Refutation of temporal logic via instant- and interval/period-based models of time

Abstract: The basic properties in seven equations are *not* tautologous. This refutes temporal logic via instant- and interval/period-based models of time and forms it as a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Galton, A.; Valentin Goranko, V. Temporal logic. (2015).
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2.1 Instant-based models of the flow of time

Some further basic properties … can be expressed with first-order sentences as follows:

reflexivity: $\forall x(x \prec x)$ (2.1.1.1)

LET p, q, r: x, y, z

#p<#p ; **FFFF FFFF FFFF FFFF** (2.1.1.2)

density (between every two precedence-related instants there is an instant):
$\forall x \forall y(x \prec y \rightarrow \exists z(x \prec z \wedge z \prec y))$ (2.1.2.1)

(#p<#q)>((#p<%r)&(%r<#q)) ;
TCTT TCTT TCTT TCTT (2.1.2.2)

no beginning: $\forall x \exists y(y \prec x);\forall x \exists y(y \prec x)$ (2.1.3.1)

%q<#p ; CTCC CTCC CTCC CTCC (2.1.3.2)

no end: $\forall x \exists y(x \prec y)$ (2.1.4.1)

#p<%q ; **F**N**FF F**N**FF F**N**FF F**N**FF** (2.1.4.2)

every instant has an immediate successor:
$\forall x \exists y(x \prec y \wedge \forall z(x \prec z \rightarrow y \leq z))\forall x \exists y(x \prec y \wedge \forall z(x \prec z \rightarrow y \leq z))$ (2.1.5.1)

(#p<%q)&((#p<#r)>~(#r<%q)) ;

FN**FF** **F**N**FF** **F**N**FF** **F**N**FF** (2.1.5.2)

every instant has an immediate predecessor:

$$\forall x \exists y (y \prec x \wedge \forall z (z \prec x \rightarrow z \leq y)) \quad (2.1.6.1)$$

(%q<#p)&((#r<#p)>~(%q<#r)) ;

CCTC CCTC CCTC CCTC 2.1.6.2)

2.2 Interval/period based models of time

Some natural basic properties of such interval-based relations and models include:

atomicity of $\sqsubseteq$ (for discrete time): $\forall x \exists y (y \sqsubseteq x \wedge \forall z (z \sqsubseteq y \rightarrow z = y))$ (2.2.1.1)

~(#p<%q)&(~(%q<#r)>(#r=%q)) ;

TCTT CCTT TCTT CCTT (2.2.1.2)

Basic properties in these seven equations are *not* tautologous. This refutes temporal logic via instant- and interval/period-based models of time.