

Philosophy and Physics

Jean Louis Van Belle, *Drs, MAEc, BAEC, BPhil*

7 June 2019

Email: jeanlouisvanbelle@outlook.com

Abstract

This paper offers a philosophical-epistemological basis for the realist interpretation of quantum mechanics on the basis of the *Zitterbewegung* model of an electron. It does so by a detailed analysis of the logic and assumptions underpinning the mainstream (Copenhagen) interpretation of quantum mechanics. For ease of reference, we use the logic and analytical pieces which Richard Feynman developed for his *Lectures* on quantum mechanics for sophomore students.

Contents

Introduction	1
Feynman's identical particles.....	3
A physical interpretation of the wavefunction	5
Real or complex amplitudes?.....	6
Theoretical spin-zero particles versus real spin-1/2 particles	8
The reality of the complex conjugate of a wavefunction	10
360° and 720° symmetries: what is real?	12
Intermezzo: Occam's Razor Principle.....	16
Interpreting state vectors and absolute squares.....	16
What's Hermiticity?	18
References	19

Philosophy and Physics

Jean Louis Van Belle, *Drs, MAEc, BAEC, BPhil*

7 June 2019

Email: jeanlouisvanbelle@outlook.com

Introduction

Richard Feynman did not think highly of philosophers¹ but might have benefited from reflecting on Occam's Razor Principle or other elementary epistemological principles while developing the logic in his *Lecture* on identical particles (Feynman's *Lectures*, Vol.III-4).

Indeed, this piece of abstract theory is oft-quoted and referred to as one of the theoretical foundations of quantum mechanics but – seen from the perspective of the realist interpretation that we've been developing based on the *Zitterbewegung* model of an electron² – it might be based on a potentially flawed idea: while all real-life particles have spin (*up* or *down*), the whole theoretical development in this founding chapter is based on the idea of a theoretical zero-spin particle which – as we all know – does not represent anything we can imagine.³ It should, therefore, not come as a surprise that the ensuing arithmetic and explanation for the behavior of bosons and fermions respectively is highly confusing.

In fact, while the mentioned lecture is a lecture for sophomore students and, hence, does not involve any of the advanced concepts that Feynman helped to pioneer after the 2nd World War (perturbation and renormalization theory), the logical flaws in it are, in our humble view, of the same order as those that made Paul Dirac, one of the founding fathers of QM, write the following in 1975:

"I must say that I am very dissatisfied with the situation because this so-called 'good theory' [perturbation and renormalization theory] involves neglecting infinities. [...] This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small – not neglecting it just because it is infinitely great and you do not want it!"

¹ There are many dismissive remarks on philosophers in his *Lectures* but the following one, in a chapter on special relativity theory, is particularly illustrative: "There is a school of philosophers who feel very uncomfortable about the theory of relativity, which asserts that we cannot determine our absolute velocity without looking at something outside, and who would say: "It is obvious that one cannot measure his velocity without looking outside. It is self-evident that it is meaningless to talk about the velocity of a thing without looking outside. The physicists are rather stupid for having thought otherwise, but it has just dawned on them that this is the case. If only we philosophers had realized what the problems were that the physicists had, we could have decided immediately by brainwork that it is impossible to tell how fast one is moving without looking outside, and we could have made an enormous contribution to physics." *These philosophers are always with us, struggling in the periphery to try to tell us something, but they never really understand the subtleties and depths of the problem.*"

² Jean Louis Van Belle, *The Emperor Has No Clothes: A Realist Interpretation of Quantum Mechanics*, 21 April 2019 (<http://vixra.org/abs/1901.0105>)

³ The attentive reader will immediately cry wolf: the *Higgs* particle is supposed to have spin zero, right? However, when Feynman wrote his lectures, there were no spin-zero particles. Also, one should probably refer to the *Higgs field* (rather than to *Higgs particles*). The *Higgs field* is a simple scalar field: each point in spacetime is associated with some (real-numbered) value. As such, one should, effectively, probably not think of this field as a collection of quanta or particles.

This is a harsh judgment – especially as Dirac himself had helped to develop the basic approach⁴ – but it is what it is, and it deserves a lot more reflection than what it is getting. In fact, it is not only Dirac and Einstein, of course, but the whole first generation of quantum physicists (including Schrödinger, Pauli and Heisenberg) who had become skeptical about the theory they had created—and not only because perturbation theory yielded those weird diverging higher-order terms.

