Relative Phase and Time States

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Abstract: It was believed that quantum system with energy ground state could not have time state. However, it is merely that absolute phase or time state does not exist. As energy difference between two quantum oscillators can be arbitrary large negative or positive, relative phase and time states between the oscillators can be constructed.

In [1], attempt to construct phase state as $\sum_n \mathrm{e}^{in\theta}|n\rangle$ failed obviously because the formula involves just a single oscillator only to be able to represent, if any, absolute phase state, which should not exist. Though relative phase is briefly discussed in [1], its importance is overlooked.

At the bottom of page 58 in section V "Phase Difference of Two Oscillators" of [1], "state with phase difference θ" is defined using number state as:

$$|R,\theta\rangle = \sum_{m=0}^{R} \mathrm{e}^{im\theta}|m\rangle\,|R-m\rangle$$

where R is the total number of excitations and m is the number of excitations in the first oscillator.

By restricting R to be even and introducing N and n as $R=2N$ and $m=N+n$, we obtain:

$$|R,\theta\rangle = |2N,\theta\rangle = \mathrm{e}^{iN\theta}\sum_{n=-N}^{N} \mathrm{e}^{in\theta}|N+n\rangle\,|N-n\rangle$$

where absolute phase of $\mathrm{e}^{iN\theta}$ may be ignored. Then:

$$\lim_{N\to\infty}\langle\theta',2N|2N,\theta\rangle = \lim_{N\to\infty}\sum_{n'=-N}^{N}\sum_{n=-N}^{N}\langle N-n'|\langle N+n'|\mathrm{e}^{i(n\theta-n'\theta')}|N+n\rangle|N-n\rangle$$

$$= \lim_{N\to\infty}\sum_{n=-N}^{N}\mathrm{e}^{in(\theta-\theta')}$$

$$= \delta(\theta-\theta')$$

Thus, $\lim_{N\to\infty}\sum_{n=-N}^{N}\mathrm{e}^{in\theta}|N+n\rangle\,|N-n\rangle$ is the relative phase state. Note that, the state is, in a sense, classical, because the state involves infinite number of quanta, which is required by number phase uncertainty that number uncertainty of phase state must be infinite. Similarly, relative time state ωt should be: $\lim_{N\to\infty} \sum_{n=-N}^{N} \mathrm{e}^{in\omega t}|N+n\rangle|N-n\rangle$. Just as absolute phase is physically meaningless, so is absolute time. Thus, it is essential that phase and time states are defined only as relative phase and time. Note that sin and cos operators in [1] implicitly assume some phase reference to distinguish sin and cos.

Reference

[1] L. Suskind, J. Glogower, "QUANTUM MECHANICAL PHASE AND TIME OPERATOR", *https://link.aps.org/pdf/10.1103/PhysicsPhysiqueFizika.1.49*, 1964.