# THE MAGNITUDE OF ELECTROMAGNETIC TIME DILATION.

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Abstract. Theories unifying gravity and electromagnetism naturally give rise to the question of whether there might be a time dilation associated with the electromagnetic 4-potential. We show here that the magnitude of EM time dilation can be computed from elementary considerations that are independent of specific unified theories. We further show that the electrostatic part of the effect is well within reach of experiment, while the magnetic part is not.

## Contents

## 1. Introduction

From the first publication of General Relativity in 1915 to about 1930, hundreds of classical theories were proposed attempting to unify gravity and electromagnetism.[?] While none of these was completely successful, some of them were very influential. For example, Weyl's Space-Time-Matter theory[?] introduced the notion of gauge invariance, while Kaluza-Klein theory[?, ?] used a compact 5th dimension and was an important precursor to string theory

Given that there is a time dilation associated with the gravitational potential in GR, it seems reasonable to wonder whether there might be a similar time dilation associated with the EM potential in such unified theories. Sadly, this question has rarely been asked, let alone answered. Even after nearly a century, we don't know whether Kaluza-Klein theory has this feature or not. Apsel in 1978-1981 gave probably the first unified theory to explicitly predict such a time dilation[?, ?, ?], and only a handful of subsequent papers[?, ?, ?, ?, ?, ?, ?, ?] mention anything similar. At least to first order, all of these theories agree on the magnitude of EM time dilation.

In this paper we show why they must. We derive the magnitudes of both gravitational and electromagnetic time dilations from elementary considerations that do not depend on the machinery of GR or any specific unified theory, and therefore demonstrate that they must be features of any unified theory that is compatible with both Special Relativity and Quantum Mechanics.

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## 2. History

Einstein first derived gravitational time dilation in his 1907 paper on the Relativity Principle[?]. He began in section 18 by using Special Relativity to show that clocks at different X-position in a reference frame accelerated in the X-direction cannot run at the same rate; then in section 19 he used the Equivalence Principle to infer that the same thing must be true for clocks at different values of a gravitational potential. He carried out all the arguments to first order to give the linear form $T_d = 1 + \Phi/c^2$, which has since become called the weak-field approximation, although he did note in passing that the actual formula must be $T_d = e^{\Phi/c^2}$.

The conclusion of the 1907 argument is that *any* acceleration causes the rate of time flow to be a function of position in the direction of the acceleration. It did not matter to Einstein whether the acceleration was caused by a rocket, or by standing on the ground in a gravitational field, or by being attached to a spinning disk. Although he didn't mention it, it is worth noting that the acceleration of a charged particle by an electric field is not immune to this argument. Neither are accelerations due to the weak and strong forces; *all* accelerations of a given magnitude *must* cause exactly the same time dilation.

After General Relativity in 1915 and the Schwarzschild solution in 1916, another view became possible, although it is still not widely appreciated. Taking the Newtonian (weak field, low speed) limit of the Schwarzschild metric leaves us with the Newtonian metric

$$ds^2 = (dx^2 + dy^2 + dz^2 - c^2dt^2) + (-\frac{2GM}{rc^2})c^2dt^2$$

which is just flat Minkowski spacetime plus the time dilation field. In this metric, space is completely flat and only time is curved, and the curved time gives geodesics that match Newtonian gravity. This pure time dilation field appears as a $1/r^2$ "force". So in Newtonian GR, matter causes time dilation and time dilation causes gravitational acceleration. The direction of causality is completely reversed from the 1907 argument.

If we accept both of these arguments, then we cannot have any acceleration without an associated time dilation, and we cannot have any time dilation without an associated acceleration. The two are inextricably linked.

## 3. Gravitational time dilation without General Relativity

In this section we use a different method to derive gravitational time dilation without invoking General Relativity. We assume only that particles have an energy associated with their mass, given by $E = mc^2$, and a frequency associated with their energy, given by $E = h\nu$.

In a uniform gravitational field of strength $g$, raising the particle by a height $z$ requires work $mgz$. Thus, to an observer at height 0, the total energy of the particle at height $z$ is given by $E(z) = mc^2 + mgz$ and its frequency by $\nu(z) = E(z)/h$.

However, an observer already at height $z$ would perceive the particle to have merely frequency $\nu(0) = mc^2/h$. This can only be true if the two observers have clocks running at different rates, in the ratio

$$T_d = \frac{\nu(z)}{\nu(0)} = \frac{E(z)}{E(0)} = \frac{mc^2 + mgz}{mc^2} = 1 + \frac{mgz}{mc^2} = 1 + \frac{gz}{c^2}$$

which is the weak-field approximation to GR's gravitational time dilation. A more careful analysis gives the exactly correct exponential form $T_d = e^{gh/c^2}$ or, for a more general potential $\Phi$, $T_d = e^{\Delta\Phi/c^2}$.

