going *faster* than light and that we, therefore, should probably introduce some concept of an *effective* Compton radius (r), as opposed to its theoretical value (a). In our paper[19], we could show this small anomaly was likely to explain the anomalous magnetic moment. Indeed, we calculated a first-order correction that was equal to $\alpha/8$, which differs from Schwinger's $\alpha/8$ by a factor that's equal $4/\pi \approx 1.27$ only, which we should be able to explain doing away with the simplifications and – importantly – noting we'll have precessional motion because of the magnetic field in a Penning trap.

---

[18] See: *The Anomalous Magnetic Moment: Classical Calculations* (http://vixra.org/abs/1906.0007).

[19] Reference above.

Figure 4: Theoretical versus effective Compton radius



Again, we have to refer to our paper for the detail but it may be useful to show that the same logic should apply. We have some charge here—be warned: a *nucleon* charge. It's *not* the electric charge, so we write it as g_N instead of e. However, if we have a charge, then the idea of some *current* should also apply. So let us go through the same logic and see where we get. We can calculate this *nucleon* current as[20]:

$$I_N = g_N \cdot \omega_N$$

Note that the current does *not* depend on the velocity or the radius: g_N is just the (naked) charge, and ω is the angular frequency $\omega_N = E_N/\hbar = v/r$. We may, in fact, assume that *v* and *r* vary but their *ratio* remains the same.[21] So far, so good. However, the next analogy is *not* so easy. In fact, it's a logical step we can't take for the moment. The point is this: we defined the charges – e and g_N – as *analogous* but now we need to calculate… Well… The magnetic moment. We know what the magnetic moment of a circular *electric* current is, but what *analogy* should we use here? We said the nucleon charge g_N was something *like* an electric charge but – when everything is said and done – it is *not the same as an electric charge*. So the analogy breaks down here, and so we should stop writing here.

We will. We will want to think about this and come back to it in our next paper. Nevertheless, before we leave you, we would just like to take through an analogous model that might or might not pave the wave for some subsequent development. Let us assume we can also define some kind of *moment* – let us *not* call it magnetic – for the nucleon, and that its formula is the same: the current times the area of the loop. We will denote this moment by μ_N even if this is very confusing because – again – we do *not* want to suggest this moment is actually magnetic. It's something else. We just don't know what right now. However, assuming the nucleon and the electromagnetic force are *structurally* the same, we write:

$$\mu_N = I_N \cdot \pi r^2 = g_N \frac{mc^2}{h} \cdot \pi r^2 = g_N c \frac{\pi r^2}{2\pi a} = g_N c \frac{r^2}{2a}$$

[20] We will no longer use the subscript p (for proton) but generalize to include both protons and neutrons here. So we write ω_N, E_N, m_N etcetera instead of ω_p, E_p, m_p etcetera.

[21] We explore this in the mentioned paper: the idea here is that the rest mass of the pointlike charge would not *exactly* zero, but *very close* to zero.

What's that assumption? Let me repeat: the assumption is that the electromagnetic and the strong force are *structurally the same*. You'll cry wolf: we started off by saying they are very different, right? Yes and no. First, we don't talk the strong force here—not yet. We're talking some force that we baptized as the *nucleon* force because it's the force that holds nucleons – protons and neutrons – together. So we can say that, in our simplified model, the $k_N = m_N \cdot \omega_N^2$ and $k_e = m_e \cdot \omega_e^2$ proportionality factors are *very* different but *they both measure the strength of a structurally similar force*.

