Some Problems About The Gravitational Wave

Ting-Hang Pei
Thpei142857@gmail.com

Abstract - The contribution of the dipole moment in the generation of the gravitational waves is reconsidered here whether it is useful or not. In this article, we find that the contribution of the dipole moment is nonzero. The gravitational wave is responsible for delivering the information of the spacetime structure and gravitational field from the source. Since the general relativity predicts the speed of gravity at most as fast as the speed of light, the change of gravity is not instantaneous and two cosmic objects must produce the gravitational waves to realize the gravitational attraction. The phenomenon that the gravitational wave cannot escape makes the black hole like an isolated system and it results in such black hole losing ability to attract other massive bodies. Therefore, we also discuss the possibility of the gravitational waves escaping the black hole.

Keywords: gravitational wave, binary system, black hole, spacetime, transverse polarization

I. Introduction

Recently, gravitational waves have been detected [1-5] that the long-term predictions of gravitational waves have been verified. In general relativity, gravitational waves are the ripples of spacetime. The gravity described by Einstein's general relativity is a phenomenon induced by the curvature of spacetime [6-11]. In principle, mass and energy can cause this curvature [6-11]. As the massive body moves in space, the nearby spacetime curvature also changes. The change in curvature induced by such movement produces a wave propagating at the speed of light. This phenomenon is that gravitational waves propagate outward from the position of the massive body. The amplitude of the gravitational wave will decrease as the propagation distance increases, and the frequency of the gravitational wave can perform red-shift or the Doppler's effect. Gravitational waves even focus at a strong gravitational field and also exhibit diffraction behavior.

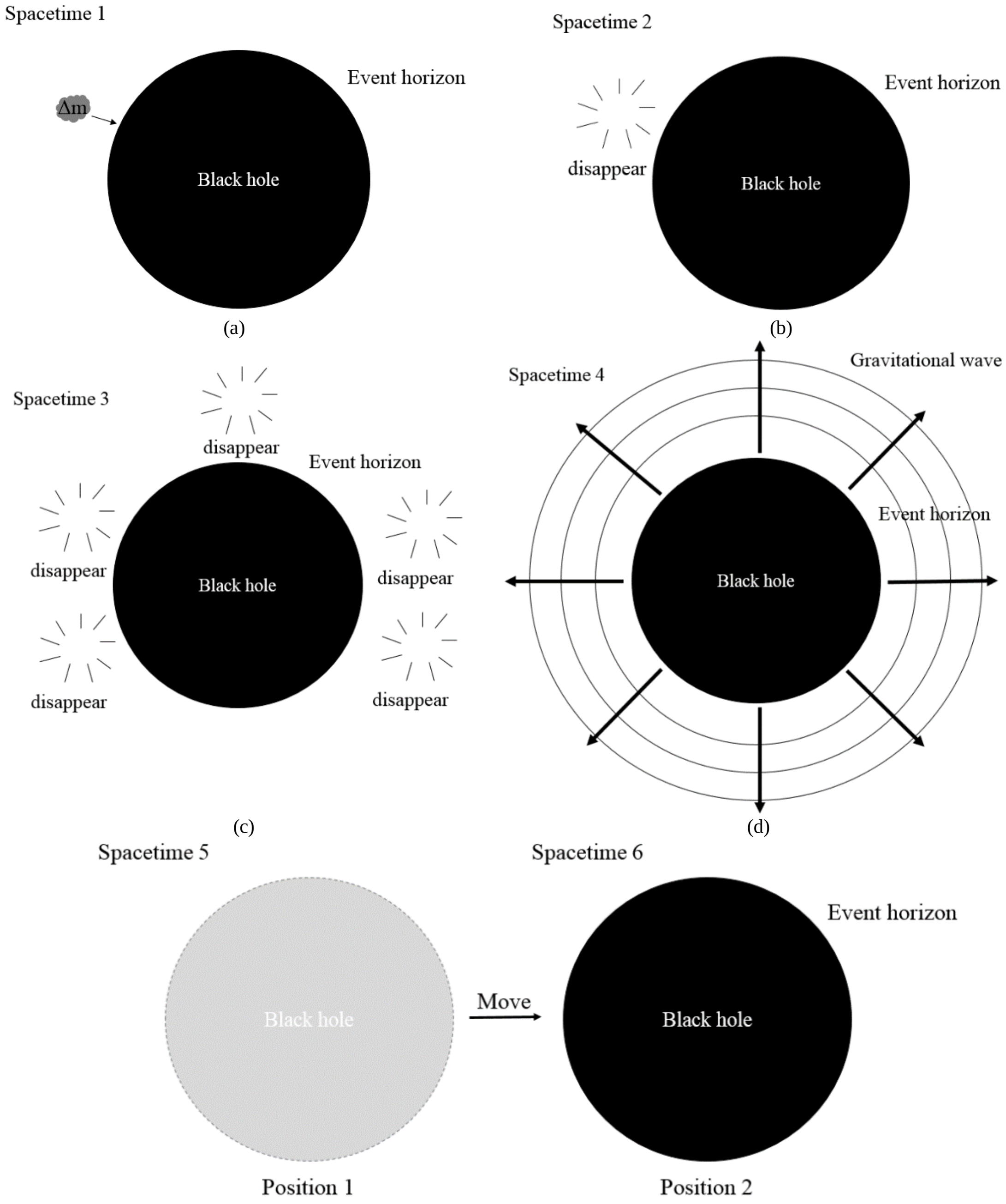
Because the interaction between gravitational waves and matter is very weak, it can truly transmit valuable information from the far source to the Earth. The study of the gravitational wave collects the data from gravitational-wave sources, such as the binary systems of white dwarfs, neutron stars, and black holes. In 1974, the Hulse–Taylor binary pulsar was discovered [6]. When the binary star system revolves around each other, they gradually lose energy and approach each other because they continuously emit gravitational waves. This phenomenon provided the first indirect evidence for the existence of the gravitational waves in 1974. In the later, scientists used the gravitational-wave detectors to observe the gravitational wave, such as the LIGO's laser interference instrument at the gravitational wave observatory [6,7,10].

