A QUANTUM DYNAMICS OF BETA DECAY

Vu B Ho

Advanced Study, 9 Adela Court, Mulgrave, Victoria 3170, Australia

Email: vubho@bigpond.net.au

Abstract: In this work we re-examine a model of the nucleons that involve the weak interaction which was once considered by Heisenberg; that is a neutron may have the structure of a dwarf hydrogen-like atom. We formulate a quantum dynamics for the associated interaction that involves the beta decay in terms of a mixed Coulomb-Yukawa potential and the More General Exponential Screened Coulomb Potential (MGESCP), which has been studied and applied to various fields of physics. We show that all the components that form the MGESCP potential can be derived from Dirac equation which in turns can be derived from a general system of linear first order partial differential equations. There are many interesting features that emerge from the MGESP potential, such as the MGESP potential can be reduced to the potential that has been proposed to describe the interaction between the quarks for strong force in particle physics, and the energy spectrum of the bound states of the dwarf hydrogen-like atom is continuous with respect to distance. This result leads to an unexpected implication that a proton and an electron may also interact strongly at short distances. We also show that the Yukawa potential when restrained can generate and determine the mathematical structures of fundamental particles associated with the strong and weak fields.

1. Introductory

Despite the mathematical formulation of quantum mechanics has been highly developed and the theory has been successfully applied into all domains of applied sciences with the most accuracies that can be achieved by experiments, many fundamental physical processes at the quantum level that involve quantum mechanics still remain a mystery. In particular, one of the profound epistemological problems that continue to exist is the question of whether microscopic phenomena are in fact continuous or progressing in quantum jumps. In an article entitled ARE THERE QUANTUM JUMPS? Schrödinger wrote: "...A great many of our educated contemporaries, not equipped with the mathematical apparatus to follow our more technical deliveries, are yet deeply concerned with many general questions; one of the most stirring among them certainly is whether actually *natura facit saltus* or no..." [1]. It seems Schrödinger himself did not believe in abrupt quantum transitions, especially when physical phenomena are not considered as real but only associated with the probability view. Fundamentally, even quantum physical processes are occurring in a deterministic manner, down to the quantum level in the process of creation of elementary particles and radiation of mediators of physical fields. In this work we will discuss a physical process that belongs to the quantum domain but the physical process can be described deterministically and continuously; that is the beta minus decay in which a neutron n is transformed into a proton p and an electron e^- and an electron antineutrino $\overline{\nu}_e$ are emitted from the system. In the beta minus decay, the electrons are emitted with a continuous spectrum of energy, which can be represented symbolically as $n = p + e^- + \overline{\nu}_e$. In 1932 Werner Heisenberg proposed a form of interaction between the neutrons and protons inside the nucleus, in which neutrons were composite particles of protons and electrons. These composite neutrons would emit electrons, creating an attractive force with the protons, and then turn into protons themselves [2]. Despite there were many issues with his theory, Heisenberg's idea of an exchange interaction between particles inside the nucleus inspired Fermi to formulate theory of beta decay by proposing the emission and absorption of the neutrino and electron, rather than just the electron as in Heisenberg's theory [3]. However, since the force associated with the neutrino and electron emission was shown not strong enough to bind the protons and neutrons in the nucleus, in his 1935 paper, Hideki Yukawa combined Heisenberg's idea of short-range interaction and Fermi's idea of an exchange particle to introduce a potential which includes an electromagnetic term and a term that decays exponentially [4]. Yukawa used the new potential to predict a massive mediator for the strong interaction. The massive mediator is called meson as its mass was in the middle of the proton and electron.

Since the energy spectrum of the emitted electron in the beta minus decay is continuous therefore Heisenberg’s model of the neutron as a dwarf hydrogen-like atom cannot be realised if we only apply the Coulomb potential to describe the system. In this work we will show that a continuous spectrum of energy can be obtained by applying a mixed Coulomb-Yukawa potential of the form

$$V(r) = -\alpha \frac{e^{-\beta r}}{r} + \frac{Q}{r}, \tag{1}$$

where Q, α and β are physical parameters that will need to be determined [5] [6]. Furthermore, in order to account for possible bound states of a dwarf hydrogen-like atom which can be identified with a neutron we will need to use a more general form of Yukawa potential, which has been studied and applied to various fields of physics, the More General Exponential Screened Coulomb Potential (MGESCP) given as

