The interior logic of inequalities

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Abstract: From a theoretical point of view, it is appropriate and necessary to distinguish science and (fantastical) pseudoscience for both practical and theoretical reasons. One specific nature of pseudoscience in relation to science and other categories of human reasoning is the resistance to the facts. In this paper, several methods are analyzed which may be of use to prevent that personal belief can be masqueraded genuinely as scientific knowledge. In particular, modus ponens, modus tollens and modus conversus are reanalyzed. Modus sine, logically equivalent to modus ponens, is developed and modus inversus and modus juris are described in detail. In point of fact, in our striving for knowledge, there is still much more scientific work to be done on the demarcation line between science and pseudoscience.

Keywords: *non strict inequality, science, pseudoscience*

1. Introduction

Acquiring long lasting and possibly generally valid scientific knowledge is concerned with problems which are closely related to central problems of science as such. At first blush, different scientific methods like inductive and deductive reasoning, systematic observation and experimentation and other methods of inquiry does not guarantee automatically the discovery and justification of new truths. Clear and sometimes formal standards or normative criteria for identifying advances and improvements in science with respect to mathematics are necessary too. In contrast to natural sciences, there is a widespread view that investigating fundamental questions concerning mathematics is to some extent problematic since the status of mathematical knowledge appears to be ultimate and therefore less open to revision than natural sciences. Narrowly speaking, even if the methods of investigation of natural sciences (more or less induction) may differ markedly from the methods of investigation in the mathematics (more or less deduction) there is usually a lot of overlap between them. both have at least on point in common, the (to many times possibly fruitless) hunt for the truth. The problem of course is, what is the truth and, in a way, easy to state, is there an absolute truth at all? The origins of the problems closely connected to the truth are traceable to ancient times and this simple statement masks a great deal of controversy. Is there at least one single knowledge, statement or axiom et cetera, which can or which must be accepted as being true by all scientist, since the axiomatic method is one of the crucial tools for mathematics? In an attempt to find a logically consistent answer to problems like this it is necessary to consider a number of distinct ways of answering questions about the nature of truth and the preservation of truth?

Strategically, proceeding axiomatically has as one of many advantages to develop a theorem or a theory in a rigorous way from some fundamental principles. This fundamental insight underpins the possibility of an axiomatic system [1] to serve, for later investigations, as a tool for discovery of errors inside a theory. As a matter of fact, Hilbert's demand to find a complete and consistent set of axioms for all mathematics was counteracted to some extent by Gödel's incompleteness theorems [2]. But one way to handle these difficulties is to reject the possibility for the premises to be true but conclusion false.

2. Material and Methods

Inequalities are widely used in many branches of physics and mathematics. In general, an inequality is a relation that holds between two values which are not equal, which are different. In mathematics, analytic number theory often operates with inequalities. Usually, an inequality is denoted by the symbols < or > or by the symbols $\leq$ or $\geq$. Furthermore, the author assumes that the readers of the present article is familiar with basic concepts connected with first-order logic and formal proof methods. Otherwise, for introduction as well as for deeper knowledge of the topic, secondary literature [3]–[5] is recommended.

2.1. Definitions

To date, mathematics is more or less a product of human thought and mere human imagination and belongs more to the world of human thought and mere human imagination then to experimentally determined sciences. In the following, it is of principal use to ground mathematics on nature grounded entities to disable the possibility to regard mathematics as a religion whose

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language are numbers, definitions et cetera than as science. In the current article, some first-order logic language symbols are used: logical constants +0 (logical false), +1 (logical true); logical connectives ¬, ∩, ∪, ¬, ←, → ; different predicate symbols; individual constants; variables et cetera.

2.1.1. Definition (Number +1)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary number [6]. The number +1 is defined as the expression

$$+(c^2 \times \varepsilon_0 \times \mu_0) \equiv \quad +1+0 \equiv \quad -i^2 \quad = \quad +1 \tag{1}$$

while "=" denotes the equals sign [7] or equality sign [8] used to indicate equality and "-" [9], [10] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" denotes the plus [7] signs used to represent the operations of addition and the notions of positive as well.

2.1.2. Definition (Number +0)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary number [6]. Let the arithmetic operation subtraction be signified by the minus sign (−). The number +0 is defined as the expression

$$+(c^2 \times \varepsilon_0 \times \mu_0) - (c^2 \times \varepsilon_0 \times \mu_0) \equiv \quad +1-1 \equiv \quad +1 \tag{2}$$

2.1.3. Definition (The sample space)

The sample space of quantum mechanical entity, a physical object, a mathematical object, an experiment, a random variable, a random trial et cetera is the set of all possible outcomes or results of that experiment or random variable or trial (*principle of superposition* of all possible outcomes or states et cetera) at a certain (period of) time t. Let A_t denote the sample space of a (Bernoulli distributed) random variable A which can take the values either +1 or +0 at a Bernoulli trial (period of time) t. Let B_t denote the sample space of another (Bernoulli distributed) random variable B which can take the values either +1 or +0 at the same Bernoulli trial (period of time) t. Let P_t denote the sample space of **the premise**, a (Bernoulli distributed) random variable P which can take the values either +1 or +0 at a Bernoulli trial (period of time) t. Let C_t denote the sample space of **the conclusion**, another (Bernoulli distributed) random variable C which can take the values either +1 or +0 at the same Bernoulli trial (period of time) t. In general, the truth value of a true statement is true and the truth value of a false statement is false. W e define the sample space A at a Bernoulli trial/period of time t as

$$A_t \quad \equiv \quad \{+0_t \quad , \quad +1_t\} \tag{3}$$

We define the sample space of B at a Bernoulli trial/period of time t as

$$B_t \quad \equiv \quad \{+0_t \quad , \quad +1_t\} \tag{4}$$

We define the sample space of a premise P at a Bernoulli trial/period of time t as

$$P_t \quad \equiv \quad \{+0_t \quad , \quad +1_t\} \tag{5}$$

We define the sample space of a conclusion C at a Bernoulli trial/period of time t as

$$C_t \quad \equiv \quad \{+0_t \quad , \quad +1_t\} \tag{6}$$

2.1.4. Definition (Strict inequalities)

A strict inequality [11] is an inequality where the inequality symbol is either < (strictly less than) or > (strictly greater than). Consequently, a strict inequality has no equality conditions. In terms of algebra, we obtain

$$A_t \quad < \quad B_t \tag{7}$$

The notation A_t is < B_t means that "A_t is strictly less than B_t". The following table (**Table 1**) may illustrate this relationship under the conditions above.

Table 1. The strict inequality $A_t < B_t$.

$A_t < B_t$		Conditioned B_t		Total
		$B_t = +1$	$B_t = +0$	
Condition A_t	$A_t = +1$	0	0	
	$A_t = +0$	**1**	0	
	Total			1

The strict inequality $A_t < B_t$ describes the complementary part of the conditio sine qua non relationship *without* A_t *no* B_t. In other words, it is $p(A_t < B_t) + p(A_t \geq B_t) = 1$. Equally there may exist conditions where

$$A_t \quad > \quad B_t \tag{8}$$

The notation $A_t > B_t$ means that "A_t is strictly greater than B_t". The following table (**Table 2**) may illustrate this relationship under the conditions above.

Table 2. The strict inequality $A_t > B_t$.

$A_t > B_t$		Conditioned B_t		Total
		$B_t = +1$	$B_t = +0$	
Condition A_t	$A_t = +1$	0	**1**	
	$A_t = +0$	0	0	
	Total			1

As can be seen, the strict inequality $A_t > B_t$ describes the complementary part of the material implication or conditio per quam relationship *if* A_t *then* B_t. In other words, in general is necessary to point out that it is $p(A_t > B_t) + p(A_t \leq B_t) = 1$.

2.1.5. Definition (Non strict inequalities)

In contrast to strict inequalities, a non-strict inequality is an inequality where the inequality symbol is $\geq$ (either greater than or equal to) or $\leq$ (either less than or equal to). Consequently, a non-strict inequality is an inequality which has an equality condition too. In terms of algebra, we obtain

$$A_t \quad \leq \quad B_t \tag{9}$$

The notation $a \leq b$ means that "*either* A_t is less than B_t *or* A_t is equal to B_t". The following table (**Table 3**) may illustrate this relationship under the conditions above.

Table 3. The non strict inequality $A_t \leq B_t$.

$A_t \leq B_t$		Conditioned B_t		Total
		$B_t = +1$	$B_t = +0$	
Condition A_t	$A_t = +1$	1	**0**	
	$A_t = +0$	1	1	
	Total			1

As can be seen, the non-strict inequality $A_t \leq B_t$ describes the **conditio per quam** relationship *if* A_t *then* B_t. In other words, it is $p(A_t \leq B_t) = 1 - p(A_t > B_t)$. The notation

$$A_t \quad \geq \quad B_t \tag{10}$$

means that "either A_t is greater than B_t or A_t is equal to B_t". The logical content of the non-strict inequality $A_t \geq B_t$ is clearly demonstrated by the following table (**Table 4**).

Table 4. The non strict inequality $A_t \geq B_t$.

$A_t \geq B_t$		Conditioned B_t		Total
		$B_t = +1$	$B_t = +0$	
Condition A_t	$A_t = +1$	1	1	
	$A_t = +0$	**0**	1	
	Total			1

As can be seen, the non-strict inequality $A_t \geq B_t$ describes the conditio sine qua non relationship *without* A_t *no* B_t. In other words, it is $p(A_t \geq B_t) = 1 - p(A_t < B_t)$.

2.1.6. Definition (Russell's paradox)

Considering Cantor's power class theorem, Russell was the first to discuss a contradiction arising in the logic of sets or classes at length in his published works [12], [13]. There are sets or classes which are members of themselves, while some other sets or classes are not members of themselves. A null or empty set or class must not be a member of itself. Thus far, according to Russell's paradox [12], [13], let R denote the class of all classes, which itself is a set or class (with certain properties). The class of all classes R itself can be an empty set or class too and like the like the null class, must not be included in itself. In other words, either R is a member of itself or R is not a member of itself. Furthermore, either R contain itself or R does not contain itself. The following table may provide a preliminary overview (**Table 5**).