It is traditionally believed that gravitational waves are generated from asymmetric movements which causes the quadrupole moments changing with time [6-11]. The general statement is that the gravitational waves can be generated as long as the shape of a system changes during motion. The two celestial bodies moving around each other can also produce gravitational waves as mentioned previously. The higher the asymmetry of a system is or the higher the speed of motion is, the stronger the gravitational waves it emits. However, we have to review the contribution of the dipole moment whether it is useful or not. In this article, we will see that the contribution of the dipole moment is nonzero. In addition, we also discuss the possibility of the gravitational waves escaping the black hole.

II. The Changes of The Spacetime Of The Black Hole

In order to figure out the production mechanism of the gravitational wave, several examples of the black holes are demonstrated as the gravitational sources. Considering the first example an object of mass Δm is very close to the black hole, and the surrounding spacetime is denoted as Spacetime 1, as shown in Figure 1(a). When this object is inhaled by the black hole and disappears outside the event horizon, the spacetime becomes to Spacetime 2, and it is redistributed with the center of the black hole as the gravitational center, instead of staying in the Spacetime 1 before the object enters the black hole, as shown in Figure 1 (b). As more objects approaches the black hole and are attracted by its gravity, the spacetime will be continuously redistributed with the center of the black hole as the gravitational center [1,2]. Then it will be a new spacetime, Spacetime 3, as shown in Figure 1(c). Whenever objects enter the black hole, the mass of the black hole changes, and the spacetime outside the horizon is also changed with the center of the black hole as the gravitational center at the same time. If no gravitational waves escape from the inside of a black hole, how does the black hole change its surrounding spacetime? It is confused that gravitational waves cannot leave the black hole to the space outside its event horizon. This induces a logical question: How can the external spacetime be reconstructed without gravitational waves escaping from black holes? It will make the black hole as an isolated system and any change of the spacetime has nothing to do with the black hole because no information about its mass leaks into the outer space. Therefore, the gravitational waves must be able to spread out from the inside of a black hole to the outer space, as shown in Figure 1(d). The black hole reconstructing the spacetime in the external space is not instantaneous, and it has to take a certain time to reach the observation point. As mentioned previously, the time for affecting the observation point is determined by the speed of the gravitational wave. That is, the change of the spacetime depends on the speed of the gravitational wave. Hence, by the gravitational waves, the spacetime away from the black hole can be influenced [6].