$$V(r) = -\frac{V_0 e^{-2\alpha r}}{r} - \frac{V_0}{r} - V_0 \alpha e^{-2\alpha r} \tag{2}$$

where V_0 is the potential depth and the parameter $\alpha \in (0, \infty)$ [7] [8]. Remarkably, we will show that the MGESCP potential can be reduced to the potential that has been proposed for the interactions between the quarks for strong force in particle physics and this result leads to an unexpected implication that a proton and an electron may also interact strongly at short distances. There are also prominent features that emerge from using the MGESCP potential to describe a neutron as a dwarf hydrogen-like atom, such as the energy spectrum of the bound states is continuous with respect to distance, and, as discussed in Section 3, the Yukawa potential can be restrained to generate and determine mathematical structures of physical objects that may be identified with the quantum mediators associated with the weak and strong interactions. With this regard, it is reasonable to suggest that functional potentials in physics may have physical mechanisms to generate mediators of associated physical fields, and these mechanisms can be formulated in terms of differentiable manifolds and their corresponding direct sums of prime manifolds as discussed in our works on the possibility to formulate physics in terms of differential geometry and topology [9].

2. Formulating potentials from Dirac equation

In the present state of physics there are four confirmed types of interactions between physical objects, which are the gravitational interaction, the electromagnetic interaction, the strong interaction, and the weak interaction. Except for the gravitational interaction, the other three types of interactions can be mathematically formulated so that they can comply with quantum mechanics, especially in the so-called standard model of particle physics [10]. In this work we will discuss a quantum dynamics of the interaction for the beta minus decay by deriving different types of potentials from Dirac relativistic equation which in turns can be derived from a general system of linear first order partial differential equations. It should be mentioned here that since Dirac relativistic equation is derived from a system of differential equations, therefore Dirac wavefunctions can be used to represent different type of physical objects rather than an exclusive mathematical method that is used to calculate the probability of the outcome of an experimental result as proposed in quantum mechanics. For example, in our work on the fluid state of Dirac quantum particles, we showed that Dirac wavefunctions can be used to represent the stream function and velocity potential of a static fluid [11]. We also showed that Dirac equation for a free particle, Dirac equation for an arbitrary field, and their corresponding solutions identified as potentials can be formulated from a general system of linear first order partial differential equations [12]. A general system of linear first order partial differential equations can be written in the form [13] [14]

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{r}\frac{\partial\psi_i}{\partial x_j} = \sum_{i=1}^{n}\left(\sum_{j=1}^{n} b_{ij}^{r} V_j + c_i^{r}\right)\psi_i + d^{r}, \quad r = 1, 2, \dots, n \tag{3}$$

The system of equations given in Equation (3) can be rewritten in a matrix form as

$$\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\psi = -i\left(\sum_{i=1}^{n} qB_iV_i + m\sigma\right)\psi + kJ \tag{4}$$

where $\psi = (\psi_1, \psi_2, \dots, \psi_n)^T$, $\partial\psi/\partial x_i = (\partial\psi_1/\partial x_i, \partial\psi_2/\partial x_i, \dots, \partial\psi_n/\partial x_i)^T$ with A_i, B_i and σ are matrices representing the quantities a_{ij}^{r}, b_{ij}^{r}, c_j^{r}, which are assumed to be constant in this work, and kJ is a matrix that represents the quantity d^{r}, where k is a dimensional constant. While the quantities q, m and kJ represent physical entities related directly to the physical properties of the particle under consideration, the quantities V_i represent the potentials of an external field, such as an electromagnetic field or the matter field of a quantum particle. In order to formulate a physical theory from the system of equations given in Equation (4), it is necessary to determine the unknown quantities A_i, B_i and σ, as well as the mathematical conditions that they must satisfy, such as commutation relations between them. The commutation relations between the matrices can be determined if we apply the operator $\sum_{i=1}^{n} A_i\, \partial/\partial x_i$ on the left on both sides of Equation (4) as follows

$$\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\left(\sum_{j=1}^{n} A_j \frac{\partial}{\partial x_j}\right)\psi = \left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\left(-i\left(\sum_{j=1}^{n} qB_jV_j + m\sigma\right)\psi + kJ\right) \tag{5}$$