Einstein, for example, was not worried about the conclusion that Nature was probabilistic (he fully agreed we cannot know everything): a quick analysis of the full transcriptions of his oft-quoted remarks reveal that he just wanted to see a theory that *explains* the probabilities. A theory that just *describes* them didn't satisfy him. As such, he was in search of a *realist* interpretation as well.⁵

In fact, we should add that even John Stewart Bell – one of the more famous third-generation physicists, we may say – did not like his own No-Go Theorem, and that he hoped that, one day, some “radical conceptual renewal”⁶ might disprove his conclusions. Indeed, we should remember Bell kept exploring alternative theories – including Bohm's *pilot wave* theory, which is a hidden variables theory – until his death at a relatively young age. We believe the day for radical conceptual renewal has come, and we hope this paper will contribute to that.

With the benefit of hindsight, one is tempted to think that Dyson, Schwinger, Feynman – the whole younger generation of mainly American scientists who dominated the *discourse* at the time – were great scientists but lacked a true leader: they kept soldiering on by inventing renormalization and other mathematical techniques to ensure those weird divergences cancel out, but they seemed to have no real direction. Whatever worked, worked for them.

Of course, they all received Nobel Prizes for their ‘discoveries’ and, hence, there is probably a vested interest now in keeping the mystery alive: no physicist in academics will want to hurt his or her career by claiming that the approach which Dyson, Schwinger, Feynman or Tomonaga helped developed might be wrong!⁷ Hence, philosophers, amateur physicists or other independent researchers may well be the only ones who can say aloud what many might be privately thinking: *The Emperor has No Clothes! Or, in regard to some of the content in Feynman's Lectures: Surely You're Joking, Mr. Feynman!*⁸

Let us analyze some of the flawed logic in Feynman's founding texts on quantum mechanics so as to make the point clearly and unambiguously, in line with Boltzmann's maxim:

“Bring forward what is true. Write it so that it is clear. Defend it to your last breath.”

⁴ Dirac's *Principles of Quantum Mechanics* introduces perturbation theory in Chapter VII.

⁵ See, for example, Lee Smolin's latest book: *Einstein's Unfinished Revolution*, April 2019.

⁶ See: John Stewart Bell, *Speakable and unspeakable in quantum mechanics*, pp. 169–172, Cambridge University Press, 1987. J.S. Bell died from a cerebral hemorrhage in 1990 – the year he was nominated for the Nobel Prize in Physics. He was just 62 years old then.

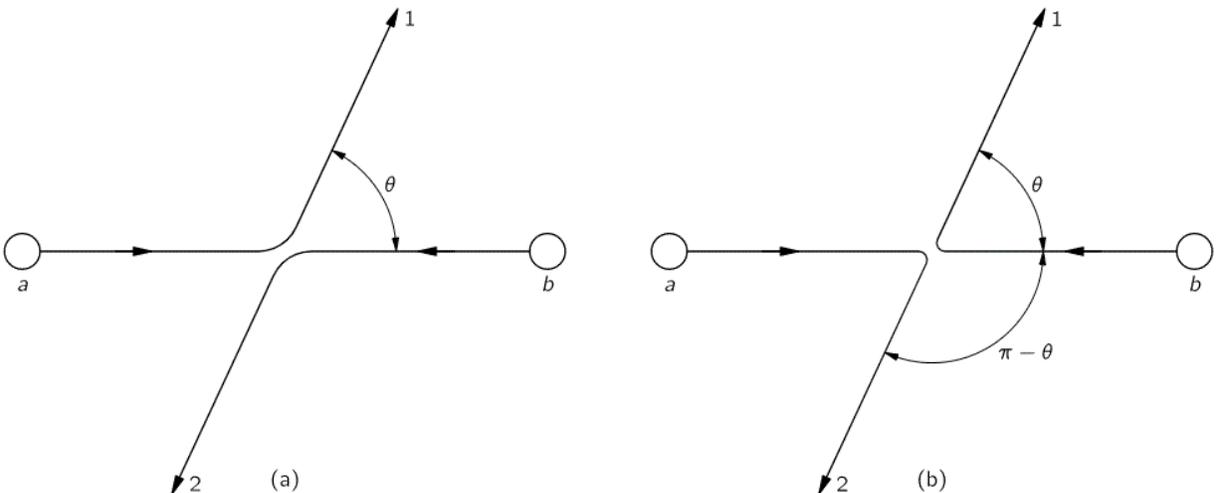
⁷ All of them have died now, except Freeman Dyson, who is 95 years old now! Richard Feynman died in 1988 from cancer. He was still quite young then (69 years). It is said that his last words were: "I'd hate to die twice. It's so boring."