Another example is to consider black holes in motion. We know that many black holes are located in the centers of the galaxies. According to the observations of the red-shift, the galaxies are leaving away from each other, and the black holes in the galaxies are also moving in space. The moving black hole also causes the changes in spacetime as shown in Figure 1(e). If there is no gravitational wave producing and spreading from the inside of a black hole, how does this moving black hole reconstruct new spacetime in space?



(a) (b)

(c) (d)

(e)

Figure 1. (a) The Spacetime 1 before something enters into the black hole. (b) The Spacetime 2 after something enters into the black hole without gravitational wave escaping from it. (c) The Spacetime 3 after a lot of things enter into the black hole without the gravitational wave escaping it. (d) The Spacetime 4 after something enters into the black hole with the gravitational wave escaping from it. (e) The Spacetime 5 before the movement of the black hole and the spacetime 6 after the movement.

III. The Equation For Producing Gravitational Waves

The gravitational wave in the nearly flat spacetime has been discussed in many textbooks [6-11], and it is also a mathematical formula to prove the previous statements. The gravitational wave evaluated at the retarded time *t-r* is [6]

$$\phi^{kl}(t,\vec{x}) = -\left[\frac{\kappa}{8\pi r}\frac{\partial^2}{\partial t^2}\int \rho(\vec{x}')\, x'^k x'^l dx'^3\right]_{t-r}, \tag{1}$$

where k, l=1,2,3, $\vec{x}'$ is the source position, and $\vec{x}$ is the observer position. It can be rearranged as

$$\phi^{kl}(t,\vec{x}) = -\frac{\kappa}{8\pi r}\frac{1}{3}\frac{\partial^2}{\partial t^2}\left[Q^{kl} + \delta_k^l \int r'^2 \rho(\vec{x}') dx'^3\right]_{t-r}, \tag{2}$$

and

$$Q^{kl} = \int \left(3x'^k x'^l - r'^2 \delta_k^l\right)\rho(\vec{x}') dx'^3. \tag{3}$$

The gravitational wave has two transverse polarizations, and one expression in terms of tensor is

$$\epsilon_{\oplus}^{kl} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

and the other one is

$$\epsilon_{\otimes}^{kl} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{5}$$

At the same time, the tensor δ_k^l is

$$\delta_k^l = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

Actually, the inner product of two matrices δ_k^l and $\epsilon_{\oplus}^{kl}$ is nonzero, and that is,

$$\langle \delta_k^l | \epsilon_{\oplus}^{kl} \rangle = \epsilon_{\oplus}^{kl}. \tag{7}$$

Hence, the second term in Eq. (2) clearly tells us that the change of mass can also induce the gravitational wave, not only from the quadruple term Q^{kl}. It also means the leading term to produce the gravitational waves is not the quadruple term. Actually, it is the mass dipole term and the center of the mass doesn't need to experience oscillation.

IV. The Cases Of The Symmetrical Change In Mass

The other case can be an explicit evidence for the second term in Eq. (2). It has been mentioned that no gravitational energy is emitted when a supernova produces a completely symmetric stellar collapse [10]. If so, then there exists some contradiction. First, we think about the movement of a single particle with mass of m. Can this particle radiate gravitational wave? If it wouldn't be, then the gravitation field established by it will be independent of its movement and position. The gravitational field established by this particle was a very long time ago since it appeared in the universe. It is maybe at ten billion light years away from the Earth. It will not change no matter how its movement is and this statement is based on the viewpoint of the quadruple term as the leading term. However, as we know, it is not true because the gravitational field really depends on the position and movement of this particle, and so does the gravitational wave. Even single particle, it must be able to radiate gravitational wave as long as it moves. Otherwise, its movement cannot affect the surrounding gravitational field because of no gravitational radiation, and the gravitational field will remain stay at the same one it has established long-time ago. Even it has left the original place ten billion years, the gravitational field is still unchanged. Hence, no gravitational wave from a moving particle results in an unreasonable phenomenon. When another test particle is added, the gravitational attraction appears between these two particles. Since the general relativity predicts the speed of gravity at most as fast as the speed of light, the change of the gravity is not instantaneous and two cosmic particles must produce gravitational waves to realize the gravitational attraction. Without the gravitational wave, the gravitational field far away from the source cannot establish or rearrange and the gravitational attraction loses its rightness.