Since the quantities A_i, B_i, σ, q and m are assumed to be constant, Equation (5) becomes

$$\begin{aligned}
&\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{n}\sum_{j>i}^{n}\left(A_iA_j + A_jA_i\right)\frac{\partial^2}{\partial x_i \partial x_j}\right)\psi \\
&= \left(-i\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\left(\sum_{j=1}^{n} qB_jV_j + m\sigma\right)\right)\psi - i\left(\sum_{i=1}^{n} qB_iV_i + m\sigma\right)\left(\left(\sum_{j=1}^{n} A_j \frac{\partial}{\partial x_j}\right)\psi\right) \\
&+ \sum_{i=1}^{n} A_i \frac{\partial (kJ)}{\partial x_i} \\
&= -i\left(\sum_{i=1}^{n}\sum_{j=1}^{n} qA_iB_j \frac{\partial V_j}{\partial x_i}\right)\psi \\
&- \left(\sum_{i=1}^{n}\sum_{j>i}^{n} q^2\left(B_iB_j + B_jB_i\right)V_iV_j - 2i\sum_{i=1}^{n} qmB_iV_i\,\sigma - m^2\sigma^2\right)\psi - i\left(\sum_{i=1}^{n} qB_iV_i + m\sigma\right)(kJ) \\
&+ \sum_{i=1}^{n} A_i \frac{\partial (kJ)}{\partial x_i}
\end{aligned} \tag{6}$$

In order to describe the dynamics of a particular physical system, undetermined parameters given in Equation (4) must be specified accordingly. To obtain Dirac equation for an arbitrary field we set $B_i = A_i = \gamma_i$, $\sigma = 1$ and $A_iA_j + A_jA_i = 0$. In this case Equation (4) becomes

$$\left(\sum_{i=1}^{4} \gamma_i \frac{\partial}{\partial x_i}\right)\psi = -i\left(\sum_{i=1}^{4} q\gamma_iV_i + m\right)\psi + kJ \tag{7}$$

where the matrices A_i and B_i have been identified with Dirac matrices γ_i as follows

$$\gamma^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{8}$$

For the case of Dirac equation, the matrices γ_i and σ satisfy the following conditions

$$\gamma_i^2 = \pm 1 \tag{9}$$

$$\gamma_i\gamma_j + \gamma_j\gamma_i = 0 \quad for \quad i \neq j \tag{10}$$

It is seen from Equation (9) that the quantity kJ represents an internal source which is associated with a dynamical process of a quantum particle. For example, a quantum particle is undergoing some form of physical evolution that changes its physical structure, such as an accumulation of mass during its formation. In fact we will show that this physical process can be formulated using the MGESCP potential in which the energy spectrum depends continuously on distance. In terms of the gamma matrices γ_i then Equation (7) can be rewritten in a covariant form as Dirac equation for an arbitrary field with an internal source kJ as [15]

$$\left(\gamma^\mu\left(i\partial_\mu - qV_\mu\right) - m\right)\psi = kJ \tag{11}$$

In this case Equation (6) also reduces to the following equation

$$\left(\sum_{i=1}^{4}\gamma_i{}^2\frac{\partial^2}{\partial x_i^2}\right)\psi = \left(-i\sum_{i=1}^{4}\sum_{j>i}^{4}q\gamma_i\gamma_j\left(\frac{\partial V_j}{\partial x_i} - \frac{\partial V_i}{\partial x_j}\right) + 2i\sum_{i=1}^{4}qm\gamma_i V_i - m^2\right)\psi - i\left(\sum_{i=1}^{n}q\gamma_i V_i + m\sigma\right)(kJ) + \sum_{i=1}^{n}\gamma_i\frac{\partial(kJ)}{\partial x_i} \tag{12}$$

If the quantities V_i are the four-potential of an electromagnetic field given by the identification $(V_1, V_2, V_3, V_4) = \left(V, A_x, A_y, A_z\right)$ then Equation (12) can be used to determine the dynamics of the components of the wavefunction $\psi = (\psi_1, \psi_2, \psi_3, \psi_3)^T$, where the term $\partial V_j/\partial x_i - \partial V_i/\partial x_j$ are the components of the electric field **E** and the magnetic field **B**.