Table 5. Russell's paradox I.

R is the set of all sets.		R contains itself		Total
		Yes	No	
R is member of itself	Yes	a	b	U
	No	c	d	<u>U</u>
	Total	W	<u>W</u>	1

Disproof of Russell's paradox.
Russell's paradox (also known as Russell's antinomy), discovered by Bertrand Russell in 1901, demands that if R is not a member of itself (case <u>U</u>), then R's definition dictates that it must contain itself (case c), and if R contains itself (case W), then R contradicts its own definition as the set of all sets that are not members of themselves.
Claim.
Russell's conclusion is not justified and incorrect.
Proof.
First of all, Russell is mismatching being member of itself and containing itself, both notions are due to Russell understanding not identical. In particular, if containing itself and being member of itself are two different and not identical or equivalent notions, then it is possible for the set of all set R to contain itself while being a member of itself or not being a member of itself (i. e. empty set). In the same respect, if containing itself and being member of itself are two different notions then it is possible for the set of all sets not being member of itself (case <u>U</u>) while containing itself (case c) or not containing itself (case d). But even in the case if R just contains itself, Russell's conclusion is incorrect. In this case we obtain the following situation (**Table 6**).

Table 6. Russell's paradox II.

R is the set of all sets.		R contains itself		Total
		Yes	No	
R is member of itself	Yes			
	No	c		<u>U</u>
	Total	W		1

Even if R contains itself, according to Russel, R must not be a member of itself otherwise being member of itself and containing itself would mean the same. *Q. e. d.*

2.1.7. Definition (Basic principles of scientific engagement)

I) The axiom principium identitatis (i. e. +1=+1) is the only principle you must respect. You shall not respect any other axioms before principium identitatis.

II) You shall not tolerate any individual unscientific behavior or any other errors in science.

III) You shall not misuse principium identitatis.

IV) You shall make sure that at least every seventh of your publications must start with or must be devoted to principium identitatis.

V) You shall honor your former scientific predecessors since the beginning of time and your present scientific competitor too. Without those, you would not be there, where you are today.

VI) You shall not put into question the reputation or the integrity of your scientific colleagues.

VII) You shall not work on two different scientific projects, articles et cetera at the same time.

VIII) You shall not forget to give credit or reference to another scientist.

IX) You shall not bear false witness against your former scientific predecessors, your scientific competitor or yourself.

X) You shall be devoted only to the discovery of the truth.

2.1.8. Definition (Theorem)

Let a *theory or a theorem* denote something, a (mathematical, physical et cetera) equation, a statement et cetera which can be shown to be true while using some basic axioms (statements, equations et cetera which are given as true), definitions, other theorems or some rules of inference. A less important theorem may be denoted as a *proposition* too.

2.1.9. Definition (Conjecture)

Let conjecture indicate a statement which is being proposed to be true. As soon, as a valid proof of a conjecture is found, the conjecture becomes a theorem. Still, it may turn out to be that a conjecture is false.

2.1.10. Definition (Lemma)

Let *lemma* denote something like a 'helping theorem' or a result which is needed to prove a theorem.

2.1.11. Definition (Corollary)

Let *corollary* denote a result which follows directly from a theorem.

2.2. Proof methods

Detecting inconsistencies and inadequacies in scientific theories and resolving contradictions is of particular importance within science itself. Experiments or experience can help us many times to decide upon the truth or falsity of natural laws but do not provide any help to trace these inconsistencies and inadequacies back to the fundamental axioms from which they spring. Unfortunately, even peer-reviewed published or proposed theorems or statements of science and mathematics are not automatically correct. Whenever we find that a system has been questioned somehow, we shall test the same again and reject it if possible, as circumstances may require. It is necessary to prove these theorems while using rigorous proof methods of science and mathematics which are acceptable beyond any shadow of doubt. In order to formulate methodological rules which, prevent us to adopt inconsistencies and inadequacies in scientific theories it necessary to consider that the results of (thought) experiments are either to be rejected, or to be accepted.

2.2.1. Direct proof

A direct (mathematical) proof demonstrates the truth or falsehood of a given equation, statement by a straightforward combination of established facts. In other words, assume that something is true (i.e. axiom 1). In the following, rules of inference, axioms, and logical equivalences et cetera are used to show that a certain conclusion is also true.

2.2.2. Proof by modus ponens

Modus ponens is a basic rule of inference. It is a strikingly simple, if (P_t implies C_t) is true and P_t is true, then we may infer that C_t is true. The first premise of a modus ponens (MP) argument is a conditional $P_t \rightarrow C_t$ or "**If *P_t is true*, then C_t *is true***" (**Table 7**). The second premise of MP is P_t is true. Under these circumstances, the conclusion of MP is C_t is true. Explicitly, modus ponens demands that ($P_t \rightarrow C_t$) = +1 and that P_t = +1 is true. Only if these two premises are given, we may infer C_t is true, otherwise not.

Table 7. Modus ponens (if P_t then C_t; P_t; therefore C_t)

$P_t \rightarrow C_t$		C_t		Total
		$C_t = +1$	$C_t = +0$	
P_t	$P_t = +1$	1	**0**	
	$P_t = +0$	1	1	
Total				1

The modus ponens demands that P_t = +1 (**Table 7**). In this case, the conclusion is that C_t = +1 too. Modus ponens demands that a deductive argument with true premises (P_t = +1) and a false conclusion (C_t = +0) is invalid. Modus ponens in the form "**If *P_t is true*, then C_t *is true***" tells the researcher what can be expected when P_t is true but is equally quiet and doesn't tell anything at all if P_t is not given or if P_t is not true. In this case, modus ponens allows that *either* C_t is true *or* C_t is false.

Table 8. Modus ponens (if P_t then C_t) is true; P_t is true; ergo C_t is true.

$P_t \rightarrow C_t$		C_t		Total
		$C_t = +1$	$C_t = +0$	
P_t	**$P_t = +1$**	**1**	0	
	$P_t = +0$	1	1	
Total				1

The proof by modus ponens in classical two-valued logic can be clearly demonstrated by the use of the following overview.

Proof by modus ponens.

Claim.

(Premise 1) $P_t \rightarrow C_t$ is true.

Proof.

(Premise 2) P_t is true.

Additional arguments.

Decision.

(Conclusion) C_t is true.

Quod erat demonstrandum.

Scholium.

However, it is not enough to assume that the rule of inference called modus ponens or $P_t \rightarrow C_t$ is true, it must be for sure true that $P_t \rightarrow C_t$ is true. Modus ponens or implication "*must give a guarantee that truth is preserved* ..." [14] . Modus ponens even if grounded on *the preservation of truth* has been criticized [15] too. Many times, modus ponens is used to proof the validity of time depended processes where an antecedent is prior in time to a consequent. An inappropriate use of modus pones under these and similar conditions can lead to contradictions. It is necessary to apply modus ponens especially on events which occur together, at the same (period of time) t. The following example may formalize modus ponens in more detail. For the sake of simplicity, we define $P_1 = (+1=+1)$, we define $P_2 = (+2=+2)$ and we define $C_1 = (+3=+3)$. The proof by modus ponens is as follows.

Proof by modus ponens I.

Claim.

Premise 1: $P_1 \rightarrow C_2$: *if* the premise $P_1 = (+1=+1)$ is true *then* the conclusion $C_1 = (+3=+3)$ is true.

Proof.

Premise 2: $\mathbf{P_1 : +1 = +1}$ is true.

Adding +2 we obtain +2 +1 = +1 +2 or +3=+3.

Decision. (Conclusion)

$C_1 = (+3=+3)$ is true.

Quod erat demonstrandum.

According to modus ponens, if a certain premise is true then the conclusion must be true. Modus ponens relies on a certain premise. But this does not exclude, that it is possible to reach at the same and true conclusion from other premises, which itself are true.

Proof by modus ponens II.

Claim.

Premise 1: P2 → C1: if the premise P2 = (+2=+2) is true then the conclusion C1 = (+3=+3) is true.

Proof.

Premise 2: **P2 : +2 = +2** is true.

Adding +1 we obtain +2 +1 = +1 +2 or +3=+3.

Decision. (Conclusion)

C1 = (+3=+3) is true.

Quod erat demonstrandum.

Modus ponens allows that one and the same conclusion $C_1 = (+3=+3)$ is true and can be deduced from different points of view, from different premisses. In the case of the premise P_1 it is true that +1=+1. The premise P_2 is because of premise P_1 not incorrect, because $P_2 : +2 = +2$ is also true, the premise P_2 is just not used for the proof performed. However, as can be seen in the second proof, the premise P_2 can used for the proof too without any restriction.

Thus far, modus ponens cannot be misused for claims that from something incorrect or non-existent ($P_1 = +0$) something correct ($C_1 = +0$) follows. This would provide evidence of *creatio ex nihilio*. Modus ponens is based on the assumption that there are many premises which can lead to the same and true conclusion.

Another example may be the following one. Define: $P_t = +1=+1$ is true. Define: C_t = It is raining. Proofing this relationship by modus ponens, we obtain: "If (+1 = +1 is true) then (it is raining is true)". Such a proof is without any sense. Theoretically, it is possible that it is not raining while (+1 = +1 is true). Thus far, the first premise of modus ponens (**(if P_t then C_t) is true**) is not given and therefore, modus ponens cannot be used to proof this relationship. Modus ponens can be used only if some certain conditions are assured and not in general and independently of any conditions. In this context, some properties of implication, which are always true, should be considered.