⁸ This refers to the title of a collection of reminiscences edited by Ralph Leighton, who was a close friend and drumming partner of Richard Feynman. Ralph Leighton was also the son of the physicist Robert B. Leighton, who worked closely with Feynman on the *Lectures*. Ralph Leighton also put the *Strange Theory of Light and Matter* together. For a rather critical review of the latter work (which is also widely quoted and referenced), see my blog article on it (<https://readingfeynman.org/2015/01/22/the-strange-theory-of-light-and-matter-iii/>).

Feynman's identical particles

Feynman's lecture on the behavior of bosons and fermions starts with Figure 1, which depicts an elastic collision between so-called identical particles. Feynman claims process (a) and (b) cannot be distinguished, because particle a and b are... Well... Identical particles.⁹

Figure 1: Feynman's identical particles



Feynman's logic is then the following:

“There is an amplitude that *either* a or b goes into counter 1, while the other goes into counter 2. This amplitude is the sum of the amplitudes for the two processes shown in the illustration. If we call the first one $f(\theta)$, then the second one is $e^{i\delta}f(\pi-\theta)$, where now the phase factor is very important because we are going to be adding two amplitudes. Suppose we have to multiply the amplitude by a certain phase factor when we exchange the roles of the two particles. If we exchange them again we should get the same factor again. But we are then back to the first process. The phase factor taken twice must bring us back where we started—its square must be equal to 1. There are only two possibilities: $e^{i\delta}$ is equal to +1, or is equal to -1. Either the exchanged case contributes with the *same* sign, or it contributes with the *opposite* sign. Both cases exist in nature, each for a different class of particles. Particles which interfere with a *positive* sign are called *Bose particles* and those which interfere with a *negative* sign are called *Fermi particles*.”

This logic triggers many questions, but the most obvious one is: how would Nature know whether it should *add* or, in the case of fermions, *subtract* amplitudes?

Now, Feynman would dismiss this as an irrelevant philosophical question, but the larger fallacy is plain logical and cannot be dismissed as plain philosophy: Feynman does not present any *real* particles here. The a and b particles are theoretical zero-spin particles: we are, somehow, supposed to be able to

⁹ The use of the same symbols (a and b) to refer to both the particles as well as the diagram is somewhat confusing, but we can live with that.

imagine them *without* their quintessential spin property to then – some time later in the theoretical development – bring their integral or half-integral spin property back in through the back door.

This is utterly strange because we *know* particles *a* and *b* will be fermions or bosons. In fact, what other property do they have besides their spin – and their *mass* (or rest energy)? In fact, already here, one may wonder how this diagram could possibly depict bosons. Think of photons: these do not usually enter into elastic collisions. Indeed, photon-photon interactions are a relatively new branch of physics and, hence, one should really wonder what Feynman had in mind here. In other words, Feynman was, for all practical purposes, thinking in terms of electrons, protons, neutrons and other simple particles¹⁰ here – and *only* about these. As such, we find the generalization he starts off with logically unacceptable.

Any particle – boson or fermion – will have spin. Hence, we should distinguish not two but eight situations:

1. Particle *a* has spin *up* and goes to 1, while particle *b* has spin *down* and goes to 2.
2. Particle *a* has spin *up* and goes to 2, while particle *b* has spin *down* and goes to 1.
3. Particle *a* has spin *up* and goes to 1, while particle *b* has spin *up* and goes to 2.
4. Particle *a* has spin *up* and goes to 2, while particle *b* has spin *up* and goes to 1.
5. Particle *a* has spin *down* and goes to 1, while particle *b* has spin *down* and goes to 2.
6. Particle *a* has spin *down* and goes to 2, while particle *b* has spin *down* and goes to 2.
7. Particle *a* has spin *down* and goes to 1, while particle *b* has spin *up* and goes to 2.
8. Particle *a* has spin *down* and goes to 2, while particle *b* has spin *up* and goes to 2.