Now we will discuss how free quantum particles can create their own physical fields in which wavefunctions can be identified as potentials. Therefore we set $\sum_{i=1}^{n} qB_iV_i = 0$. Equations (7) and (12) for free particles reduce to

$$\gamma^\mu\partial_\mu\psi = -im\psi + kJ \tag{13}$$

$$\gamma_i{}^2\frac{\partial^2\psi}{\partial x_i^2} = -m^2\psi - imkJ + \gamma_i\frac{\partial(kJ)}{\partial x_i} \tag{14}$$

In the following we will consider two cases depending on the conditions that are applied to the internal source kJ in which either $kJ = 0$ or $J \neq 0$. For the case $kJ = 0$, Equation (13) reduces to Dirac equation for a free particle

$$\gamma^\mu\partial_\mu\psi = -im\psi \tag{15}$$

For massive particles in which $m \neq 0$, all components of Dirac wavefunction ψ_μ satisfy the Klein-Gordon equation

$$\frac{\partial^2\psi_\mu}{\partial t^2}-\frac{\partial^2\psi_\mu}{\partial x^2}-\frac{\partial^2\psi_\mu}{\partial y^2}-\frac{\partial^2\psi_\mu}{\partial z^2}=-m^2\psi_\mu \tag{16}$$

And, in particular, if the wavefunctions are time-independent then we obtain

$$\frac{\partial^2\psi_\mu}{\partial x^2}+\frac{\partial^2\psi_\mu}{\partial y^2}+\frac{\partial^2\psi_\mu}{\partial z^2}=m^2\psi_\mu \tag{17}$$

In this case the wavefunctions ψ_μ can be viewed as static Yukawa potential

$$\psi_\mu(r)=-\alpha\frac{e^{-\beta r}}{r} \tag{18}$$

where α and β are undetermined dimensional constants [10]. The identification of the wavefunctions ψ_μ can be viewed either as static Yukawa potential or Coulomb potential is similar to the identification that we discussed in our work on the fluid state of Dirac quantum particles in which Dirac wavefunctions are identified either with a velocity potential or a stream function [11]. According to Yukawa work, the wavefunctions given in Equation (18) can be associated with the strong interaction between nuclear particles.

For massless time-independent particles, the Klein-Gordon equation given in Equation (17) reduces to Laplace equation

$$\frac{\partial^2\psi_\mu}{\partial x^2}+\frac{\partial^2\psi_\mu}{\partial y^2}+\frac{\partial^2\psi_\mu}{\partial z^2}=0 \tag{19}$$

Solutions to Laplace equation can be written in the form

$$\psi_\mu(x,y,z)=\frac{Q}{r} \tag{20}$$

In this case the wavefunctions ψ_μ can be viewed as static Coulomb potential, where Q is an undetermined dimensional constant, which is associated with the electromagnetic interaction between elementary particles.

As mentioned in the introduction, we will discuss possible bound states of a dwarf hydrogen-like atom which can be identified with a neutron therefore we will need to use a more general form of Yukawa potential, which is the MGESCP potential given as in Equation (2). Since the MGESCP potential has an extra term of the form $\psi_\mu = ae^{-mr}$, therefore we now need to show how to derive this form of potential from Dirac equation with an internal source given in Equation (13). Now, Dirac wavefunctions ψ_μ satisfy the following Klein-Gordon equation

$$\frac{\partial^2\psi_\mu}{\partial t^2}-\frac{\partial^2\psi_\mu}{\partial x^2}-\frac{\partial^2\psi_\mu}{\partial y^2}-\frac{\partial^2\psi_\mu}{\partial z^2}=-m^2\psi_\mu-imkJ+\gamma_i\frac{\partial(kJ)}{\partial x_i} \tag{21}$$

In particular, if the wavefunctions are time-independent then we obtain

$$\frac{\partial^2\psi_\mu}{\partial x^2}+\frac{\partial^2\psi_\mu}{\partial y^2}+\frac{\partial^2\psi_\mu}{\partial z^2}=m^2\psi_\mu+imkJ-\gamma_i\frac{\partial(kJ)}{\partial x_i} \tag{22}$$

It can be verified that a solution of the form $\psi_\mu = ae^{-mr}$, where a and m are constants, satisfies the following equation

$$\frac{\partial^2\psi_\mu}{\partial x^2}+\frac{\partial^2\psi_\mu}{\partial y^2}+\frac{\partial^2\psi_\mu}{\partial z^2}=m^2\psi_\mu-\frac{2mae^{-mr}}{r} \tag{23}$$

By comparing Equation (23) to Equation (22), we obtain the following equation for the internal quantity kJ

$$imkJ-\gamma_i\frac{\partial(kJ)}{\partial x_i}=-\frac{2ame^{-mr}}{r} \tag{24}$$