Totality.

$$\left((A_t \rightarrow B_t) \cup (B_t \rightarrow A_t) \right) = +1 \tag{11}$$

Transitivity.

$$\left((A_t \rightarrow B_t) \rightarrow (B_t \rightarrow C_t) \right) \rightarrow (A_t \rightarrow C_t) \tag{12}$$

Distributivity.

$$\left(C_t \rightarrow (A_t \rightarrow B_t) \right) \rightarrow \left((C_t \rightarrow A_t) \rightarrow (C_t \rightarrow B_t) \right) \tag{13}$$

2.2.3. Proof by modus secus (anti modus ponens)

In point of fact, modus ponens demands that "**if P_t is true then C_t is true**" or $P_t \rightarrow C_t$. Thus far, if the negation of modus ponens is true i. e. $\neg(P_t \rightarrow C_t)$ = true, then the original modus ponens proposition is false. The following table (**Table 9**) shows the case, when modus ponens is false.

Table 9. Modus secus – negation of modus ponens

$\neg(P_t \rightarrow C_t)$		C_t		Total
		$C_t = +1$	$C_t = +0$	
P_t	$P_t = +1$	0	**1**	
	$P_t = +0$	0	0	
	Total			1

The proof by modus secus: **premise P_t is true and conclusion C_t is false $\neg(P_t \rightarrow C_t)$** in classical two-valued logic can be demonstrated in the following way.

Proof by modus secus.

Claim.

(Premise 1) $\neg(P_t \rightarrow C_t)$

Proof.

(Premise 2) P_t.

Additional arguments.

Decision.

(Conclusion) $\neg C_t$

Quod erat demonstrandum.

Modus ponens guarantee a true conclusion when the premises are true but offers no guarantee when the premises are false. In general, *either* modus ponens ($P_t \rightarrow C_t$) is true *or* modus secus ($\neg(P_t \rightarrow C_t)$) is true but not both. Thus far, under conditions under which technical or other errors are absent, it is not possible to reach at a false conclusion C_t as long as the premise P_t is true. More specifically, since a conclusion is either true or not true, modus ponens demands that *from a true premise a true conclusion must follow*. In a slightly different way, even if this focus on modus ponens might seem myopic, modus ponens is merely preserving the truth.

2.2.4. Proof by modus sine

Modus ponens or *if* P_t *then* C_t, even if generally valid, is able to generate only a limited set of facts. Despite modus ponens' capacities to render some exceedingly clear and well-verified central truth, its broader uses endorse competing views. The preservation of the truth is one but not the only side of modus ponens. Modus ponens and modus sine are more than only closely related, both are logically equivalent. The logical equivalence between modus ponens and modus sine means that they are true together or false together. In other words, if a statement is true according to modus ponens, then the same statement is true according to modus sine (**Table 10**), and vice versa.

Table 10. Modus sine – *Without* premisse P_t is false *no* conclusion C_t is false ($\neg P_t \leftarrow \neg C_t$). $P_t = +0$. Ergo: $C_t = +0$.

$\neg P_t \leftarrow \neg C_t$		C_t		Total
		$C_t = +0$	$C_t = +1$	
P_t	$P_t = +0$	1	1	
	$P_t = +1$	**0**	1	
	Total			1

Modus sine as the other side of modus ponens doesn't allow us to draw a false conclusion form a true premise. A brief methodological remark intended to clarify the basics of modus ponens has the potential to undermine today's unquestioned traditional views on modus ponens. The other fundamental side of modus ponens (i.e. modus sine) is that *without* a false premise P_t *no* false conclusion C_t. In toto, it is no longer necessary to reduce modus ponens only to the "**if P_t is true then C_t is true**" point of view. The proof by modus sine in classical two-valued logic is not completely identical with the proof by contraposition and can be demonstrated by use of illustration.

Proof by modus sine.
Claim.
(Premise 1) $\neg P_t \leftarrow \neg C_t$
Proof.
(Premise 2) $\neg P_t$
Additional arguments.
Decision.
(Conclusion) $\neg C_t$
Quod erat demonstrandum.

Table 11.

					Implication	Modus sine	Contrapositive
t	P_t	C_t	$\neg P_t$	$\neg C_t$	($P_t \rightarrow C_t$)	($\neg P_t \leftarrow \neg C_t$)	($\neg C_t \rightarrow \neg P_t$)
1	1	1	0	0	1	1	1
2	1	0	0	1	0	0	0
3	0	1	1	0	1	1	1
4	0	0	1	1	1	1	1

In general, we obtain *the logical equivalence* of

$$(P_t \rightarrow C_t) = (\overline{P_t} \cup C_t) = (\neg P_t \leftarrow \neg C_t) = (\neg C_t \rightarrow \neg P_t) \quad (14)$$

Because modus sine has always the same truth value (truth or falsity) as modus ponens itself, it can be a powerful tool for proving mathematical theorems from another point of view.

2.2.5. Proof by modus tollens

Modus tollens and modus ponens are closely related. Theophrastus was the first to explicitly describe the argument form modus tollens [16]. A proof by modus tollens is determined by the secured relationship $P_t \rightarrow C_t$. The logical consequence is that the negation of C_t implies the negation of P_t is valid. Following Popper, "… *it is possible by means of purely deductive inferences (with the help of the modus tollens of classical logic) to argue from the truth of singular statements to the falsity of universal statements*." ([17], p. 19). In other words, "*By means of this mode of inference we falsify the whole system (the theory as well as the initial conditions) which was required for the deduction of the statement p, i.e. of the falsified statement*." ([17], p. 56). In particular and in contrast to a proof by contrapositive, the modus tollens statement demands that $P_t \rightarrow C_t$ or that the premise "**If *P_t is true*, then C_t *is true***" is given.

Proof by modus tollens.
Claim.
(Premise 1) $P_t \rightarrow C_t$
Proof.
(Premise 2) $\neg C_t$
Additional arguments.
Decision.
(Conclusion) $\neg P_t$
Quod erat demonstrandum.

In other words, modus tollens demands that $(P_t \rightarrow C_t) = +1$ and that $\neg C_t = +1$ is true. In this case, the conclusion is justified that $\neg P_t = +1$ and is clearly demonstrated by use of the following **table 12**.

Table 12. Modus tollens: *if* the premisse P_t is true *then* the conclusion C_t is true (P_t -> C_t). $C_t = +0$. Ergo: $P_t = +0$.

$P_t \rightarrow C_t$		C_t		Total
		$C_t = +1$	$C_t = +0$	
P_t	$P_t = +1$	1	0	
	$P_t = +0$	1	**1**	
	Total			1

The modus tollens rule may be written as a theorem of propositional logic as

$$((P_t \rightarrow C_t) \cap \neg C_t) \rightarrow (\neg P_t) \quad (15)$$

The following **table 13** may illustrate modus tollens from another point of view.

Table 13.

					Implication		Modus tollens
t	P_t	C_t	$\neg P_t$	$\neg C_t$	$(P_t \rightarrow C_t)$	$((P_t \rightarrow C_t) \cap \neg C_t)$	$((P_t \rightarrow C_t) \cap \neg C_t) \rightarrow (\neg P_t)$
1	1	1	0	0	1	1	1
2	1	0	0	1	0	0	0
3	0	1	1	0	1	1	1
4	0	0	1	1	1	1	1

2.2.6. Proof by contraposition

The contrapositive of $P_t \rightarrow C_t$ is known to be $\neg C_t \rightarrow \neg P_t$. In general, if a statement is true, then its contrapositive is true and vice versa. If a single statement is false, then its contrapositive is false too. A statement and its contrapositive are logically equivalent. A proof by contraposition is based on the fact that the statement "*if* the premisse P_t is true *then* the conclusion C_t is true" is **logically equivalent** to the statement "*if* the conclusion C_t is not true *then* the premisse P_t is not true". (**Table 14**). The proof by contraposition is not identical with the proof by modus tollens. In contrast to modus tollens, in a proof by contraposition we show that C_t is false and then conclude that P_t is false too.

Table 14. Proof by contraposition: *if* the conclusion C_t is not true *then* the premisse P_t is not true ($\neg C_t \rightarrow \neg P_t$).

$\neg C_t \rightarrow \neg P_t$		P_t		
		$P_t = +0$	$P_t = +1$	Total
C_t	$C_t = +0$	1	0	
	$C_t = +1$	1	1	
Total				1

Modus ponens and the proof by contrapositive are *logically equivalent* and are determined by the minimum demand that

$$(P_t \cap C_t) = (\neg P_t \cap \neg C_t) = +1 = TRUE \quad (16)$$

The case $P_t = +1$ and $C_t = +0$ is neither compatible with the equation

$$(P_t \cap C_t) = (1 \cap 0) = +0 \quad (17)$$

nor with the equation

$$(\neg P_t \cap \neg C_t) = (0 \cap 1) = +0 \quad (18)$$

In general, logical equivalence doesn't care whether a glass is half-full or half-empty. This is a matter of personal taste. A statement and its contrapositive are logically equivalent or it is

$$(P_t) \rightarrow (C_t) \equiv (\neg C_t) \rightarrow (\neg P_t) = +1 = TRUE \quad (19)$$

The contrapositive of a certain statement has the same truth value (truth or falsity) as the statement itself. If a contrapositive is true, then its statement is true (and vice versa). If a contrapositive is false, then its statement is false (and vice versa).

Proof by contrapositive.
Claim.
(Premise 1) $\neg C_t \rightarrow \neg P_t$
Proof.
(Premise 2) $\neg C_t$
Additional arguments.
Decision.
(Conclusion) $\neg P_t$
Quod erat demonstrandum.

2.2.7. Proof by modus inversus

A valid argumentation, in the absence of technical errors or incorrectly applied rules of reasoning et cetera, preserves the truth-value of its premises with the logical necessity that *if* a premise P_t is false, *then* conclusions C_t derived from it should be false as well. *The proof by inversion* (modus inversus) is a valid rule of inference or a proof method "*by which from a given proposition another is derived having for its subject the contradictory of the original subject and for its predicate the contradictory of the original predicate*." ([18], p. 51). In general, the inverse of the modus ponens statement $P_t \rightarrow C_t$ ("**If *P_t* is true, then *C_t* is true**") is known to be the statement or the equation $\neg P_t \rightarrow \neg C_t$ or in spoken language: "**If *P_t* is false, then *C_t* is false**" (**Table 15)** while the basic relationship $(P_t \rightarrow C_t) = (\neg\neg P_t \rightarrow \neg\neg C_t)$ is valid.