This derivation has several interesting features. First, it appears in some sense to be quantum, since it utilizes $E = h\nu$. But because time dilation is a ratio, $h$ cancels out and its precise value doesn't matter. This means that the classical ($h \to 0$) limit is exactly the same as the "quantum" result.

Since both this derivation and Einstein's 1907 one avoid almost all the assumptions of GR, each of them implies that any other theory that predicts a gravitational time dilation must have the same ratio of dilation to potential as GR, as long as $E = mc^2$ and $E = h\nu$ are both still true. From this viewpoint, the existence and magnitude of gravitational time dilation cannot be viewed as a confirmation of GR specifically, but only of a class of theories of which GR is the best known example.

## 4. Electromagnetic time dilation by the same method

We now consider the case of a particle with mass $m$ and charge $q$ in an electrostatic potential $V$. The change in energy is $q\Delta V$, so the corresponding time dilation (to first order) must be

$$T_d = \frac{mc^2 + q\Delta V}{mc^2} = 1 + \frac{q\Delta V}{mc^2}$$

Unlike in the gravitational case, here both charge and mass matter. Uncharged particles should be completely unaffected. For a given non-zero $q$, lighter particles will be dilated more strongly than heavier particles. The electron, being the lightest charged particle and having the highest charge-to-mass ratio, should be affected the most. But since electrons have infinite lifetime, the only observable effect on them is the shift in phase frequency. Although this is universally observed, most physicists would not consider it proof of or even evidence for time dilation.

Thus, for experimental testing, we are lead to the muon. With a mass-energy of $m_\mu c^2 = 105.7$MeV, it is still light enough to have its mean lifetime of $2.2\mu$S affected by a modest potential. For example, a potential of 1.057 MV should alter its lifetime by about 1%; such a potential could be achieved by a Van de Graaff generator with a sphere of about 76 cm diameter in air, which is well within reach of a serious hobbyist. Apsel first proposed this kind of experiment in 1979[?]; 40 years later it still has never been performed.

Muon bound to low-Z nuclei are also known to have lengthened lifetimes. The normal explanation for this is that the muon has a kinetic energy given by the quantum virial theorem, and an average velocity corresponding to that kinetic energy, and a special-relativistic time dilation corresponding to that velocity. However, Apsel has argued that this calculation does not match the experimental data very well, and that adding electromagnetic time dilation gives a better fit[**?**]. If so, we may have already been seeing evidence for decades. The effect should be more obvious for higher-Z. Unfortunately, as Z increases, nuclear capture by a proton begins to dominate, and we don't have good data on non-capture decay rates for most elements.

For magnetic interactions, the muon's measured magnetic moment is $-4.49 \times 10^{-26}$ J/T. To get the same 1% level of time dilation, say between spin-up and spin-down muons, we would need to place them in a field of

$$1.057 \text{ MeV} \times \frac{1 \text{ J}}{6.24 \times 10^{12} \text{ MeV}} \times \frac{1 \text{ T}}{2 \times (4.49 \times 10^{-26} \text{ J})} = 1.89 \times 10^{12} \text{ T}$$

Given that the current world record magnetic field is only $1.2 \times 10^3$ T, this seems beyond the reach of current experiment. Van Holten thought that 5 GT might suffice and could be found in the vicinity of a magnetar[**?**, **?**].

## 5. Discussion

## 6. Counterarguments

I am aware of five classes of counterarguments to Apsel-style theories. Some of them claim that no such effect can possibly exist; others, that even if it exists it would not constitute a time dilation.

### 6.1. **Naive Electromagnetic Gauge Invariance.**

Many physical theories, such as classical EM and Van Holten's theory mentioned in the previous section, have a property that I will call Naive Electromagnetic Gauge Invariance. In NEGI theories, everything can be expressed in terms of fields acting locally; potentials can be viewed as having no physical reality but being merely aids to computation. NEGI would of course rule out any time dilation effects from an EM potential in a field free region, such as inside the sphere of a Van De Graaff generator. Many physicists seem to think that this is sufficient to disprove the theory.

The problem with this viewpoint is that it is flat-out wrong. The universe does *not* have the NEGI property; the Aharonov-Bohm effect[**?**, **?**] suffices as a counterexample. The importance of this is often glossed over. For example, Jackson and Okun[**?**, p.24] write:

> … gauge invariance is a manifestation of non-observability of $A_\mu$. However integrals … are observable when they are taken over a closed path, as in the Aharonov-Bohm effect … The loop integral

> of the vector potential there can be converted by Stokes's theorem into the magnetic flux through the loop, showing that the result is expressible in terms of the magnetic field, albeit in a nonlocal manner. It is a matter of choice whether one wishes to stress the field or the potential, but the local vector potential is not an observable.