A differential equation for the quantity kJ can be determined by using the matrices γ_i given in Equation (8), which is written in an explicit form as

$$\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\frac{\partial(kJ)}{\partial x}+\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}\frac{\partial(kJ)}{\partial y}+\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}\frac{\partial(kJ)}{\partial z}$$
$$=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\left(imkJ+\frac{2ame^{-mr}}{r}\right) \tag{25}$$

From Equation (25) we obtain the following equations for the quantity kJ

$$imkJ+\frac{2ame^{-mr}}{r}=0 \tag{26}$$

$$\frac{\partial(kJ)}{\partial z}=0,\qquad \frac{\partial(kJ)}{\partial x}-i\frac{\partial(kJ)}{\partial y}=0,\qquad \frac{\partial(kJ)}{\partial x}+i\frac{\partial(kJ)}{\partial y}=0 \tag{27}$$

The equations given in Equation (27) show that the source kJ is constant and from Equation (29) this also results in the constancy of the Yukawa potential which can be written as

$$\frac{e^{-mr}}{r}=-\frac{ikJ}{2a} \tag{28}$$

Now we apply the results that have been obtained into the MGESP potential given in Equation (2), one of whose components has the form $\psi_\mu = V_0\alpha e^{-2\alpha r}$. By comparing this potential to $\psi_\mu = ae^{-mr}$ we have $a = V_0\alpha$ and $m = 2\alpha$. If V_0 and α are real then the function $\psi_\mu = V_0\alpha e^{-2\alpha r}$ represents a real but decreasing function with distance. In this case it is implied from Equation (28) that the quantity kJ must be imaginary, in which the source J can be identified with a real source and the dimensional constant k is an imaginary number.

On the other hand, if we require that the quantity kJ must be real, because it represents a real physical entity such as energy density, then the quantity α must be imaginary. In this case if we let $\alpha = i\beta$, where β is a real number, then we obtain $\psi_\mu = i\beta V_0 e^{-i2\beta r} = \beta V_0 e^{-i(2\beta r - \frac{\pi}{2})}$. This is the familiar oscillating function in physics that describes a harmonic motion. In our present interpretation of the function ψ_μ we conclude that the potential $\psi_\mu = V_0 \alpha e^{-2\alpha r}$ is an oscillating potential which can be applied to a physical system with bound states. In particular, with the Yukawa potential that is restrained by the condition given in Equation (28) then the MGESP potential given in Equation (2) is reduced to

$$V(r) = -\frac{V_0}{r}(1 + (1 + \alpha r)e^{-2\alpha r}) = -\frac{V_0}{r} + \frac{kJ}{2\alpha}(1 + \alpha r) = -\frac{V_0}{r} + \frac{kJ}{2}r + \frac{kJ}{2\alpha} \qquad (29)$$

Except for the constant $kJ/2\alpha$, the potential given in Equation (29) has the form that is similar to the potential that describes an interaction between two fundamental quarks as proposed in the theory of quantum chromodynamics, namely, $V(r) = A/r + Br$. This type of potential describes interactions between two quarks that can be represented in the following picture

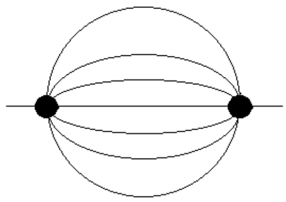

Figure 1: For small r ***V = A/r***

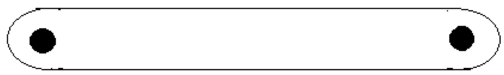

Figure 2: For large r ***V = Br***

For small values of the distance r the potential manifests as a Coulomb potential $V = A/r$, however, for large values of distance the potential acts as a linear potential with respect to the distance $V = Br$. The linear potential shown in Figure 2 is a flux tube of energy in which the quantity B has the dimension of a cross-sectional energy therefore by comparison we may also interpret the quantity kJ in Dirac equation given in Equation (7) also as a cross-sectional energy. The reduced form of the MGESP potential also indicates that a proton and an electron can attract strongly at very short distances so that they can bind and form a dwarf hydrogen-like atom.