Table 15. Proof by modus inversus: *if* the premisse P_t is not true *then* the conclusion C_t is not true ($\neg P_t \rightarrow \neg C_t$).

$\neg P_t \rightarrow \neg C_t$		C_t		
		$C_t = +0$	$C_t = +1$	Total
P_t	$P_t = +0$	1	0	
	$P_t = +1$	1	1	
Total				1

In contrast to modus ponens, modus inversus demands that it is not possible to draw a true conclusion from a false premise (**Table 16**).

Table 16. Proof by modus inversus: *without* premisse P_t is true *no* conclusion C_t is true ($P_t \leftarrow C_t$).

$P_t \leftarrow C_t$		C_t		
		$C_t = +1$	$C_t = +0$	Total
P_t	$P_t = +1$	1	1	
	$P_t = +0$	0	1	
Total				1

The logical equivalent of the equation $\neg P_t \rightarrow \neg C_t$ or in spoken language: "**If P_t *is false*, then C_t *is false***" is "**Without P_t *is true* no C_t *is true***" and viewed by table 17 (**Table 17**).

Table 17. Modus inversus.

t	P_t	C_t	$\neg P_t$	$\neg C_t$	Modus ponens $(P_t \rightarrow C_t)$	Modus inversus $(\neg P_t \rightarrow \neg C_t)$	Without P_t is true no C_t is true. $(P_t \leftarrow C_t)$
1	1	1	0	0	1	1	1
2	1	0	0	1	0	1	1
3	0	1	1	0	1	0	0
4	0	0	1	1	1	1	1

In other words, a direct proof provided without any technical errors which is grounded on something false must end up in something false.

Proof by modus inversus.
Claim.
(Premise 1) $\neg P_t \rightarrow \neg C_t$
Proof.
(Premise 2) $\neg P_t$
Additional arguments.
Decision.
(Conclusion) $\neg C_t$
Quod erat demonstrandum.

2.2.8. Proof by modus juris (anti modus inversus)

In point of fact, modus inversus demands that "*if* the premisse P_t is false *then* the conclusion C_t is flase $(\neg P_t \rightarrow \neg C_t)$. P_t =+0. Ergo: C_t =+0. Thus far, if the negation of modus inversus is true i. e. $\neg(\neg P_t \rightarrow \neg C_t)$ = true, then the original modus inversus is false.

Table 18. Proof by modus juris (negation of modus inversus): $\neg(\neg P_t \rightarrow \neg C_t)$. $P_t = 0$. $C_t = +1$.

$\neg(\neg P_t \rightarrow \neg C_t)$		C_t		Total
		$C_t = +0$	$C_t = +1$	
P_t	$P_t = +0$	0	1	
	$P_t = +1$	0	0	
	Total			1

Either modus inversus $(\neg P_t \rightarrow \neg C_t)$ is true *or* modus juris $(\neg(\neg P_t \rightarrow \neg C_t))$ is true but not both.

Proof by modus juris.
Claim.
(Premise 1) $\neg(\neg P_t \rightarrow \neg C_t)$
Proof.
(Premise 2) $P_t = +0$
Additional arguments.
Decision.
(Conclusion) $C_t = +1$
Quod erat demonstrandum.

2.2.9. Proof by modus conversus

The inverse of $P_t \rightarrow C_t$ is $\neg P_t \rightarrow \neg C_t$ and logically equivalent to the contrapositive $(\neg C_t \rightarrow \neg P_t)$ of the converse $(C_t \rightarrow P_t)$. If $(P_t \rightarrow C_t)$ is true, then its contrapositive $(\neg C_t \rightarrow \neg P_t)$ is true (and vice versa). In other words, the contrapositive $(\neg C_t \rightarrow \neg P_t)$ always has the same truth value (truth or falsity) as $(P_t \rightarrow C_t)$ itself, both are logically equivalent. The converse of $(P_t \rightarrow C_t)$ is the result of reversing the two parts of $P_t \rightarrow C_t$ to $C_t \rightarrow P_t$ (**Table 19**).

Table 19. Proof by modus conversus: *if* conclusion C_t is true *then* premisse P_t is true $(C_t \rightarrow P_t)$.

$C_t \rightarrow P_t$		P_t		Total
		$P_t = +1$	$P_t = +0$	
C_t	$C_t = +1$	1	0	
	$C_t = +0$	1	1	
	Total			1

The converse as the contrapositive of the inverse has always the same truth value as the inverse. A truth table (**Table 20**) is able to provide evidence that $(P_t \rightarrow C_t)$ and its own converse $(C_t \rightarrow P_t)$ are not logically equivalent unless both terms imply each other.

Table 20. Modus conversus.

t	P_t	C_t	$\neg P_t$	$\neg C_t$	Modus ponens $(P_t \rightarrow C_t)$	Modus conversus $(C_t \rightarrow P_t)$	Modus inversus $(\neg P_t \rightarrow \neg C_t)$
1	1	1	0	0	1	1	1
2	1	0	0	1	0	1	1
3	0	1	1	0	1	0	0
4	0	0	1	1	1	1	1

Under conditions where $(P_t \rightarrow C_t)$ is treated as being equivalent to its own converse $(C_t \rightarrow P_t)$ without being the case, the fallacy of affirming the consequent is committed. However, there are circumstances where $(P_t \rightarrow C_t)$ is true and its own converse $(C_t \rightarrow P_t)$ is equally true (i.e., if P_t is true if and only if C_t is also true). The dominance of modus ponens over the other rules of inference is not justified. In particular, it is possible to approach to a methodological proof of a theory, of a theorem et cetera from different points of view and it is completely a matter of personal taste whether a glass of water is treated as being either half full (modus ponens: $P_t \rightarrow C_t$) or half empty (modus inversus: $\neg P_t \rightarrow \neg C_t$). One disadvantage of modus ponens is that conclusions with false antecedents are potentially considered true. In opposite to modus ponens, modus conversus does not allow to conclude a true conclusion $(C_t = +1)$ from a false premise $(P_t = +0)$.

2.2.10. Proof by modus securus

The various rules of inferences differ in many aspects but have at least one point in common. **Besides of all differences, the implication, the converse, the contrapositive and the inverse agree all completely at the trial t=1 and the trial t=4**. In the following, **Table 21** provides us with an overview.

Table 21. Overview

			Implication	Converse			Contrapositive	Inverse
Trial t	P_t	C_t	$(P_t \rightarrow C_t)$	$(C_t \rightarrow P_t)$	$\neg P_t$	$\neg C_t$	$(\neg C_t \rightarrow \neg P_t)$	$(\neg P_t \rightarrow \neg C_t)$
1	1	1	1	1	0	0	1	1
2	1	0	0	1	0	1	0	1
3	0	1	1	0	1	0	1	0
4	0	0	1	1	1	1	1	1

Note that the converse of $P_t \rightarrow C_t$ is $C_t \rightarrow P_t$. The contrapositive of $P_t \rightarrow C_t$ is $\neg C_t \rightarrow \neg P_t$ and has the same truth values as $P_t \rightarrow C_t$ or it is $P_t \rightarrow C_t = \neg C_t \rightarrow \neg P_t$. The inverse of $P_t \rightarrow C_t$ is $\neg P_t \rightarrow \neg C_t$. The converse of $P_t \rightarrow C_t$ and the inverse of $P_t \rightarrow C_t$ have the same truth values or it is $C_t \rightarrow P_t = \neg P_t \rightarrow \neg C_t$. In general, the inverse of premise $P_t \rightarrow C_t$ is the same as the contrapositive of the converse.

Proof by modus securus I.

Claim.

(Premise 1) $P_t \longleftrightarrow C_t$

Proof.

(Premise 2) P_t

Additional arguments.

Decision.

(Conclusion) C_t

Quod erat demonstrandum.

or

Proof by modus securus II.

Claim.

(Premise 1) $P_t \longleftrightarrow C_t$

Proof.

(Premise 2) $\neg P_t$

Additional arguments.

Decision.

(Conclusion) $\neg C_t$

Quod erat demonstrandum.

2.2.11. Proof by contradiction (Reductio ad absurdum)

Science had always been conducted through step-by-step experiments or (logical) proofs but still requires a careful attention to (classical) logic otherwise various confusions and contradictions may appear. In classical logic, a contradiction is something which is always absurd. A theorem, a theory et cetera which includes a contradiction is logically inconsistent. In point of fact, even if it is difficult for scientists prove a theorem, a theory et cetera to be true for ever, it is necessary to prevent the outbreak of epidemic contradictions. Regardless of how many positive examples appear to support a theorem or a theory, *one single counter-example* or one single contradictory instance to a theory is sufficient enough to falsify the general validity of a theorem or of a theory et cetera. A proof by contradiction [19], [20] is a very commonly used scientific proof technique which is able to proof in general the falsity or the truth of a statement, an equality, a principle (P) et cetera. The principal concern, then, is that if contradictions are not absurd and if a theory has contradictions in it then reductio ad absurdum is not possible. However, reduction to the impossible is a style of reasoning which has been used by many authors and can be found repeatedly in Aristotle's Prior Analytics [21]. Throughout the history of philosophy and mathematics from classical antiquity onwards there have been circumstances where a thesis had to be accepted because its rejection would be untenable. "*The proof ... reductio ad absurdum, which Euclid loved so much, is one of the mathematician's finest weapons*" ([22], p. 94).

