Refutation of self-referential sentences and provability: antimony of the liar

Abstract: The seven definitions evaluated are *not* tautologous. The first four equations refute the author's abstract that "a sentence cannot be denominated by p and written as p is not true". The next three refute that "in a system in which q denominates the sentence q is not provable it is not provable that q is true and not provable". The net result is refutation of the liar's antimony as a paradox (contradiction) and concludes that self-referential sentences are in fact *provable* as *not* tautologous. These findings provide a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ferreira, J.C. (2008). The antimony of the liar and provability.
ia800401.us.archive.org/28/items/arxiv-0806.0635/0806.0635.pdf

Abstract: This work evidences that a sentence cannot be denominated by p and written as p is not true. It demonstrates that in a system in which q denominates the sentence q is not provable it is not provable that q is true and not provable.

3 Self-referential sentences and provability: … what happens when the self-referential sentence is of the form p is not α where α is different from true or not true or from something equivalent to either true or not true. (10.0.1)

Remark 10.0.1: We take "different" to mean not "equivalent". The word "something" implies invocation of the existential quantifier, but the truth table result below is not affected.

LET p, q: p, α.

q@((q=q)+(q@q)) ; TT**FF** TT**FF** TT**FF** TT**FF** (10.0.2)

Let us substitute recursively p in the sentence for the sentence denominated by p:

p is not α (10.1.1)

p@q ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F** (10.1.2) (p is not α) is not α (10.2.1)

(p@q)@q ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (10.2.2)

((p is not α) is not α) is not α (10.3.1)

(((p@q)@q)@q ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F** (10.3.2)

Remark 11.0: We combine the two sentences in Eq. 10.0 as an implication in Eqs. 11 et seq.

Eqs. 10.0.1 implies 10.1.1. (11.1.1)

(q@((q=q)+(q@q)))>(p@q) ;
FTTT **F**TTT **F**TTT **F**TTT (11.1.2)

Eqs. 10.0.1 implies 10.2.1. (11.2.1)

(q@((q=q)+(q@q)))>((p@q)@q) ;
FTTT **F**TTT **F**TTT **F**TTT (11.2.2)

Eqs. 10.0.1 implies 10.3.1. (11.3.1)

(q@((q=q)+(q@q)))>(((p@q)@q)@q) ;
FTTT **F**TTT **F**TTT **F**TTT (11.3.2)

Remark 11.1: Eqs 11.1.2-11.3.2 return the same truth table, *not* tautologous, which refutes the liar's antimony as a paradox (contradiction) and concludes that self-referential sentences are in fact provable as *not* tautologous.