3. Topological structures of elementary particles generated by Yukawa potential

In this section we discuss further the restraint to the Yukawa potential given in Equation (28) which has been shown to reduce the MGESP potential to the potential that is proposed for the interaction between the quarks for strong force in particle physics. We now show that in fact the restrained Yukawa potential actually generates and determines mathematical structures of physical objects that may be identified with quantum mediators of the weak and strong interactions. Instead of giving a mathematical analysis of the restrained Yukawa potential given in Equation (28), as an illustration, we simply plot possible shapes that can be generated and determined by a restrained Yukawa potential from the relation given in Equation (28), namely, $e^{-mr} = (-ikJ/2a)r$, with different values given to the parameters m and $-ikJ/2a$. Together, they possess a remarkable difference in their topological structures that may underlie physical effects that are associated with elementary quantum particles [16].

For the case $m = 0.001$ with $-ikJ/2a = \gamma = 1, 2, 3, 4$ we have the following possible shapes for elementary quantum particles


Figure 3: $\boldsymbol{\gamma} = \mathbf{1}$


Figure 4: $\boldsymbol{\gamma} = \mathbf{2}$


Figure 5: $\boldsymbol{\gamma} = \mathbf{3}$

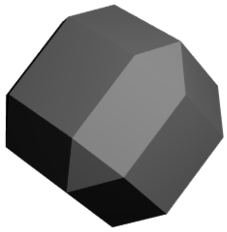
Figure 6: $\boldsymbol{\gamma} = \mathbf{4}$

For the case $m = 0.01$ with $-ikJ/2a = \gamma = 1, 2, 3, 4$ we have the following possible shapes for elementary quantum particles


Figure 7: $\boldsymbol{\gamma} = \mathbf{1}$


Figure 8: $\boldsymbol{\gamma} = \mathbf{2}$


Figure 9: $\boldsymbol{\gamma} = \mathbf{3}$

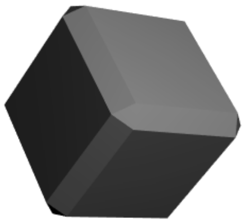
Figure 10: $\boldsymbol{\gamma} = \mathbf{4}$

For the case $m = 0.1$ with $-ikJ/2a = \gamma = 0.1, 1, 2, 3$ we have the following possible shapes for elementary quantum particles


Figure 11: $\boldsymbol{\gamma} = \mathbf{0.1}$


Figure 12: $\boldsymbol{\gamma} = \mathbf{1}$


Figure 13: $\boldsymbol{\gamma} = \mathbf{2}$

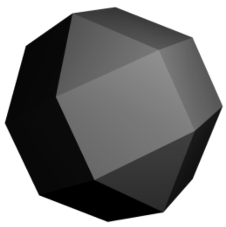
Figure 14: $\boldsymbol{\gamma} = \mathbf{3}$

For the case $m = 1$ with $-ikJ/2a = \gamma = 0.001, 0.01, 0.1, 1$ we have the following possible shapes for elementary quantum particles


Figure 15: $\boldsymbol{\gamma = 0.001}$

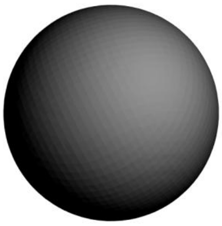
Figure 16: $\boldsymbol{\gamma = 0.01}$

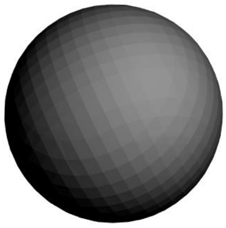
Figure 17: $\boldsymbol{\gamma = 0.1}$


Figure 18: $\boldsymbol{\gamma = 1}$

For the case $m = 5$ with $-ikJ/2a = \gamma = 10^{-11}, 10^{-3}, 0.01, 0.1$ we have the following possible shapes for elementary quantum particles


Figure 19: $\boldsymbol{\gamma = 10^{-11}}$


Figure 20: $\boldsymbol{\gamma = 10^{-3}}$


Figure 21: $\boldsymbol{\gamma = 0.01}$


Figure 22: $\boldsymbol{\gamma = 0.1}$

For the case $m = 10$ with $-ikJ/2a = \gamma = 10^{-26}, 10^{-21}, 10^{-18}, 10^{-13}$ we have the following possible shapes for elementary quantum particles


Figure 23: $\boldsymbol{\gamma = 10^{-26}}$

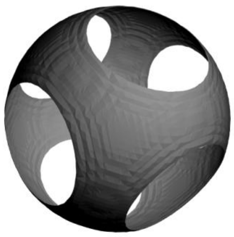
Figure 24: $\boldsymbol{\gamma = 10^{-21}}$