Classical logic, as we all know, cannot tolerate the presence of a contradiction without collapsing. In this context, the Principle of Pseudo-Scotus (PPS), also known since medieval times as *ex contradictione sequitur quodlibet* [23] (and also called the Principle of Explosion), states that from any theory or formal axiomatic system which includes a contradiction A and not A any other desired theorem B (correct or incorrect) can be derived. In other words, a contradiction in a formal axiomatic system can prove any theorem true. Thus far, even if there are trials advocated especially by the Peruvian philosopher Francisco Miró Quesada [24] and other [25] authors in modeling something like a system of *paraconsistent logic* which attempts to *deal with contradictions* [26], the progress has been very slow. Many times, dialectical logic has been treated as a particular case [23] of paraconsistent logic. This is clearly a mistake. In contrast to paraconsistent logic, dialectical logic is an extension of classical logic and goes beyond classical logic. *Dialectical logic* [27] starts there, where classical logic ends and vice versa. The relationship between classical logic and dialectical logic [28]–[30] is similar to the relationship between Newtonian mechanics and Einstein's special theory of relativity, the one pass over into its own other [26] without any contradiction. Thus far and in short, a proof by contradiction demands to assume that P_t is false. In the following assume that $\neg P_t$ is true and derive a contradiction. Since P_t cannot be both true and false, P_t is false.

Proof by contradiction.

Claim.

(Premise 1) P_t is false.

Proof.

(Premise 2) $\neg P_t$ is true.

Additional arguments.

Decision.

(Conclusion) Derive a contradiction from $\neg P_t$ is true.

Quod erat demonstrandum.

Something impossible or incorrect cannot be derived from something correct as long as there are no technical or other errors inside a proof.

2.2.12. Proof by counterexample

Can we learn anything from scientific theories or from experiments at all? Theoretically, one single experiment [31] has the potential to refute a whole theory even if historically, no theory has been refuted by one single experiment. In philosophy, mathematics, physics or in science as such, it is not all the time possible to prove all scientific claims in time beyond any doubt. The proof by a counterexample [32]–[35] is a valid proof methodology to infer consequences of scientific claims or theories and to demonstrate clearly that a certain scientific position is wrong by showing that it does not apply in certain cases. A counterexample which is able to derive a logical contradiction in the absences of technical and other errors out of a theorem or a theory refutes the same.

2.3. Axioms

An axiom as simple as possible taken to be true has the potential to serve as the foundation for consistency and completeness [1] in mathematics and as the premise or starting point for further reasoning and arguments in all other sciences too besides of Gödel's incompleteness theorems [2]. In light of Kurt Gödel's most extreme view [2] that consistency is incompatible with completeness we are forced to accept that an inconsistent foundation for mathematics appears to be the only remaining candidate for completeness. Gödel's incompleteness theorems are illustrated by the following table.

Table 22. Gödel's first incompleteness theorem [2].

A set of axioms is		consistent		Total
		yes	no	
complete	yes	0	1	
	no	1	0	
Total				1

The starting point of Gödel's theorems is an *either* completeness *or* consistency logical fallacy. The key to diagnosing Gödel's error in reasoning is to point out that Gödel is unnecessarily and unfairly limiting our possibilities only two choices, *either* complete *or* consistent. Gödel's incompleteness theorems are placing us unjustifiably between a hard place and a rock. A proper challenge to Gödel's incompleteness theorems fallacy could be to say, everything is either complete or not complete, but not everything is *either* complete *or* inconsistent. Theoretically, it is possible that a certain and special set of axioms is complete and consistent. However, since the problem of indeterminate forms is not solved, the division of a tensor by a tensor is not solved and many other problems too, a set of axioms valid for all system is to be achieved.

Example.
Let as set of axioms describe the construction of a big house (or a scientific theory) perfectly. A house under construction may be complete or incomplete. The statics of the same house can be consistent or not consistent. However, the statics of a house, even if the same is not complete, can but must not be consistent. Thus far, the limited menu of choices as advocated by Gödel's incompleteness theorems is not fair, it is unfair and completely worthless.

2.3.1. Axiom I (principium identitatis)

$$+1 = +1 \tag{20}$$

Remark 1.
The identity law has a very long history in philosophy and science. The earliest recorded use of the identity law is ascribed to Plato in his dialogue Theaetetus. In one of his dialogues concerning the nature of knowledge, Plato wrote circa 369 BCE that "*each is different from each, and the same with itself*" ([36], p. 177). The identity law can be found in Aristotle's Metaphysics (Book IV, Part 4) and has been discussed by several other ([37], p. 113) authors too. A more restrained, but still unorthodox, view on the identity law is expressed especially Gottfried Wilhelm Leibniz (1646-1716). Leibniz wrote, "*Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra A est A, B est B*" [38] or "… everything is that what it is ... A equals A, B equals B"

2.3.2. Axiom II (principium contradictionis)

Contradictions are an objective and important feature [26] of objective reality. Still, contradictions in theorems, arguments and theories would allow us to conclude everything desired. In contrast to religion and other domains of human culture, one very important and at the end to some extent normative criteria to achieve some advances and progress in science is depended on detecting contradictions in science and eliminating the same too. The most important point is that even if we are surrounded by contradictions a co-moving observer [26] will always find that something is *either* +1=+1 *or* +0=+0 but not both, i. e. it is not +1 = +0. The simplest form of Aristotle's law of contradiction is defined as

$$+0 = +1 \tag{21}$$

Multiplying this equation by +0, we obtain [26]

$$(+0) \times (+0) = (+1) \times (+0) \tag{22}$$

Aristotle's law of contradiction follows according to Boolean algebra [39] according to today's laws of algebra and mathematics, as

$$(+0) = +1 \times (+0) \tag{23}$$

Boole is of the position that "*... the principle of contradiction ... affirms that it is impossible for any being to possess a quality, and at the same time not to possess it ...* "([39], p. 49). Accordingly, "*Hence x(1 -x) will represent the class whose members are at once 'men,' and 'not men,' and the equation (1) thus express the principle, that a class whose members are at the same time men and not men does not exist. In other words, that it is impossible for the same individual to be at the same time a man and not a man.*" ([39], p. 49). Thus far, and no wonder that, according to Popper, a philosopher of science of the 20th century, contradiction is at the end a demarcation line between science and 'non-science' too. "*We see from this that if a theory contains a contradiction, then it entails everything, and therefore, indeed, nothing[...]. A theory which involves a contradiction is therefore entirely useless as a theory*". ([17], p. 429). In particular, to face the threat of a logical or scientific *Armageddon* and the breakdown of any logical coherence posed by accepting the contradiction +0 = +1, it is necessary to formulate the same clearly. Far from reduced to the silence of deep space given due to the explosive effect of *ex contradictione quodlibet*, there are circumstances of special theory of relativity where it is possible to allow a kind of inconsistency without logical incoherence [26]. A proof can be based on *principium contradictionis*, the premise like +0 = +1 can justify further conclusions. A sound argument would follow if the conclusions were logically derived from the premises [40] without any technical errors. The result would have to be a mistake, since we started with a mistake. In contrast to a sound argument a valid argument is a sound argument while all the premises are true. If anything after the false premise is true and logically consistent then in the absence of any technical errors the conclusion itself must be false too.

2.3.3. Axiom III (principium negationis)

$$\frac{+1}{+0} \approx +\infty \tag{24}$$

while +∞ denotes the positive infinity.

3. Results

Theorem 3.1 (The preservation of the truth)

Claim.

In the absence of technical errors or incorrectly applied rules of reasoning et cetera, the truth is preserved.

Proof by counterexample.

In contrast to our theorem above, we assume that in the absence of technical errors or incorrectly applied rules of reasoning *it is generally valid that the truth is not preserved.* In this case, it is not allowed to find one single case which provides evidence that the truth is preserved.

$$+1 = +1 \tag{25}$$

Adding +1, we obtain

$$+1+1 = +1+1 \tag{26}$$

or

$$+2 = +2 \tag{27}$$

which is true and equally a contradiction.

In general, it is not possible to derive something incorrect from something correct in the absence of technical errors or incorrectly applied rules of reasoning and other errors. In the absence of technical errors or incorrectly applied rules of reasoning et cetera, it is not generally valid that the truth is not preserved.

Quod erat demonstrandum.

Theorem 3.2 (The preservation of the contradiction)

We inevitably make mistakes and have false beliefs. To prevent lapsing into absurdity, hypothesis can be tested only on clear foundations. In general, it is claimed that from a contradictory premise, anything follows, which contradicts the principle of the preservation of contradictions. However, in the absence of technical errors, if something contains a contradiction then everything else derived from such a contradiction should obtain a contradiction too (preservation of contradiction) otherwise it must be possible without one exemption the derive a true conclusion form a false premise.

Claim.

In the absence of technical errors or incorrectly applied rules of reasoning et cetera, the contradiction or falsehood is preserved.

Proof by counterexample.

In contrast to our theorem above, we assume that *it is generally valid that the falsehood is not preserved.* In this case, it is not allowed to find one single case which provides evidence that the falsehood is preserved. The equation

$$+0 = +1 \tag{28}$$

is obviously false or self-contradictory. Adding +1, we obtain

$$+0+1 = +1+1 \tag{29}$$

or

$$+1 = +2 \tag{30}$$

which is false.

In general, it is not possible to derive something correct from something incorrect in the absence of technical errors or incorrectly applied rules of reasoning and other errors. In the absence of technical errors or incorrectly applied rules of reasoning et cetera, it is not generally valid that the falsehood or contradiction is not preserved. In contrast to the starting point of this proof, the contradiction *is* preserved.

Quod erat demonstrandum.

Remark 2.

We started the proof above with a false premise (P_t: +0=+1). Our expectation was, that this contradiction will not be preserved with the consequence that we should be able to derive a true conclusion but we failed. In the absence of technical errors, it was not possible to derive a true conclusion (C_t: +1=+2 is false) from a false premise, which completes our proof. From a contradiction, a contradiction follows. It was mentioned before that modus ponens allows to derive a true conclusion from different premises. If one concrete and single premise (i. e. P_t is true) is used to derive a true conclusion, then the rest (i. e. P_t is false) of many other possible but *true* premises is equally not used for these purposes. Modus pones just don't care about the rest of all other possible premises to derive a true conclusion, modus ponens considers only one single premise and insists that from such a single and true premise a true conclusion must be drawn. To be precise, the conclusion that modus ponens allows to derive a true conclusion from a false premise is incorrect. As a result, obtaining true and long-lasting scientific knowledge conducted through most simple step-by-step proofs has the potential to overcome obscurity and confusion in science. Searching for true scientific knowledge is a risky gesture.

