Mapping the Real Numbers to the Set of Natural Numbers

by Jim Rock

Abstract. The terminating decimal fractions in the open interval \((0, 1)\) are put in one to one correspondence with the set of positive integers. This shows that attempting to map the set of real numbers to the natural numbers by listing them as infinite decimal fractions is futile. The real numbers are represented as the limit of partial decimal sums. We show how the real numbers can be represented as a denumerable set. We create a new (Level Set Theory), where all infinite sets have the same cardinality as the set of natural numbers.

The positive integers can be put in one to one correspondence with the terminating decimal fractions in the open interval \((0, 1)\). \(1\rightarrow.1, 2\rightarrow.2, \ldots, 10\rightarrow.01, \ldots\) Each terminating decimal is the mirror image reflection through the decimal point of a positive integer. The mapping does not include any repeating decimal fractions. From this mapping the set of all rational numbers would appear to be uncountable. This shows that attempting to map the real numbers in the closed interval \([0, 1]\) to the natural numbers, by listing them as infinite decimal fractions is futile. Cantor’s diagonal proof that the real numbers are an uncountable set can never even get started.

The problem is in defining the real numbers as the set of all infinite decimal expansions. That’s vague. Most non-algebraic real numbers cannot be explicitly referenced. We need a new definition.

Let \(p\) be an integer. Each real number \(S\) is:

the limit \(m\rightarrow\infty\) \(n = 1\) to \(m\), \(0 \leq a_n \leq 9\) \(\sum p + a_n/10^n = S.\)

Start with an infinite row of all the algebraic numbers. We allow each algebraic number to have a column above it with the number having all the irrational algebraic numbers as exponents, leaving out any duplicate numbers. This infinite grid can be matched to the natural numbers by a diagonalization technique. Let the grid element in row \(a\) and column \(b\) be \((a, b)\).

Then \(0\rightarrow(1, 1)\) \(1\rightarrow(2, 1)\) \(2\rightarrow(1, 2)\) \(3\rightarrow(3, 1)\) \(4\rightarrow(2, 2)\) \(5\rightarrow(1, 3)\) \(6\rightarrow(4, 1)\) \(7\rightarrow(3, 2)\) \(8\rightarrow(2, 3)\) \(9\rightarrow(1, 4)\) ...

We use the six arithmetic processes (addition, subtraction, multiplication, division, exponentiation, and root extraction) on all the transcendental numbers just created together with the algebraic numbers. This is done repeatedly in any sequence to create all possible new transcendental numbers from the original set of real numbers. The diagonalization technique can be applied to an infinite number of grids. Each formed from an infinite number of rows of transcendental numbers created from the six arithmetic processes.

Then the infinite number of grids are each diagonalized and the transcendental numbers just created are listed as infinite columns above the elements of the originally created infinite diagonalized row. The diagonalization process is then repeated. This new set of real numbers is listed as an infinite row with each row element having a column above it of the row element with all the irrational numbers from the row just created as exponents, leaving out any duplicates. The diagonalization process is then repeated. All possible additional transcendental numbers are created by applying the six arithmetic processes. These additional transcendental numbers are diagonalized in the same manner as described above, and listed as infinite columns above the elements of the infinite diagonalized row just created. The diagonalization process is repeated.

This process of creating ever larger new sets of real numbers can be continued indefinitely. Eventually, every real number \((S\) as defined above) will be generated. We conjecture that no other transcendental (trigonometric, logarithmic, power series or other) functions need be performed to generate the real numbers. Each time after the six arithmetic processes are performed and the diagonalization technique is completed the new infinite row of real numbers just created is added to a composite infinite grid forming a new row, leaving out any numbers that are in any previous row of this composite infinite grid. The diagonalization technique is applied to the composite infinite grid to map all the real numbers to the natural numbers.
A new non-hierarchical (Level Set Theory) is created when we let $I/\mathcal{X} = 0$.

Then the limit $x \to \infty f(x) = \mathcal{X}$, $\leftrightarrow$ the limit $x \to \infty 1/f(x) = 0$.

The limit $n \to \infty n = \mathcal{X}$, and the limit $n \to \infty 2^n = 2^{\mathcal{X}} = \mathcal{X}$. Thus, $|2^{\mathcal{X}}| = |\mathcal{X}|$.

The limit $n \to \infty n^n = \mathcal{X}^{\mathcal{X}} = \mathcal{X}$. Thus, $|\mathcal{X}^{\mathcal{X}}| = |\mathcal{X}|$.

There is no hierarchy of infinites. The limit $n \to \infty 2^{n^n} = 2^{\mathcal{X}^{\mathcal{X}}} = \mathcal{X}$. $|2^{\mathcal{X}^{\mathcal{X}}}| = |\mathcal{X}|$.

Explore the detailed proofs and fascinating consequences of $|2^{\mathcal{X}}| = |\mathcal{X}|$ in https://arxiv.org/abs/1002.4433

Addressing mathematical inconsistency: Cantor and Gödel refuted by J. A. Perez.

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