Refutation of linear temporal logic (LTL)

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Abstract: We evaluate two composite modal atoms which turn out to be reductions into single modal operators. This refutes the notion that “new temporal modalities are obtained” and forms a non tautologous fragment of the universal logic $VŁ4$.

We assume the method and apparatus of Meth8/$VŁ4$ with $T$autology as the designated proof value, $F$ as contradiction, $N$ as truthity (non-contingency), and $C$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

\[
\begin{align*}
\text{LET} & \quad \sim \text{ Not, } \neg \quad ; \\
& \quad + \text{ Or, } \lor, \cup, \sqcup ; \\
& \quad - \text{ Not Or; } \& \text{ And, } \land, \cap, \sqcap ; \\
& \quad \text{ Not And; } \\
& \quad \rightarrow \text{ Imply, greater than, } \Rightarrow, \\
& \quad \leftarrow \text{ less than, } \Leftarrow, \prec, \preceq; \\
& \quad = \text{ Equivalent, } \equiv, \equiv, \\
& \quad \% \text{ possibility, for one or some, } \exists, \Diamond, M ; \\
& \quad \# \text{ necessity, for every or all, } \forall, \Box, L ; \\
& \quad (z=z) \text{ T as tautology, } T, \text{ ordinal 3; } \\
& \quad (z@z) \text{ F as contradiction, } \emptyset, \text{ Null, } \bot, \text{ zero; } \\
& \quad (%z>#z) \text{ N as non-contingency, } \Delta, \text{ ordinal 1; } \\
& \quad (%z<#z) \text{ C as contingency, } \nabla, \text{ ordinal 2; } \\
& \quad \neg(y<x) \text{ ( } x \subseteq y \text{), } (x \subseteq y); \quad (A=B) \text{ (A=B). } \\
& \quad \text{Note for clarity, we usually distribute quantifiers onto each designated variable. }
\end{align*}
\]


Syntax, Slide (4):

There are additional temporal operators:

- $\Diamond$ “eventually” (eventually in the future) \hspace{1cm} [often]
- $\Box$ “always” (now and forever in the future) \hspace{1cm} [forever]

By combining the temporal modalities $\Diamond$ and $\Box$, new temporal modalities are obtained.

\[
\begin{align*}
\Box\Diamond \phi \quad & \text{“infinitely often } \phi \text{”} \quad \text{(4.1.1)} \\
\Box\Diamond \phi \text{ reduces to } \Diamond \phi, \text{“often } \phi \text{”} \quad \text{(4.1.2)} \\
\Diamond\Box \phi \quad & \text{“eventually forever } \phi \text{”} \quad \text{(4.2.1)} \\
\Diamond\Box \phi \text{ reduces to } \Box \phi, \text{“forever } \phi \text{”} \quad \text{(4.2.2)}
\end{align*}
\]

Eqs. 4.1.2 and 4.2.2 as rendered are reductions of composite modal operators and hence refute the notion that “new temporal modalities are obtained”.