Still, *either* modus inversus ($\neg P_t \rightarrow \neg C_t$) is generally true *or* modus juris ($\neg(\neg P_t \rightarrow \neg C_t)$) is generally true but not both. The proof demonstrated that modus juris, the negation of modus inversus, is not generally true. Consequently, we must accept that modus inversus is generally true and of use for further purposes. Nonetheless, contemporary approaches taken to develop *a system of paraconsistent logic* [24]–[26], [41]–[44] which we have so far seen need to ensure the preservation of truth. Despite the fact that paraconsistent logic is to some extent the rejection of classical logic, even a system of paraconsistent logic cannot avoid the explosion principle (*ex contradictione quodlibet*) when faced with a contradiction, where a contradiction is present. The contradiction is preserved especially according *modus sine* ($\neg P_t \leftarrow \neg C_t$) too which is the logical equivalent of *modus ponens* ($P_t \rightarrow C_t$). *Modus sine* as the other side of modus ponens demands that *without* a false premisse P_t *no* false conclusion C_t.
A theorem which assures both, something and equally its own negation, is logically inconsistent, includes a contradiction and is entirely useless. However, this does not exclude real existing contradictions in objective reality [26].

Theorem 3.3 ($A_t < B_t$ and disjunction are not equivalent)

The strict inequality $A_t < B_t$ is not identical with an inclusive (logical) disjunction $A_t \cup B_t$, also known as alternation.

Proof by contradiction.

In contrast to our theorem before, we assume that logical disjunction and the strict inequality $A_t < B_t$ are logically equivalent. The logical equivalence between strict inequality $A_t < B_t$ and an inclusive (logical) disjunction means that both are true together or false together. In other words, if a statement is true according to the strict inequality $A_t < B_t$, then the same statement is true according an inclusive (logical) disjunction too and vice versa. Any difference with respect to this point determines a contradiction and cannot be accepted. The logical disjunction ($A_t \cup B_t$) is defined as follows (**Table 23**).

Table 23. Logical disjuntion.

$A_t \cup B_t$		B_t		Total
		$B_t = +1$	$B_t = +0$	
A_t	$A_t = +1$	**a_t = yes**	**b_t = yes**	
	$A_t = +0$	**c_t = yes**	d_t = no	
Total				1

The strict inequality $A_t < B_t$ itself is defined as follows (**Table** 24).

Table 24. The strict inequality $A_t < B_t$.

$A_t < B_t$		B_t		Total
		$B_t = +1$	$B_t = +0$	
A_t	$A_t = +1$	a_t = no	b_t = no	
	$A_t = +0$	**c_t = yes**	d_t = no	
Total				1

The logical disjunction $A_t \cup B_t$ and the strict inequality $A_t < B_t$ are identical only at the case $c_t = ((A_t = +0) \cap (B_t = +1))$ but not in general. The strict inequality $A_t < B_t$ is not identical with *inclusive* disjunction $A_t \cup B_t$.

Quod erat demonstrandum.

Remark 3.
There are many ways how to proof the correctness of something. The above method uses truth tables to establish a valid proof. This proof is of importance especially for quantum theory too.

Theorem 3.4 ($A_t > B_t$ and disjunction are not equivalent)

The strict inequality $A_t > B_t$ is not identical with an inclusive (logical) disjunction $A_t \cup B_t$, also known as alternation.

Proof by contradiction.

In contrast to our theorem before, we assume that logical disjunction and the strict inequality $A_t > B_t$ are logically equivalent. Again, a logical equivalence between strict inequality $A_t > B_t$ and an inclusive (logical) disjunction means that both are false together or true together. In other words, if a statement is true according to an inclusive (logical) disjunction, then the same statement is true according the strict inequality $A_t > B_t$ too and vice versa. Any difference means a contradiction and cannot be accepted. The logical disjunction ($A_t \cup B_t$) again is defined as follows (**Table 25**).

Table 25. Logical disjuntion $A_t \cup B_t$.

$A_t \cup B_t$		B_t		Total
		$B_t = +1$	$B_t = +0$	
A_t	$A_t = +1$	**a_t = yes**	**b_t = yes**	
	$A_t = +0$	**c_t = yes**	d_t = no	
Total				1

The strict inequality $A_t > B_t$ itself is defined as follows (**Table 26**).

Table 26. The strict inequality $A_t > B_t$.

$A_t < B_t$		B_t		Total
		$B_t = +1$	$B_t = +0$	
A_t	$A_t = +1$	a_t = no	**b_t = yes**	
	$A_t = +0$	c_t = no	d_t = no	
Total				1

The logical disjunction $A_t \cup B_t$ and the strict inequality $A_t > B_t$ are identical only at the case $b_t = ((A_t = +1) \cap (B_t = +0))$ but not in general. The strict inequality $A_t > B_t$ is not identical with *inclusive* disjunction $A_t \cup B_t$ and vice versa.

Quod erat demonstrandum.

Remark 4.
Throughout history or at least since the French Revolution, both equality and inequality are complex, multifaceted and very contested concepts in science and in our everyday life. Despite of the widespread and controversial misconceptions about the meaning equality and inequality as great social ideals, this article is not concerned with social and political equality or inequality. The role and correct account of the relationship between an inequality and a logical disjunction is itself a difficult issue. For this reason, and to clarify this, the theorems above are of use.

Theorem 3.5 ($A_t \leq B_t$ and material implication)

The non-strict inequality $A_t \leq B_t$ is identical with material implication.

Proof.

The material implication ($A_t \rightarrow B_t$) is defined as follows (**Table 27**).

Table 27. Material implication ($A_t \rightarrow B_t$).

$A_t \rightarrow B_t$		B_t		Total
		$B_t = +1$	$B_t = +0$	
A_t	$A_t = +1$	**a_t = yes**	b_t = no	
	$A_t = +0$	**c_t = yes**	**d_t = no**	
Total				1

The strict inequality $A_t \leq B_t$ itself is defined as follows (**Table 28**).

Table 28. The non-strict inequality $A_t \leq B_t$.

$A_t \leq B_t$		B_t		Total
		$B_t = +1$	$B_t = +0$	
A_t	$A_t = +1$	**a_t = yes**	b_t = 0	
	$A_t = +0$	**c_t = yes**	**d_t = yes**	
Total				1

Material implication ($A_t \rightarrow B_t$) and non-strict inequality $A_t \leq B_t$ agree in all cases. The non-strict inequality $A_t \leq B_t$ is identical with material implication ($A_t \rightarrow B_t$).

Quod erat demonstrandum.

Remark 5.

This theorem is of use for continuously distributed random variables too.

Theorem 3.6 ($A_t \geq B_t$ and conditio sine qua non)

The non-strict inequality $A_t \geq B_t$ is logically identical with the conditio sine qua non relationship.

Proof.

The conditio sine qua non ($A_t \leftarrow B_t$) is defined as follows (**Table 29**).

Table 29. Conditio sine qua non ($A_t \leftarrow B_t$).

$A_t \leftarrow B_t$		B_t		Total
		$B_t = +1$	$B_t = +0$	
A_t	$A_t = +1$	**a_t = yes**	**b_t = yes**	
	$A_t = +0$	c_t = no	**d_t = no**	
Total				1

The strict inequality $A_t \geq B_t$ itself is defined as follows (**Table 28**).

Table 30. The non-strict inequality $A_t \geq B_t$.

$A_t \geq B_t$		B_t		Total
		$B_t = +1$	$B_t = +0$	
A_t	$A_t = +1$	**a_t = yes**	**b_t = yes**	
	$A_t = +0$	c_t = no	**d_t = yes**	
Total				1

Conditio sine qua non ($A_t \leftarrow B_t$) and non-strict inequality $A_t \geq B_t$ agree in all cases. The non-strict inequality $A_t \geq B_t$ and the conditio sine qua non ($A_t \leftarrow B_t$) relationship are logically equivalent.

Quod erat demonstrandum.

Remark 6.

This proof demonstrates clearly the equivalence of the non-strict inequality $A_t \geq B_t$ and conditio sine qua non. Especially, it is not true, that the non-strict inequality $A_t \geq B_t$ is identical with (an inclusive/exclusive) disjunction. The non-strict inequality $A_t \geq B_t$ can be simplified as *either* $(A_t = B_t)$ *or* $(A_t > B_t)$ but not both at the same trial. Furthermore, both cases *either* $(A_t = B_t)$ *or* $(A_t > B_t)$ are a determining part of the non-strict inequality $A_t \geq B_t$. If the case $(A_t > B_t)$ is not allowed, then the use of the non-strict inequality $A_t \geq B_t$ is not allowed too, since the same demands that it must be possible that $(A_t > B_t)$ too. Using the non-strict inequality $A_t \geq B_t$ without allowing the case $(A_t > B_t)$ implies a mis-use of the non-strict inequality $A_t \geq B_t$ and can be the source of many contradictions. The variance $\sigma(X_t)^2$ is defined as $\sigma(X_t)^2 = E((X_t)^{2)} - E(X_t)^2$ and can take the values $\sigma(X_t)^2 \geq 0$, where E denotes the expectation values. In principle, it is allowed that *either* $\sigma(X_t)^2 > 0$ *or* $\sigma(X_t)^2 = 0$, but not both at the same trial t / (period of) time t.

