Some Problems About The Gravitational Wave

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Abstract - We think about the contribution of the dipole moment whether it is useful or not. In this article, we will see that the contribution of the dipole moment nonzero. In addition, we also discuss the possibility of gravitational waves escaping the black hole.

Keywords: gravitational wave, binary system, black hole

I. Introduction

Recently, gravity waves have been detected [1-5] that the long-term predictions of gravity waves have been proven. In general relativity, gravity waves are the ripples of spacetime. The gravity described by Einstein's general relativity is a phenomenon induced by the curvature of spacetime [6-11]. Mass and energy can cause this curvature [6-11]. As the massive body moves in space, the nearby spacetime curvature changes. The change in curvature induced by such movement produces a wave propagating at the speed of light. This phenomenon is that gravity waves propagate outward from the position of the massive body. The amplitude of the gravity wave will decrease with the increase of the propagation distance, and the frequency of the gravity wave will be red-shifted due to the expansion of the universe, the interaction of gravity or the Doppler's effect. Gravity waves even focus at a strong gravitational field and also exhibit diffraction behavior.

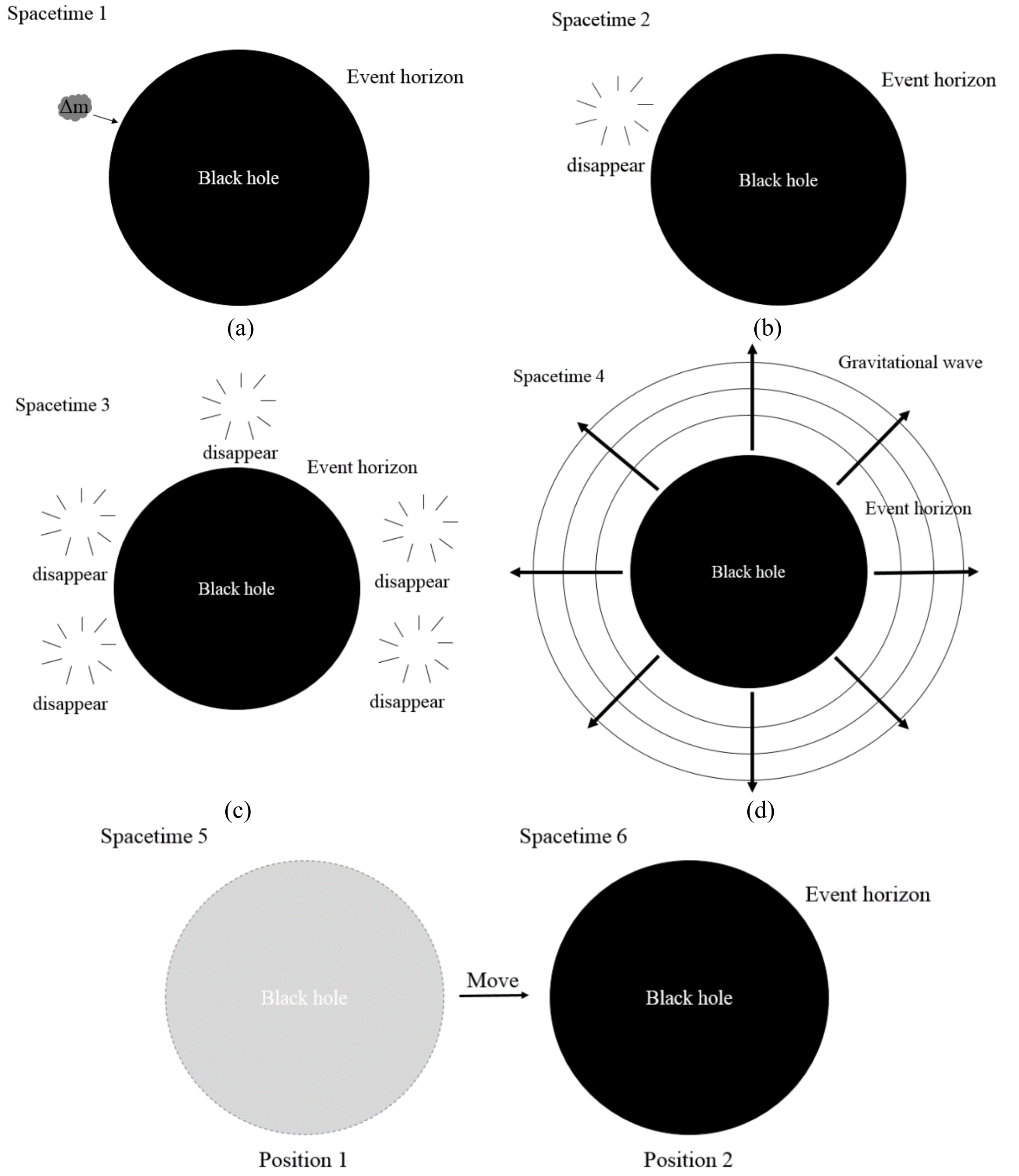
Because the interaction between gravitational waves and matter is very weak, it can truly transmit valuable information from the far universe to the Earth. The astronomy studying gravitational wave collect the data from gravitational-wave sources, such as the binary systems of white dwarfs, neutron stars, and black holes. In 1974, the Hulse–Taylor binary pulsar was discovered [6]. When the binary star system revolves around each other, they lose energy because they continuously emit gravitational waves, so they gradually approach each other. This phenomenon provides the first indirect evidence for the existence of gravitational waves. In the later, scientists used the gravitational-wave detectors to observe gravitational wave, such as the LIGO's laser interference gravitational wave observatory [6,7,10].