For the second example of a symmetrically explosive supernova, we consider a point A inside and the other point B outside this star before the supernova. When the supernova takes place, the radius of the star changes from r_o to r_i as shown in Fig. 2. Due to the supernova, the mass of the star is changed, and the position of point A becomes outside the star. Obviously, the gravitational fields at the two observation points are significantly different before and after the supernova. This change in the gravitational field, and the change in the spacetime structure, comes from the change of the star mass. This change is not instantaneous and must be delivered by gravitational waves to these two observation points. Therefore, a symmetrically explosive supernova also generates gravitational waves in order to establish the variation of the surrounding spacetime structure due to the change of mass.

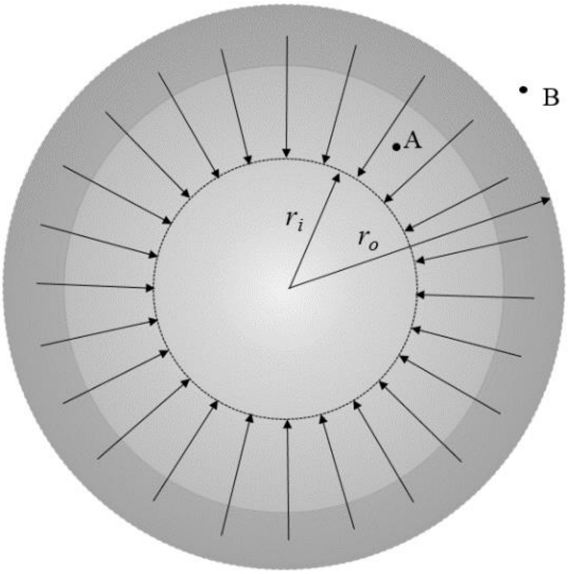


Figure 2. The radius changes during the supernova. Two points A and B experiences different gravitational fields respectively before and after the supernova.

The third example is to consider the planets in the solar system. When the mass of a certain planet symmetrically increases in a very short time, the radius increases from r_1 to r_2 at the same time as shown in Fig. 3. After that, the earth will feel the gravitational field changes originated from this planet. However, this change is not instantaneous, and it exists a time delay. The change in the gravitational field or the spacetime structure transmitted by gravitational waves is due to the increase in mass of this planet. The delivered time from the planet to the earth is the propagation time of the gravitational waves.

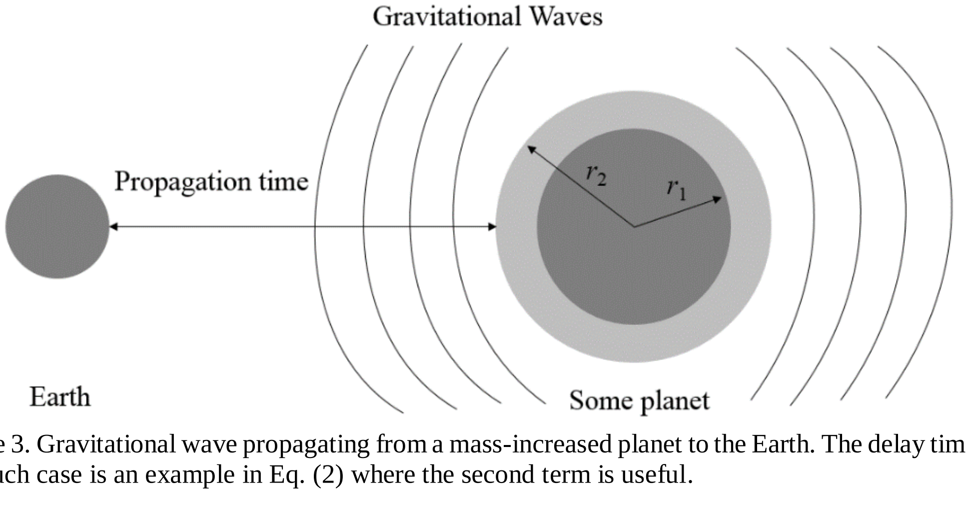


Figure 3. Gravitational wave propagating from a mass-increased planet to the Earth. The delay time exists and such case is an example in Eq. (2) where the second term is useful.