Contrast this with the discussion in Feynman Vol. II[?] lecture 15-5, where the central importance of the potential is emphasized:

> The fact that the vector potential appears in the wave equation of quantum mechanics (called the Schrödinger equation) was obvious from the day it was written. That it cannot be replaced by the magnetic field in any easy way was observed by one man after the other who tried to do so. This is also clear from our example of electrons moving in a region where there is no field and being affected nevertheless. But because in classical mechanics $\boldsymbol{A}$ did not appear to have any direct importance and, furthermore, because it could be changed by adding a gradient, people repeatdly said that the vector potential had no direct physical significance — that only the magnetic and electric fields are "right" even in quantum mechanics. It seems strange in retrospect that no one thought of discussing this experiment until 1956 ... The implication was there all the time, but no one paid attention to it. ... It is interesting that something like this can be around for thirty years but, because of certain prejudices of what is and is not significant, continues to be ignored.

In either case, the NEGI idea (that fields acting locally on particles can explain everything) is admitted to be false.

6.2. **CPT Invariance.** It is often stated (e.g. in [?, ?, ?]) that the CPT theorem guarantees that particle and antiparticle masses and lifetimes are identical. However, this conclusion is only justified at zero potential, or with the further assumption of NEGI (which renders potential irrelevant). A true CPT reflection must invert all charges in the universe, which necessarily inverts all electric potentials as well. Therefore, the CPT theorem only *really* proves that a particle's mass and lifetime at 4-potential $A$ must equal its antiparticle's mass and lifetime at 4-potential $-A$. This holds true under EM time dilation, since the dilations for those two cases are identical. Thus, the CPT theorem does not contradict the claim that particles and antiparticles will be time-dilated oppositely at a non-zero potential and that their lifetimes will differ there. EM time dilation is completely compatible with the notion of CPT invariance.

6.3. **The S-Matrix and Accessible States.** An argument due to M. Gelfand[?] is based on the muon decay time being given by the S-matrix of the standard model. Since this matrix couples the muon initial state to possible final states, and placing the muon in a potential reduces or increases the number of accessible final states, Gelfand notes that one would expect an alteration in muon decay rate from this alone. So, while the previous arguments all claim that there can be no alteration in muon lifetime, this argument predicts that there *will* be an alteration, but explains it as being due to the mechanics of the S-matrix and not due to any time dilation. (Obviously, since this argument disagrees with the others about whether or not there will be an effect, at least one of them must be wrong.)

While this argument is subtle, I do not find it compelling. It is completely general and applies to any potential, including gravitational ones. Therefore, if we accept it, we must also accept that alteration of lifetime by a gravitational potential is entirely due to the S-matrix and not due to any time dilation. But we have substantial experimental evidence that gravitational time dilation exists, and our current best theory is that it is explained by GR alone.

This does not mean that the S-matrix approach is wrong; it merely means that the S-matrix is capable of expressing a time dilation. Thus, the ability to calculate accurate decay lifetimes entirely from S-matrix considerations does not preclude the existence of a potential-related time dilation.

One characteristic of a pure time dilation is that, all other things being equal, it must necessarily slow down (or speed up) all decay modes equally. Since muons have 3 known decay modes[**?**], this can be used as a test for whether lifetime alterations can reasonably be viewed as solely due to time dilation, or whether other factors must be invoked.

6.4. **Consequences of Linearity.** In the above first order formula, the energy of a charged particle is a linear function of potential, and the time flow is likewise linear with voltage, and thus has many of the same problems as the weak-field gravitational approximation. In particular, for any particle there should be a potential at which the absolute phase frequency goes to zero; for example, the frequency of a $\mu^-$ should go to zero in a potential of $m_\mu c^2/q_\mu$ =+105.658 MV. The time flow at this potential must also be zero, so the predicted muon lifetime is infinite. Even worse, at higher potentials the time flow is predicted to be negative, and so is the lifetime. It is not clear what this could possibly mean.

One could conceive that the "real" theory, as in the case of gravitational time dilation, might be exponential, or otherwise non-linear. That would solve the problems of the previous paragraph. However, finding such a formula is not as trivial as in the gravitational case, and it would also appear to require abandoning either $E = h\nu$, $E = mc^2$, or $\Delta E = q\Delta V$. Since our derivation assumes all of these as universally applicable principles, this is not easy to work around.