It is traditionally believed that gravitational waves are generated from asymmetric movements which causes the quadrupole moments changing with time [6-11]. The general statement is that gravitational waves can be generated as long as the shape of a system changes during motion. The two celestial bodies moving around each other can also produce gravitational waves. The higher the asymmetry of a system is and the higher the speed of motion is, then the stronger the gravitational waves it emits. However, we have to think about the contribution of the dipole moment whether it is useful or not. In this article, we will see that the contribution of the dipole moment nonzero. In addition, we also discuss the possibility of gravitational waves escaping the black hole.

II. The Changes of The Spacetime Of The Black Hole

Considering the first example an object of mass Δm is very close to the black hole, and the surrounding space and time is denoted as Spacetime 1, as shown in Figure 1(a). When this object is inhaled by the black hole and disappears outside the event horizon, the space and time changes to Spacetime 2, and it is redistributed with the center of the black hole as the gravitational center, instead of staying at the Spacetime 1 before the object enters the black hole, as shown in Figure 1 (b). When more objects are close to the black hole and are attracted by its gravity, the space and time will be redistributed with the center of the black hole as the gravitational center again [1,2]. Then it is a new spacetime, Spacetime 3, as shown in Figure 1(c). Whenever objects enter the black hole, the mass of the black hole changes, and the time and space outside the horizon is also changed with the center of the black hole as the gravitational center at the same time. If no gravitational waves escape from the inside black hole, how does the black hole change the surrounding space and time? This makes us feel unsatisfied that gravity waves cannot leave the black hole to the space outside the event horizon. Without gravitational waves escaping the black hole, how does the outside spacetime reconstruct? Therefore, the gravitational waves have to be able to spread out from the inside black hole to the outer space, as shown in Figure 1(d). The black hole reconstructing the spacetime in the external space is not instantaneous, and it has to take a certain time to reach the observation point. The time for affecting the observation point is determined by the speed of the gravitational wave. That is, the change of the spacetime depends on the speed of the gravitational wave. For example, if one of the nine planets doubles in mass, the influence of gravity on the earth is not instantaneous, and the change of the spacetime structure is delivered through gravity waves where the reach time is the propagation time of the gravity waves. Hence, by the gravitational waves, the spacetime away from the black hole can be influenced [6].

