Refutation of two variants of noncontingency operator

Abstract: Four axiomatizations of extensions of L(dot-box) over special frames are *no*t tautologous. Those for symmetry and qe&pe are different, but the respective, second-order renditions are equivalent. This refutes the two variants of noncontingency operator. Therefore the conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Fan, J. (2019). Two variants of noncontingency operator. arxiv.org/pdf/1906.03091.pdf

7.2 Extensions

In this section, we study the axiomatizations of L(box-dot) over special frames. The following table lists extra axioms and proof systems, and the frame properties that the corresponding systems characterize.

B ϕ → ((ϕ ∧ (ϕ → ψ) ∧ ¬ ψ) → χ) B = K + B symmetry (7.2.2.1)

LET p, q, r, s: ϕ, ψ, χ, box-cross

p>(s&(((s&p)&(s&(p>q)))&((~s&q)>r))) ;
T**F**T**F** T**F**T**F** T**F**TT T**F**TT (7.2.2.2)

5 ¬ ϕ → (¬ ϕ ∨ ψ) K5 = K + 5 qe&pe (7.2.4.1)

(~s&p)>(s&((~s&p)+q)) ; T**F**T**F** T**F**T**F** TTTT TTTT (7.2.4.2)

In the above table, qt, pt, qe, pe abbreviate quasi-transitivity, pseudo-transitivity, quasi-Euclidicity and pseudo-Euclidicity, respective, which are formalized by

respectively, where i, j ∈ {1, 2}: [as antecedent] (7.9.1)

LET s, r, u, v, x, y, z: s, R, i, j, x, y, z

(u&v)<((%s>#s)&(%s<#s)) ; (7.9.2)

∀xyz(xRiy ∧ yRjz → xR1z ∧ xR2z), pt (7.6.1)

```
((u&v)<((%s>#s)&(%s<#s)))>((((#x&r)&(u&#y))&((#y&r)&(v&#z)))>
(((#x&(r&(%s>#s)))&#z)&((#x&(r&(%s<#s)))&#z))) ;
                    TTTT TTTT TTTT TTTT(112)
                    TTTT TTTT TTTT TTTT(  6)}x2
                    TTTT CCCC TTTT CCCC(  2)}      (7.6.2)
```

$\forall xyz(xRiy \land xRjz \rightarrow yR1z \land yR2z)$, pe (7.8.1)

```
((u&v)<((%s>#s)&(%s<#s)))>((((s&#x)&(u&#y))&((#x&r)&(v&#z)))>
((#y&((%s>#s)&#z))&(#y&((%s<#s)&#z)))) ;
                    TTTT TTTT TTTT TTTT(112)
                    TTTT TTTT TTTT TTTT(  6)}x2
                    TTTT CCCC TTTT CCCC(  2)}      (7.8.2)
```

Four axiomatizations of extensions of L(dot-box) over special frames are *no*t tautologous. Those for symmetry and qe&pe are different, but the respective, second-order renditions are equivalent. This refutes the two variants of noncontingency operator.