Let us continue the development. We want to calculate some gyromagnetic ratio – or the *equivalent* of that for the strong force, I should say – so we need the angular momentum. Angular momentum is calculated using (angular) mass and (angular) frequency, so we shouldn't hesitate to use the same formula:

$$\mathrm{L_N} = I_\mathrm{N} \cdot \omega_\mathrm{N} = \frac{\mathrm{m_N} \cdot r^2}{2} \cdot \frac{v}{r} = \frac{\mathrm{m_N} \cdot r \cdot v}{2} = \mathrm{m}_\gamma \cdot r \cdot v$$

The m_N is the *rest* mass of the nucleon, which is *twice* that of the pointlike nucleon charge (m_γ), so we write nothing new here. You should note, however, that we substituted *c* for *v* and *a* for *r*: the idea here is that the angular frequency ω remains the same ($\omega = E/\hbar = v/r$) because the rest mass (or rest energy) of the nucleon is what it is and, therefore, the radius *r* and *v* may be different from *a* and *c* but they are still related through the tangential velocity formula: $v = r \cdot \omega = r \cdot E/\hbar = r \cdot m \cdot c^2/\hbar$.

We can now calculate this equivalent moment – some *nucleon* moment or whatever you want to call it. In fact, what we are going to calculate is a gyromagnetic ratio.[22] Dropping the subscript N because we now need to use the subscript for *r* (the *effective* Compton radius), we write:

$$g_r = \frac{\mu_r}{\mathrm{L}_r} = \frac{\mathrm{I} \cdot \pi r^2}{\mathrm{m}_\gamma \cdot r \cdot v} = \frac{\mathrm{I} \cdot \pi \cdot \mathrm{r}}{\mathrm{m}_\gamma \cdot v}$$

What can we do with this? Nothing much. However, note that we introduced a subscript (g_r) to distinguish the *actual* value for *g* from its theoretical value, which we get from equating *r* to *a* and *v* to *c*. The symbols get quite confusing now because we have *g* for the gyromagnetic ratio and g_N for the nucleon charge, but you should be able to understand the formula below:

$$g = \frac{\mu}{\mathrm{L}} = \frac{\mathrm{I} \cdot \pi a^2}{\mathrm{m}_\gamma \cdot a \cdot c} = \frac{\mathrm{g_N} \cdot c \cdot \frac{a^2}{2a}}{\mathrm{m} \cdot a \cdot c/2} = \frac{\mathrm{g_N}}{\mathrm{m}}$$

You will say this doesn't look like the *g*-factor for a pure spin moment, and you are right. The convention is to write the g-factor as a multiple of $q_e/2m$, so it is a pure number. In this case, we should write it as a multiple of $g_N/2m$, so we get this:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{g_N}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{g_N}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{g_N}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

---

[22] In electromagnetics, and in quantum mechanics as well, we define the gyromagnetic ratio as the ratio of the magnetic moment and the angular momentum. That is why the anomalous magnetic moment is actually a misnomer. First, it is *not* a magnetic moment: it is the g-ratio. Second, as we try to show here, it may actually not be anomalous at all!

Looks good, you'll say. Yes, but we actually are not so happy with this convention because we think it obscures the matter hugely[23], so we'll just stick with our ratio – which is a *real* gyromagnetic ratio instead of some pure number – and let's see what happens. The anomaly is usually defined as the *difference* between real gyromagnetic ratio and the theoretical value ($g_r - g$). However, we think it's more useful to also write it as a ratio:

$$\frac{g_r}{g} = \frac{\dfrac{\mathrm{g_N} c \dfrac{r^2}{2a}}{\mathrm{m} \cdot r \cdot v/2}}{\dfrac{\mathrm{g_N} c \dfrac{a^2}{2a}}{\mathrm{m} \cdot a \cdot c/2}} = \frac{r^2}{a^2}\frac{a}{r} = \frac{r}{a}$$

This is a wonderful result: the anomaly is just the ratio between the effective and theoretical Compton radius of a nucleon. We can write it very simply:

$$g_r = (r/a) \cdot g$$

Looks great, doesn't it?

Not really. All we did was to establish an analogy. However, that analogy doesn't give us any answer to the obvious question: what *is* that nucleon charge? And what is that g-ratio that we associate with it? We'll need to relate both the electromagnetic charge, somehow. We'll think about that in the coming days and weeks and, hopefully, arrive at some meaningful conclusions.

Jean Louis Van Belle, 15 June 2019

[23] See: Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018 (http://vixra.org/abs/1812.0233).