Another example is to consider black holes in motion. We know that many black holes are located in the centers of the galaxies. According to the theory of the universal expansion, the galaxies are leaving away from each other, and the black holes in the galaxies are also moving in space. The moving black hole also cause the changes in spacetime as shown in Figure 1(e). If there is no gravitational wave spreading from the inside black hole, how does this moving black hole reconstruct new spacetime in space?



(e)

Figure 1. (a) Spacetime before something enters into the black hole. (b) Spacetime after something enters into the black hole without gravitational wave escaping from it. (c) Spacetime after a lot of things enter into the black hole without the gravitational wave escaping it. (d) The spacetime 4 after something enters into the black hole with the gravitational wave escaping from it. (e) The spacetime 5 before the movement of the black hole and the spacetime 6 after the movement.

The gravitational wave in the nearly flat spacetime has been discussed in many textbooks, and it is a mathematical formula to prove the previous statements. The gravitational wave evaluated at the retarded time *t-r* is [6]

$$\phi^{kl}(t,\vec{x}) = -\left[\frac{\kappa}{8\pi r}\frac{\partial^2}{\partial t^2}\int \rho(\vec{x}')\, x'^k x'^l dx'^3\right]_{t-r}, \qquad (1)$$

where k, l=1,2,3. It can be rearranged as

$$\phi^{kl}(t,\vec{x}) = -\frac{\kappa}{8\pi r}\frac{1}{3}\frac{\partial^2}{\partial t^2}\left[Q^{kl} + \delta_k^l \int r'^2 \rho(\vec{x}')dx'^3\right]_{t-r}, \tag{2}$$

and

$$Q^{kl} = \int \left(3x'^k x'^l - r'^2 \delta_k^l\right)\rho(\vec{x}')dx'^3. \tag{3}$$

The gravitational wave has two transverse polarizations, and one expressed in terms of tensor is

$$\epsilon_{\oplus}^{kl} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

and the other is

$$\epsilon_{\otimes}^{kl} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{5}$$

At the same time, the tensor δ_k^l is

$$\delta_k^l = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

Actually, the inner product of two tensors δ_k^l and $\epsilon_{\oplus}^{kl}$ is nonzero, and that is,

$$\left\langle \delta_k^l \middle| \epsilon_{\oplus}^{kl} \right\rangle = \epsilon_{\oplus}^{kl}, \tag{7}$$

so the second term in Eq. (2) clearly tells us that the change of mass can also induce the gravitational wave.

The other case can be an explicit evidence for the second term in Eq. (2). It has been mentioned that no gravitational energy is emitted when a supernova produces a completely symmetric stellar collapse [10]. If so, then there exists some contradiction. First, we think about the movement of a single electron with mass of m_e. Can this electron radiate gravitational wave? If it wouldn't be, then the gravitation field established by it will be independent of its movement and position. The gravitational field established by this electron was a very long time ago since it appeared in the universe. It is maybe at ten billion light years away from the Earth. However, as we know, it is not true because the gravitational field depends on the position and movement of the electron, and so does the gravitational wave. Even single electron, it can radiate gravitational wave as long as it moves. Otherwise, its movement cannot affect the surrounding gravitational field because of no gravitational radiation.

For an example of a symmetrically explosive supernova, we consider a point A inside and the other point B outside this star before the supernova explosion. When the supernova goes off, the radius of the star changes from r_o to r_i as shown in Fig. 2. Due to supernova explosion, the mass of the star is changed, and the position of point A becomes outside the star. Obviously, the gravitational fields at the two observation points are significantly different before and after supernova explosion. This change in the gravitational field, and the change in the spacetime structure, comes from the change of the star mass. This change is not instantaneous and must be delivered by gravity waves to these two observation points. Therefore, a symmetrically explosive supernova also generates gravity waves in order to make the variation of the surrounding spacetime structure due to the change of mass.

