

## Refutation of linear temporal logic (LTL)

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**Abstract:** We evaluate two composite modal atoms which turn out to be reductions into single modal operators. This refutes the notion that “new temporal modalities are obtained” and forms a *non* tautologous fragment of the universal logic  $\forall\exists\forall$ .

We assume the method and apparatus of Meth8/ $\forall\exists\forall$  with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET  $\sim$  Not,  $\neg$ ; + Or,  $\vee$ ,  $\cup$ ,  $\sqcup$ ; - Not Or; & And,  $\wedge$ ,  $\cap$ ,  $\square$ ,  $;$ ; \ Not And;  
> Imply, greater than,  $\rightarrow$ ,  $\Rightarrow$ ,  $\mapsto$ ,  $>$ ,  $\supset$ ,  $\Rightarrow$ ; < Not Imply, less than,  $\in$ ,  $<$ ,  $\subset$ ,  $\neq$ ,  $\neq$ ,  $\ll$ ,  $\lesssim$ ;  
= Equivalent,  $\equiv$ ,  $:=$ ,  $\Leftrightarrow$ ,  $\leftrightarrow$ ,  $\hat{=}$ ,  $\approx$ ,  $\simeq$ ; @ Not Equivalent,  $\neq$ ;  
% possibility, for one or some,  $\exists$ ,  $\diamond$ , **M**; # necessity, for every or all,  $\forall$ ,  $\square$ , **L**;  
( $z=z$ ) **T** as tautology, **T**, ordinal 3; ( $z@z$ ) **F** as contradiction,  $\emptyset$ , Null,  $\perp$ , zero;  
(% $z>\#z$ ) **N** as non-contingency,  $\Delta$ , ordinal 1; (% $z<\#z$ ) **C** as contingency,  $\nabla$ , ordinal 2;  
 $\sim(y < x)$  ( $x \leq y$ ), ( $x \subseteq y$ ), ( $x \sqsubseteq y$ ); ( $A=B$ ) ( $A\sim B$ ).  
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ashari, R.; Habib, S. (presenters). (date unknown). Linear temporal logic (LTL). Chapter 5. [cs.colostate.edu/~france/CS614/Slides/Ch5-Summary.pdf](http://cs.colostate.edu/~france/CS614/Slides/Ch5-Summary.pdf)

**Syntax**, Slide (4):

There are additional temporal operators:

$\diamond$	“eventually” (eventually in the future)	[often]
$\square$	“always” (now and forever in the future)	[forever]

By combining the temporal modalities  $\diamond$  and  $\square$ , new temporal modalities are obtained.

$$\square\diamond\varphi \quad \text{“infinitely often } \varphi\text{”} \quad (4.1.1)$$

$$\square\diamond\varphi \text{ reduces to } \diamond\varphi, \text{ “often } \varphi\text{”} \quad (4.1.2)$$

$$\diamond\square\varphi \quad \text{“eventually forever } \varphi\text{”} \quad (4.2.1)$$

$$\diamond\square\varphi \text{ reduces to } \square\varphi, \text{ “forever } \varphi\text{”} \quad (4.2.2)$$

Eqs. 4.1.2 and 4.2.2 as rendered are reductions of composite modal operators and hence refute the notion that “new temporal modalities are obtained”.