Theorem 3.7 (The general validity of modus inversus)

We appear to rely on some scientific methods ubiquitously in our daily life, and it is also generally thought that the same are at the very foundation of science as such and generally valid. However, what exactly is meant by the notion general validity of a theorem, of a theory et cetera? Much has been written attempting to work out the meaning of this notion but without a clear and definite resolution in sight. Thus far, the notion "*general validity*" carries to some extent a negative connotation and the question is justified does it make any sense at all to claim that a theorem or a mathematical procedure et cetera is generally valid? Perhaps unsurprisingly, the challenge indeed is to find a way of living with such a radical-seeming notion like general validity of a theorem, a theory et cetera. The generation of scientific knowledge depends on the historical background too and is associated with errors and the diversity of scientific methods used across the sciences makes it to a greater or lesser extent difficult to find anything that separates sciences from something other not typically considered scientific. In order to distinguish scientific knowledge from dubious, nonscientific, or merely ideological and illegitimate forms of knowledge or pseudoscience it is necessary to develop and use scientific methods which ensure the perseveration of the truth and the preservation of the contradiction in the same respect. Thus far, it is worth noting, however, that even in the absence of unintentionally or intentionally (in order to deceive other scientist) created logical fallacies and other circumstances of fallacious human reasoning a generally valid theorem or theory is valid only preliminary. In principle, it is difficult to exclude that in the future one single counter-example can be presented which is able to refute the general validity of such a theorem or theory.

Claim.

The proof by modus inversus is generally valid.

Proof by contradiction.

Either the proof by modus inversus *is generally valid or* the proof by modus inversus *is not generally valid*, a third is not given, *tertium not datur*. In contrast to our theorem above, we are of the opinion that the proof by modus inversus is <u>not</u> generally valid. Thus far, if the proof by modus inversus is not generally valid, then it is no possible to provide one single case where the proof by modus inversus is valid. We are starting this proof with a false premise. In general, it is false that

$$+0 = +1 \tag{31}$$

Adding +1 to this equation, it is

$$+0+1 = +1+1 \tag{32}$$

and we obtain

$$+1 = +2 \tag{33}$$

Quod erat demonstrandum.

Remark 7.

We started with something *incorrect* (P_t: +0=+1) and derived something *incorrect* (C_t :+1=+2) which is contradiction. The theorem above provides evidence that the proof by modus inversus (*if* P_t (+0=+1) is false *then* C_t (+1=+2)) is false) is valid.

One of the central notions of (classical) logic is the notion of logical consequence. Thus far, taken as a whole, we have been able to present one single case where the assumption that the proof by modus inversus is <u>not</u> generally valid breaks down. Since *either* the proof by modus inversus *is generally valid or* the proof by modus inversus *is not generally valid*, a third is not given, *tertium not datur* we are left with the logical consequence that the proof by modus inversus *is generally valid*. In other words, from something incorrect or from a contradiction, something incorrect or a contradiction must follow.

Theorem 3.8 (The general validity of modus ponens)

Claim.

The proof by modus ponens is generally valid.

Proof by contradiction.

Either the proof by modus ponens *is generally valid or* the proof by modus ponens *is not generally valid*. In contrast to this theorem, we are of the opinion that the proof by modus ponens is <u>not</u> generally valid. The logical consequence is, that if the proof by modus ponens is not generally valid, then it is not allowed or possible to provide one single case where the proof by modus ponens itself is valid. We are starting the proof of *if* P_t (+1=+1) is true *then* C_t (+2=+2) is true with a true premise. In general, it is true that

$$+1 = +1 \tag{34}$$

Adding +1 to this equation, it is

$$+1+1 = +1+1 \tag{35}$$

and we obtain

$$+2 = +2 \tag{36}$$

Quod erat demonstrandum.

Remark 8.

This is a contradiction. Our opinion was that *modus ponens is not generally valid*. In this case, it is not allowed to present one single case where this assumption breaks down. However, this theorem provides evidence, that modus ponens is valid. Since *either* the proof by modus ponens *is generally valid or* the proof by modus inversus *is not generally valid*, we must anodon our opinion that modus ponens is not generally valid, which completes our proof.

Theorem 3.9 (The general validity of mods securus)

Claim.

Modus securus is generally valid.

Proof.

Modus ponens or $P_t \rightarrow C_t$ is proofed as generally valid. Modus sine or $(\neg P_t \leftarrow \neg C_t)$ is the logical equivalent of modus ponens with the consequence that *modus sine is generally valid* too. A theorem before provided evidence that *modus inversus or* $(\neg P_t \rightarrow \neg C_t)$ *is generally valid* too. The logical consequence is that modus securus as the logical conjunction of modus sine and modus inversus or

$$Modus\ securus:\quad (\neg P_t \leftarrow \neg C_t) \quad \cap \quad (\neg P_t \rightarrow \neg C_t) \quad = \quad 1 \tag{37}$$

is generally valid too which is illustrated by the following table (**Table 31**).

Table 31. The general validity of modus securus.

			Modus ponens	Modus sine			**Modus inversus**	**Modus securus**
Trial t	P_t	C_t	$(P_t \rightarrow C_t)$	$(\neg P_t \leftarrow \neg C_t)$	$\neg P_t$	$\neg C_t$	$(\neg P_t \rightarrow \neg C_t)$	$(\neg P_t \leftarrow \neg C_t) \cap (\neg P_t \rightarrow \neg C_t)$
1	**1**	**1**	**1**	**1**	**0**	**0**	**1**	**1**
2	1	0	0	0	0	1	1	0
3	0	1	1	1	1	0	0	0
4	0	0	**1**	**1**	1	1	**1**	**1**

Quod erat demonstrandum.

Remark 9.

The various rules of inferences may differ in several aspects but have at least one point in common. **Modus securus is the common foundation of the presented rules of inferences.** To recognize and to reduce unjustified non-scientific influence exerted upon scientific research and the dissemination of that research as well, modus securus is of strategic importance to disable that any non-scientific influence can warp science.

Theorem 3.10 (The rule of multiplication by zero is refuted)

Claim.

In the absence of technical errors, today's rule of the multiplication by zero is self-contradictory and logically inconsistent.

Proof by modus inversus.

The proof by modus inversus is logically structured as follows.

$$\begin{array}{lllll} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & (+3 & = & +2) \end{array} \tag{38}$$

$Premise\ 1\colon\ \ if\ \ \ P_t = (+1 = +0)\ is\ false,\ \ \ then\ \ \ C_t = (+3 = +2)\ is\ false.$

The premise 2 of this proof by modus inversus is

$$+1 \quad = \quad +0 \tag{39}$$

which is of course incorrect. Adding +2 to something incorrect, it is

$$+1 + 2 \quad = \quad +0 + 2 \tag{40}$$

and we obtain

$$+3 \quad = \quad +2 \tag{41}$$

which is correct too according to *the proof by modus inversus* (*if* P_t (+1=+0) is false *then* C_t (+3=+2) is false). The following changes of this equation are mathematically without any technical error and correct. These changes must preserve this contradiction too. Multiplying by zero, it is

$$(+3) \times (+0) \quad = \quad (+2) \times (+0) \tag{42}$$

According to our today's rules of the multiplication by zero, this equation is equivalent with

$$(+0) \quad = \quad (+0) \tag{43}$$

In other words, our today's rules of the multiplication by zero equalizes differences and are able to change something false to something true without any logical necessity. Rearranging equation before, it is

$$(+1 - 1) \quad = \quad (+1 - 1) \tag{44}$$

Simplifying equation, we obtain

$$(+1) \quad = \quad (+1) \tag{45}$$

Quod erat demonstrandum.

Remark 10.

The proof above started with something *incorrect* and derived something *correct*, which is a contradiction. Thus far, one conclusion could be that from contradictory premises, anything follows, even something which is true, which is not convincing. Modus inversus has been proofed as generally valid with the consequence that a mathematical rule like the rule of the multiplication by zero must assure that (*if* P_t (+1=+0) is false *then*

C_t (+1=+1) is false) which is not the case. Today's rule of the multiplication by zero which changes +3 = +2 to +0=+0 and at the end to +1=+1 is self-contradictory and must be abandoned. We must reject today's rule of the multiplication by zero as incorrect.

Theorem 3.11 (The old rule of addition of zeros is refuted.)

Nicomachus of Gerasa (ca. 60 –ca. 120 AD), born in Gerasa, aformer Roman province of Syria, is best known for his book *Introduction to Arithmetic*. Nicomachus of Gerasa ([45], pp. 48, 120, 237-238) claimed that the sum of nothing added to nothing was nothing or in other words it is 0+0+0+ ...+ 0= 0.

Claim.

Today's rule of the addition of zero's (+0+0+... +0 = +0) is self-contradictory and based on a logical contradiction.

Proof by modus inversus.

In general, the premise

$$+(1)+(1)+\cdots+(1) \quad = \quad +(1) \tag{46}$$

is of course not true and a non-acceptable contradiction. Multiplying this equation by 0, we obtain according to our today's rules of mathematics that

$$\big(+(1)+(1)+\cdots+(1)\big)\times(+0) \quad = \quad +(1)\times(+0) \tag{47}$$

or

$$\begin{aligned}\big(+(1)+(1)+\cdots+(1)\big)\times(+0) \quad &= \quad +(1)\times(+0)\\ (n-times)\times(+0) \quad &= \quad +(1)\times(+0)\end{aligned} \tag{48}$$

or

$$\big(+(1\times 0)+(1\times 0)+\cdots+(1\times 0)\big) \quad = \quad +1\times 0 \tag{49}$$

and according to our today's rule of the addition of zero's

$$+(0)+(0)+\cdots+(0) \quad = \quad +(0) \tag{50}$$

Quod erat demonstrandum.

Remark 11.
According to modus inversus, if (+1+1+…+1 = +1) is false, then (+0+0+…+0 = +0) is false. Thus far, (+1+1+…+1 = +1) is false. Ergo: (+0+0+…+0 = +0) is false.

Theorem 3.12 (The new rule of addition of zeros.)

Claim.

The correct rule of the addition of zero's is (+0+0+... +0 = n×0).

Proof by modus inversus.