## 7. Summary

We reviewed two early derivations of gravitational time dilation and gave a new elementary derivation of it. Both the 1907 Einstein derivation and this new method can be trivially modified to give a simple derivation of electromagnetic time dilation as well, which agrees in magnitude with the handful of prior theories predicting such an effect. That such an effect seems so inescapably implied, and is yet so widely rejected (with multiple counter-arguments, albeit some flawed) points perhaps to a deep paradox in current physical thought. Since testing for the electrostatic effect would be quite easy and cheap, it seems worthwhile to actually perform that experiment.

## References

[1] V.P. Vizgin (tr.J.B. Barbour), "Unified Field Theories in the first third of the 20th century", Birkhäuser Verlag (1994)
[2] H. Weyl, *Space-Time-Matter* (1922) (quote on p.283)
[3] T. Kaluza, "Zum Unittsproblem in der Physik", *Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.)*: 966972.
[4] O. Klein, "Quantentheorie und fnfdimensionale Relativittstheorie", *Zeitschrift fr Physik A.* 37 (12): 895906 doi:10.1007/BF01397481.
[5] D. Apsel, "Gravitational, electromagnetic, and nuclear theory", *International Journal of Theoretical Physics* v.17 #8 643-649 (Aug 1978) DOI: 10.1007/BF00673015
[6] D. Apsel, "Gravitation and electromagnetism", *General Relativity and Gravitation* v.10 #4 297-306 (Mar 1979) DOI: 10.1007/BF00759487
[7] D. Apsel, "Time dilations in bound muon decay", *General Relativity and Gravitation* v.13 #6 605-607 (Jun 1981) DOI: 10.1007/BF00757247
[8] W.A. Rodrigues Jr., "The Standard of Length in the Theory of Relativity and Ehrenfest Paradox", *Il Nuovo Cimento* v.74 B #2 199-211 (11 April 1983)
[9] L.C.B. Ryff, "The Lifetime of an Elementary Particle in a Field", *General Relativity and Gravitation* v.17 #6 515-519 (1985)
[10] R.G. Beil, "Electrodynamics from a Metric", *Int. J. of Theoretical Physics* v.26 #2 189-197 (1987)
[11] J.W. van Holten, "Relativistic Time Dilation in an External Field", NIKHEF-H/91-05 (1991)
[12] J.W. van Holten, "Relativistic Dynamics of Spin in Strong External Fields"", `arXiv:hep-th/9303124v1`, (24 March 1993)
[13] A.K. Dubey, A.K. Sen, "Gravitational Redshift in Kerr-Newman Geometry"", `arXiv:1503.03833v5`, (16 October 2015)
[14] Piotr Ogonowski, "Time dilation as field", Journal of Modern Physics, 3, 200-207 (2012)
[15] P. Ogonowski, P. Skindzier, "Maxwell-like picture of General Relativity and its Planck limit", `https://arxiv.org/pdf/1301.2758.pdf`
[16] A. Einstein, "Über das Relativitätsprinzip und die aus demselben gezogenenn Folgerungen", *Jahrbuch der Radioaktivität und Elektronik* 4, 411-462 (4 December 1907); English translation, "On the relativity principle and the conclusions drawn from it", *The Collected Papers*, v.2, 433-484 (1989), available at `https://einsteinpapers.press.princeton.edu/vol2-trans/319`
[17] W. Ehrenberg, R. E. Siday,. "The Refractive Index in Electron Optics and the Principles of Dynamics", *Proc. Phys. Soc.* B62: 8-21 (1949). DOI: 10.1088/0370-1301/62/1/303
[18] Y. Aharonov, D. Bohm, "Significance of Electromagnetic Potentials in the Quantum Theory", *Phys. Rev.* 115, 485-491 (1959). `http://link.aps.org/doi/10.1103/PhysRev.115.485` DOI: 10.1103/PhysRev.115.485

[19] J. D. Jackson & L. B. Okun, "Historical roots of gauge invariance", LBNL-47066 (12 March 2001)
[20] R. P. Feynman, *Lectures On Physics* v.II, Addison Wesley (1964)
[21] V.A. Andreev et al., "Measurement of the Rate of Muon Capture in Hydrogen Gas and Determination of the Protons Pseudoscalar Coupling $g_P$", submitted to Phys.Rev.Lett arXiv:0704.2072v1
[22] H. Murayama, "CPT Tests: Kaon vs Neutrinos", arXiv:hep-ph/0307127
[23] R.G. Sachs, *The Physics of Time Reversal*, U. Chicago Press (1987), p.175
[24] M. Gelfand, personal communication (2013)
[25] K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) `http://pdg.lbl.gov/2014/listings/rpp2014-list-muon.pdf`