V. Conclusion

About the generation of gravity wave, we first discuss whether the gravitational wave can escape the black hole or not from the viewpoint of the mass absorption by the black hole. If the gravitational wave cannot escape the black hole, the space-time structure around the black hole does not change although the mass of the black hole changes. Moreover, we know that the most black holes are located in the center of the galaxies, and it is well-known the redshift phenomenon that the galaxies are moving away from the Earth. This also means that the black hole is in moving. If the moving black hole cannot radiate gravitational waves, the space-time structure around it cannot be changed. In addition, from the example of a symmetrical supernova burst, it logically proves that the symmetrical change in mass can produce gravitational waves, which affects the surrounding space-time structure. Another example is the symmetrical increase in mass of a planet in the solar system. It always exists a delay time for the gravitational wave influencing the earth. This delay time is the fact that the change of the space-time structure delivering by the gravitational wave is not instantaneous. Without the information transmitted by the gravitational wave, the earth will not feel the change of the gravity. These are the examples of the generation of gravitational waves based on the Einstein’s general relativity.

We also derive the formula of gravitational waves generating from the weak gravity sources and review the radiation mechanism of gravitational wave. Not only the quadrupole radiation, but the gravitational waves can also be generated as long as the mass changes no matter it is symmetrical or not. Even more, the gravitational waves can be generated by moving massive objects, because the corresponding space-time structure changes depending on the position of the object. For example, the Earth's gravitational field is different for the sun at the near and far points, and the space-time structure is also slightly different at these two points.

In conclusion, gravitational waves are generated as long as the position or mass of the object changes. The gravitational waves are not only from the merge of a binary black holes, but also from the motion of a single black hole. The gravitational wave consists in transmitting information about the change of the spacetime structure. It also coincides with the results of our logical deduction, which proves that the leading term in the generation of the gravitational wave is not from the quadrupole source.

Acknowledgement

This research is under no funding.

Reference:

[1]. B. P. Abbott et. al. (LIGO Scientific Collaboration and Virgo Collaboration), “Observation of Gravitational Waves from a Binary Black Hole Merger,” Phys. Rev. Lett. **116**, 061102 (2016).

[2]. B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence,” Phys. Rev. Lett. **116**, 241103 (2016).

[3]. B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “GW170104: Observation of A 50 Solar-Mass Binary Black Hole Coalescence At Redshift 0.2,” Phys. Rev. Lett.

118, 221101 (2017).

[4]. B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “GW170814: a Three-Detector Observation of Gravitational Waves From A Binary Black Hole Coalescence,” Phys. Rev. Lett. **119**, 141101 (2017).

[5]. B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “GW170817: Observation of Gravitational Waves From A Binary Neutron Star Inspiral,” Phy. Rev. Lett. **119**, 161101 (2017).

[6]. Hans C. Ohanian and Remo Ruffini, *Gravitation and Spacetime* (W. W. Norton & Company, 2nd ed., New York, 1994), p.445.

[7]. Hans Stephani, *Relativity – An Introduction to Special And General Relativity* (Cambridge, 3rd Edition, Cambridge), p.238.

[8]. Bernard F. Schutz, *A First Course In General Relativity* (Cambridge University Press, Cambridge, 1985), p.291.

[9]. F. De Felice & C. J. S. Clarke, *Relativity On Curved Manifolds* (Cambridge University Press, Cambridge, 1990), p. 355 & p.362.

[10]. Richard A. Mould, *Basic Relativity* (Springer, New York, 2002), p.324 & p. 383.

[11]. L. D. Landau and E. M. Lifshitz. *The Classical Theory Of Fields*. (Pergamon Press LTD., Fourth Revised English Edition, 1975), p.309.