In general, the premise

$$+(1)+(1)+\cdots+(1) \quad = \quad +(1)+(1)+\cdots+(1) \tag{51}$$

is true. Let n= (1+1+...+1) and define n×0 = n_0 [46]. Multiplying this equation by 0, we obtain according to our today's rules of mathematics that

$$\big(+(1)+(1)+\cdots+(1)\big)\times(+0) \quad = \quad n\times 0 = n\_0 \tag{52}$$

or

$$\big(+(0)+(0)+\cdots+(0)\big) \quad = \quad n\times 0 = n\_0 \tag{53}$$

Quod erat demonstrandum.

Remark 12.
In other words, n×0 is not equivalent to 1×0.

Theorem 3.13 (The factorial operation is logically inconsistent)

Claim.

The factorial operation

$$(+1)! \quad = \quad (+0)! \tag{54}$$

is grounded on a logical contradiction and refuted.

Proof by modus inversus.
The proof by *modus inversus* is logically sound and demands that *if* P_t (+1=+0) is false *then* C_t (+1! = +0!) is false too. We define in this context

$$\begin{array}{ccccc} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & ((+1)! & = & (+0)!) \end{array} \tag{55}$$

$$Premise\ 1:\quad if\quad P_t=(+1=+0)\ is\ false,\quad then\quad C_t=\big((1!=0!)\big)\ is\ false.$$

The premise 2 of this proof by modus inversus is

$$+1 \quad = \quad +0 \tag{56}$$

which is of course false. Following Christian Kramp (1760 – 1826), the factorial [47] of a positive integer n is denoted by *n!*. Today, the value of 0! is 1, and the value of 1! is 1 too. Thus far, taking the factorial of the equation before, we obtain

$$(+1)! \quad = \quad (+0)! \tag{57}$$

or

$$+1 \quad = \quad +1 \tag{58}$$

which is a contradiction because +1=+1 is true but it cannot be true.

Quod erat demonstrandum.

Remark 13.
In the proof above according to the logic of modus inversus, it is $P_t = (+1=+0)$ = false, ergo C_t $(+1! = +0!)$ must be false too but it is not. A technical error is not apparent. This contradiction is due to the fact that the factorial has the potential to equalize differences. It is a rule in mathematics which is self-contradictory and rejected. This proof demonstrated the definitions are only of preliminary values and must be correct as such.

Theorem 3.14 (The equation (1/0) = (0/0) is refuted.)

Claim.

The equation

$$+\left(\frac{1}{0}\right) = +\left(\frac{0}{0}\right) \tag{59}$$

is grounded on a logical contradiction and refuted.

Proof by modus inversus.
This proof by modus inversus is logically structured as follows.

$$\begin{array}{ccccc} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & +\left(\frac{1}{0}\right) & = & +\left(\frac{0}{0}\right) \end{array} \tag{60}$$

$Premise\ 1:\ \ if\ \ \ P_t = (+1 = +0)\ is\ false,\ \ \ then\ \ \ C_t = +\left(\frac{1}{0}\right) = +\left(\frac{0}{0}\right)\ is\ false.$

The premise 2 of this proof by modus inversus is

$$+1 = +0 \tag{61}$$

and false. Dividing this equation by plus zero, i.e. +0, we obtain

$$+\left(\frac{1}{0}\right) = +\left(\frac{0}{0}\right) \tag{62}$$

Quod erat demonstrandum.

Remark 14.
Modus inversus $(\neg P_t \rightarrow \neg C_t)$ demands that *if* false premisse P_t (in our case +1=+0) *then* false conclusion C_t (+(1/0) = +(0/0)). In the proof above, it is $P_t = (+1=+0)$ = false. There are no technical errors inside the proof. Ergo: C_t = (+(1/0) = +(0/0)) is false too.

Theorem 3.15 (Anderson's Nullity is logically inconsistent.)

Modus ponens demands that the premise "**If P_t *is true*, then C_t *is true***" or $(P_t \rightarrow C_t)$ = true is given. Modus inversus as the inverse of the modus ponens statement $P_t \rightarrow C_t$ ("**If P_t *is true*, then C_t *is true***") is characterized by the statement or the equation $\neg P_t \rightarrow \neg C_t$ or in spoken language: "**If P_t *is false*, then C_t *is false***". Thus far, under circumstances where P_t is false, the conclusion is that C_t must be false too.

Claim.

Anderson's et al. definition of Nullity [48] is grounded on a logical contradiction, self-contradictory and refuted.

Proof by modus inversus.
This proof by modus inversus is logically structured as follows.

$$\begin{array}{ccccc} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & Nullity & = & Nullity \end{array} \tag{63}$$

$Premise\ 1:\ \ if\ \ \ P_t = (+1 = +0)\ is\ false,\ \ \ then\ \ \ C_t = Nullity = Nullity\ is\ false.$

The premise 2 of this proof by modus inversus is

$$+1 = +0 \tag{64}$$

and false. Multiplying this equation by Andersons's Nullity we obtain

$$(+1) \times (Nullity) = (+0) \times (Nullity) \tag{65}$$

According to Anderson's Axiom 15 [48], it is Nullity × 1 = Nullity. We obtain

$$(Nullity) = (+0) \times (Nullity) \tag{66}$$

According to Anderson's Axiom 15, it is Nullity × 0 = Nullity [48]. We obtain a conclusion C_t which is correct as

$$(Nullity) = (Nullity) \tag{67}$$

Quod erat demonstrandum.

Remark 15.
A consistent logical or mathematical operation is one that does not entail any contradiction. Modus inversus as generally valid demands that (*if* P_t (+1=+0) is false *then* C_t (Nullity = Nullity)) is false). Thus far, the premise P_t (+1=+0) is false but not the conclusion. The conclusion Nullity = Nullity is correct, which is a contradiction.

Theorem 3.16 (*Ex contradictione sequitur quodlibet* is refuted.)

The principle ex contradictione sequitur quodlibet is said to be contradiction intolerant and prevent us from falling or lapsing into absurdity. According to the principle *ex contradictione sequitur quodlibet* [23], [49] or the Principle of Explosion from a contradictory premise or statement (+1=+0), anything follows. Historically, ex contradictione sequitur quodlibet is ascribed to William of Soissons, a 12th century French logician who lived in Paris. However, a detailed proof, whether ex contradictione sequitur quodlibet is generally valid, is still not provided.

Claim.

Ex contradictione sequitur quodlibet (ECSQ) is not generally valid.

Proof by contradiction.
Either ex contradictione sequitur quodlibet is generally valid *or* ex contradictione sequitur quodlibet is not generally valid. In contrast to the theorem, we are of the opinion that ex contradictione sequitur quodlibet is generally valid. The logical consequence is, that if ex contradictione sequitur quodlibet is generally valid, then it is not allowed or possible to provide one single case where ex

contradictione sequitur quodlibet is not valid. We are starting the proof of *if* P_t (+0=+1) is false *then* C_t (+1=+2) is false. A thought experiment is performed while starting this proof with a contradiction. In general, it is false or a contradiction that

$$+0 = +1 \tag{68}$$

Adding +1 to this equation, it is

$$+0+1 = +1+1 \tag{69}$$

and we obtain

$$+1 = +2 \tag{70}$$

Quod erat demonstrandum.

Remark 16.
In the case above, from a contradiction (+0 = +1) does not everything follow. Under the conditions above, from a contradiction +0 = +1 follows exactly that +1 = +2. Even if this experiment is repeated independently many times more and adding or using other numbers (i. e. +2 or +3 or …), the result will be the same. In the absence of technical errors and other errors of human reasoning from a contradiction does not everything follow. The principle ex contradictione sequitur quodlibet is not generally valid and refuted. A principle like *ex contradictione sequitur quodlibet* should help us and not prevent us from gaining knowledge through repeated experiment or observation. To be convincing, *ex contradictione sequitur quodlibet* needs to provide evidence that the same is true. This evidence might come from repeating the whole experiment independently several times. However, the result will not change. In toto, the refutation of the principle *ex contradictione sequitur quodlibet* does not provide evidence that paraconsistent logic [24], [25], [50], [51] is correct as such.

4. Discussion

The change of objective reality appears to be a consistent process, to date, an end is not in sight. The nature of scientific inquiry of objective reality by which scientific knowledge is generated varies much across disciplines and often also invoke the incompatibility of opposed properties. The scientific success achieved depend to a very great extent on the scientific methods used. Therefore, considerations accounting for the very nature of truth and falsity must be able to rely on logically sound scientific methods too.
Modus ponens is one of the basic rules of inference and demands something like "If P_t, then C_t". In other words, from P_t, we can infer C_t. If it were possible to have P_t true and C_t false then modus ponens inference would be invalid. What we think, what we write, what we talk, our everyday reasoning is supported by modus ponens too. Loosing modus ponens in science would indicate a severe loss. Philosopher's aimed to show that modus ponens is not a generally valid [15] rule of inference. Many times, similar to other paradoxes, in one or other way, such trials rest on confusions and are easily circumvented.

Example.
Premise 1: If it is raining today on the street X (P_t is true), then the street X is wet (1000000 light years later).
Premise 2: It is raining on the street X today (P_t is true),
Conclusion: The street X is wet (1000000 light years later).

Such a conclusion is of course not justified. But this does not disprove modus ponens. We just don't know today, whether the street X is still existing 1000000 light years later. But even if the street X is still existing 1000000 light years later, what has today's rain to do with the street which is wet 1000000 light years later. Modus ponens can lead to inconsistencies if some basic assumptions are not considered by the user. It is necessary to make sure that events analysed occur at the same (period of) time t.

To date, the misuse of non-strict inequalities finds its own melting point in the mathematical formulations of Heisenberg's uncertainty principle [52], of Bell's theorem/inequality [53], in CHSH inequality [54] et cetera of quantum mechanics. It is more than strange to ground any scientific knowledge on such an inconsistency [26].

5. Conclusion

Non-strict inequalities have their own interior logic which must be respected in detail otherwise a theory or theorem will end up at a contradiction.

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