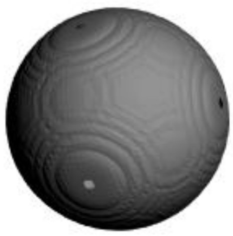
Figure 25: $\boldsymbol{\gamma = 10^{-18}}$


Figure 26: $\boldsymbol{\gamma = 10^{-13}}$

4. A quantum dynamics of the weak and strong interactions

In this section we will discuss whether a neutron can be modelled as a dwarf hydrogen-like atom with the two different mixed potentials given in Equations (1) and (2) so that it can be used to explain the physical processes associated with the beta minus decay. As shown in

Section 2, these potentials can be formed from the three forms of potentials that have been derived from the Dirac equations. First we consider the mixed potential that is formed from the Coulomb potential and the Yukawa potential as given in Equation (1). As shown below, this form of potential can be used to explain how an electron can be repelled from a dwarf hydrogen-like atom composed of a proton and an electron. By differentiating Equation (1), we can obtain the following equations

$$\frac{dV}{dr} = \alpha e^{-\beta r}\left[\frac{\beta}{r} + \frac{1}{r^2}\right] - \frac{Q}{r^2}, \qquad \frac{d^2V}{dr^2} = -\alpha e^{-\beta r}\left[\frac{\beta^2}{r} + \frac{2\beta}{r^2} + \frac{2}{r^3}\right] + \frac{2Q}{r^3} \tag{30}$$

From Equation (30), the corresponding force of interaction $F(r) = -dV/dr$ is obtained as

$$F(r) = -\alpha e^{-\beta r}\left[\frac{\beta}{r} + \frac{1}{r^2}\right] + \frac{Q}{r^2} \tag{31}$$

According to classical electrodynamics, the net force acting on the electron must be zero when it is circulating in stable orbits. If we assume the net force acting on the electron to vanish when it moves in a stationary orbit of finite radius $r = R = 1/\beta$, i.e., $F(1/\beta) = 0$, then from Equation (31) we obtain the relation

$$\beta^2\left(Q - \frac{2\alpha}{e}\right) = 0 \tag{32}$$

Since $\beta \neq 0$, we require $Q = 2\alpha/e$. The mixed potential given in Equation (1) now takes the form

$$V(r) = -\frac{eQ}{2}\frac{e^{-\frac{r}{R}}}{r} + \frac{Q}{r} \tag{33}$$

And the corresponding force of interaction $F(r) = -dV/dr$ is

$$F(r) = -\frac{eQ}{2}e^{-\frac{r}{R}}\left[\frac{1}{Rr} + \frac{1}{r^2}\right] + \frac{Q}{r^2} \tag{34}$$

In order to investigate further we need to know the nature of the stationary point at $r = R$. From Equation (30), the second derivative of $V(r)$ at $r = R$ is found as

$$\frac{d^2V}{dr^2} = -\frac{Q}{2R^3} \tag{35}$$

Since we are considering the case for which the mixed potential given in Equation (35) is applied to the bound system of two charges of opposite signs, namely a proton and an electron, therefore the quantity Q can be written as $Q = -Kq_1q_2$, where $K > 0$ is a coupling constant. If q_1 is the charge of the proton, $q_1 = q$, and q_2 is the charge of the electron, $q_2 = -q$, then we have $Q = Kq^2 > 0$. Then from Equation (35) we obtain the condition $d^2V/dr^2 < 0$, therefore *V(r)* has a local maximum at $r = R$. Since $F(r) = -dV/dr$, the force is attractive for $r < R$ and repulsive for $r > R$. This situation is seen similar to the case of weak interaction of beta minus decay in which a neutron turns into a proton and emits an electron and an anti-neutrino. First the neutron turns into a dwarf hydrogen-like atom, whose bound states will be described below using the MGESCP potential, then the electron moves in an orbit of zero force and then it is repelled from the dwarf hydrogen atom by a repulsive force. The force given in Equation (34) is rewritten as follows

$$F(r) = -\frac{eKq^2}{2} e^{-\frac{r}{R}} \left[\frac{1}{Rr} + \frac{1}{r^2}\right] + \frac{Kq^2}{r^2} \tag{36}$$

The force given in Equation (36) is assumed to be a weak force. Since R is the radius of the stationary orbit therefore we can assume that the electron is ejected from the stationary orbit because the equilibrium of this system at $r = R$ is unstable.