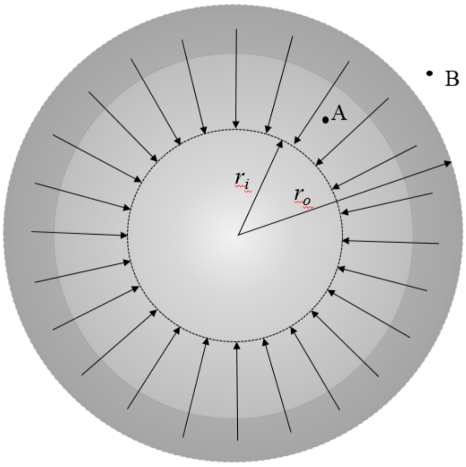


Figure 2. The radius changes during supernova explosion. Two points A and B experiences different gravitational fields respectively before and after supernova explosion.

Another example is to consider the planets in the solar system. When the mass of a certain planet symmetrically increases in a very short time, the radius increases from r_1 to r_2 at the same time as shown in Fig. 3. After that, the earth will feel the gravitational field changes originated from this planet. However, this change is not instantaneous, and it exists a time delay. The change in the gravitational field or the spacetime structure transmitted by gravity waves is due to the increase in mass of this planet.

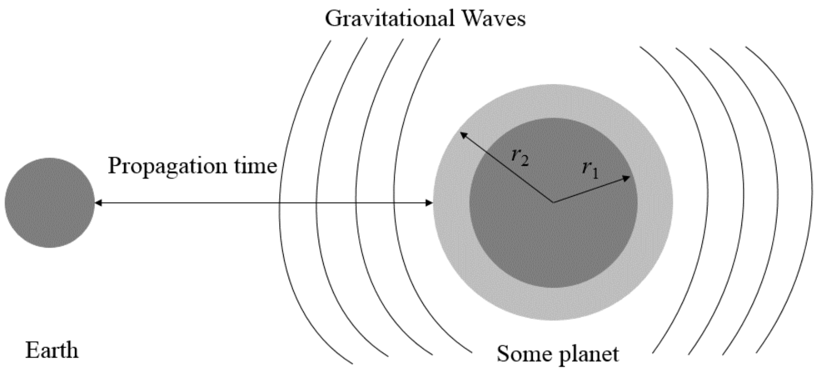


Figure 3. Gravitational wave propagating from a mass-increased planet to the Earth. The delay time exists and such case is an example in Eq. (2) where the second term is useful.

Acknowledgement

This research is under no funding.

Reference:

[1]. B. P. Abbott et. al. (LIGO Scientific Collaboration and Virgo Collaboration), "Observation of Gravitational Waves from a Binary Black Hole Merger," Phys. Rev. Lett. **116**, 061102 (2016).
[2]. B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence," Phys. Rev. Lett. **116**, 241103 (2016).
[3]. B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "GW170104: Observation of A 50 Solar-Mass Binary Black Hole Coalescence At Redshift 0.2," Phys. Rev. Lett. **118**, 221101 (2017).
[4]. B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "GW170814: a Three-Detector Observation of Gravitational Waves From A Binary Black Hole Coalescence," Phys. Rev. Lett. **119**, 141101 (2017).
[5]. B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "GW170817: Observation of Gravitational Waves From A Binary Neutron Star Inspiral," Phy. Rev. Lett. **119**, 161101 (2017).
[6]. Hans C. Ohanian and Remo Ruffini, *Gravitation and Spacetime* (W. W. Norton & Company, 2nd ed., New York, 1994), p.445.
[7]. Hans Stephani, *Relativity – An Introduction to Special And General Relativity* (Cambridge, 3rd Edition, Cambridge), p.238.
[8]. Bernard F. Schutz, *A First Course In General Relativity* (Cambridge University Press, Cambridge, 1985), p.291.
[9]. F. De Felice & C. J. S. Clarke, *Relativity On Curved Manifolds* (Cambridge University Press, Cambridge, 1990), p. 355 & p.362.
[10]. Richard A. Mould, *Basic Relativity* (Springer, New York, 2002), p.324 & p. 383.
[11]. L. D. Landau and E. M. Lifshitz. *The Classical Theory Of Fields*. (Pergamon Press LTD., Fourth Revised English Edition, 1975), p.309.