It can be seen that the whole process of beta decay is a complicated physical process that is actually undergoes through many different physical configurations of the system, therefore it can only be described approximately by many different dynamics, only if we can formulate the whole physical process under a mathematical formulation that can give rise to each physical configuration by some form of limit associated with mathematical parameters that is used to describe the whole system. With this in mind in the following we will discuss in terms of Schrödinger wave mechanics a more complete structure of a neutron as a dwarf hydrogen-like atom using a more complete MGESCP potential given in Equation (2). Consider the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \tag{37}$$

in which $V(r)$ is the More General Exponential Screened Coulomb Potential (MGESCP) given in Equation (2). Since the MGESCP potential is spherically symmetric, Equation (37) can be written in the spherical polar coordinates as

$$-\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{\mathbf{L}^2}{\hbar^2 r^2}\right)\psi(\mathbf{r}) + \left(-\frac{V_0}{r}(1 + (1 + \alpha r)e^{-2\alpha r})\right)\psi(\mathbf{r}) = E\psi(\mathbf{r}) \tag{38}$$

where the orbital angular momentum operator $\mathbf{L}^2$ is given by

$$\mathbf{L}^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial^2\phi}\right) \tag{39}$$

Solutions of Equation (38) can be found using the separable form

$$\psi_{nl}(\mathbf{r}) = R_{nl}(r)Y_{lm}(\theta, \phi) \tag{40}$$

where R_{nl} is a radial function and Y_{lm} is the spherical harmonic. Applying Equation (40), Equation (38) is reduced to the system of equations

$$\mathbf{L}^2 Y_{lm}(\theta, \phi) = l(l + 1)\hbar^2 Y_{lm}(\theta, \phi) \tag{41}$$

$$\left(-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)+\frac{l(l+1)\hbar^2}{2\mu r^2}-\frac{V_0}{r}\left(1+(1+\alpha r)e^{-2\alpha r}\right)\right)R_{nl}(r)=ER_{nl}(r) \tag{42}$$

It has been shown that the radial solution $R_{nl}(r)$ to Equation (42) can be obtained as

$$R_{nl}(r)=N_{nl}r^{\left(-1+\sqrt{4l(l+1)+1}\right)/2}e^{-\beta r}L_n^{1+\sqrt{4l(l+1)+1}}(2\beta r) \tag{43}$$

and the corresponding energy spectrum E_{nl} is given by

$$E_{nl}=-V_0\alpha e^{-2\alpha r}-\frac{\mu}{2\hbar^2}\left(\frac{(V_0+V_0e^{-2\alpha r})}{n+l+1}\right)^2 \tag{44}$$

where $\beta^2=\left(2\mu(V_0+V_0e^{-2\alpha r})/\hbar^2\left(2n+1+\sqrt{4l(l+1)+1}\right)\right)^2$ [8]. Although this energy spectrum is discrete with respect to the quantum numbers n and l, it depends continuously on the radial distance r. In order to interpret the energy spectrum given in Equation (44) as some of energy spectrum associated with the beta minus decay we need to apply the restraint condition applied to the Yukawa given in Equation (28) so that the MGESCP potential is reduced to the potential that is used to describe strong interaction at very short distances so that a proton and an electron can form a dwarf hydrogen-like atom. Then we obtain

$$E_{nl}=\frac{ikJ}{2}r-\frac{\mu}{2\hbar^2}\left(\frac{(V_0-(ikJ\alpha/2)r)}{n+l+1}\right)^2 \tag{45}$$

Now we may interpret this continuous spectrum of energy with respect to distance as the energy spectrum of massive mediators associated with strong force described by the potential given in Equation (29). When a physical particle is created it is being created continuously until it reaches the size that is required for the system. This process happens in a very short time therefore it seems like an instantaneous creation. In particle physics, the parameter α of the exponential term is expressed in terms of the mass m of a force carrier as $\alpha=mc/2\hbar$. Therefore when the mass of the force carrier is being continuously created the parameter α is being getting larger, at the same time the radius r is also getting bigger, therefore the term $e^{-2\alpha r}\to 0$ and also the term $\alpha e^{-2\alpha r}\to 0$. The mass that is accumulated by the force carrier must be supplied by the neutron. When the force carrier with required mass hits the electron, the latter will move further from the proton. On the other hand the MGESCP potential is reduced to the mixed Coulomb-Yukawa potential when the process of creation of the force carrier is complete. This form of potential provides a repulsive force to move the electron away.

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