The situation is simple and logically clear: particle *a* can have its spin up or down, and can go into counter 1 or 2. If it goes into counter 1, then particle *b* will go into counter 2, and vice versa. So we have 8 possibilities. Of course, the particles are identical but for their spin and our counters should be able to distinguish between spin *up* or spin *down*. Also, we should know with what spin particle *a* and *b* left before they headed for collision because in experiments like this, the beams are usually prepped so they are polarized. Hence, the situation can be summed by the following table:

Table 1: Initial and final states of the elastic collision experiment

	Spin of <i>b</i> is up	Spin of <i>b</i> is down
Spin of <i>a</i> is up	D1 detects a spin-up particle D2 detects a spin-up particle	D1 detects a spin-up particle D2 detects a spin-down particle
		D1 detects a spin-down particle D2 detects a spin-up particle
Spin of <i>a</i> is down	D1 detects a spin-up particle D2 detects a spin-down particle	D1 detects a spin-down particle D2 detects a spin-down particle
	D1 detects a spin-down particle D2 detects a spin-up particle	

¹⁰ His lecture includes the example of α -particles, which are helium nuclei (particles with two protons and two neutrons). The exchange here is even more complicated because the two nuclei might exchange a nucleon in the collision.

Hence, from an epistemological point of view, the situation that is to be modeled here is quite simple: we have four initial states, and we have four final states. Hence, we need to make sure the mathematical description of the initial and final states is unambiguous. That's kids' stuff, right?

For physicists, it isn't. They start off with ambiguous descriptions and – *surprise!* – end up with equally ambiguous results. The point is rather simple: mainstream physicists will tell you they don't *really* think of the elementary wavefunction as representing anything real but, in fact, they do. Of course! And, if you insist, they will tell you, rather reluctantly because they are not so sure about what is what, that it might represent some theoretical spin-zero particle. Now, as mentioned above, we all know spin-zero particles do not exist. All *real* particles – electrons, photons, anything (all bosons and all fermions) – have spin, and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. It is, therefore, completely odd that the plus (+) or the minus (–) sign of the imaginary unit (i) in the $a \cdot e^{\pm i\theta}$ function is *not* being used to include the spin *direction* in the mathematical description.

Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i\theta} = a \cdot e^{-i(\omega t - kx)}$ and $a \cdot e^{+i\theta} = a \cdot e^{+i(\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). One would expect that the professors would then proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that “the choice is a matter of convention” and, that “happily, most physicists use the same convention.”¹¹

Well... This is where we think the second-generation of quantum physicists (and the first generation too, actually) went wrong. The *Lectures* that follow this one – on the transformation rules for amplitudes – are equally non-sensical. We will show why, how and where exactly. However, before we do so, we want to make a few remarks on why a realist interpretation of quantum mechanics is possible.

A physical interpretation of the wavefunction

The structural similarities between the classical electromagnetic theory and QED inspire easy geometric and physical interpretations of the wavefunction. Here we need to specify what we mean with a physical interpretation because any course in quantum mechanics will state that the interpretation of $|\psi(\mathbf{x}, t)|^2$ as the probability to find a particle at \mathbf{x} and t amounts to a *physical* interpretation.¹² However, this is *not* what we mean. A veritable *physical* interpretation should explain these probabilities in terms of something *real* (mass or energy densities, for example). That is where formal courses leave the student mystified: can or can it not be done?

We think it can be done and, more importantly, we also think our papers show *how*, exactly.¹³ In our *realist* or physical interpretation of the wavefunction, we interpret the real and imaginary part of the elementary wavefunction $a \cdot e^{i\theta}$ as *real* field vectors driven by the same function but with a phase difference of 90 degrees:

¹¹ In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1x). We quote this example for the same reason as why we use Feynman's *Lectures* as a standard reference: it is an authoritative course, and it's available online so the reader can check and explore for himself.

¹² See, for instance, the MIT OCW courses 8.04 and 8.05.

¹³ For the list, see: http://vixra.org/author/jean_louis_van_belle.

$$a \cdot e^{i\theta} = a \cdot (\cos\theta + i \cdot \sin\theta) = a \cdot \sin(\theta + \pi/2) + i \cdot a \cdot \sin\theta$$

This fundamental idea inspired a consistent electron model¹⁴, but the ramifications are more general.¹⁵

What is the nature of the force field? It must be electromagnetic¹⁶ and, hence, we associate the real and imaginary part of the wavefunction with the force per unit charge dimension (*newton per coulomb*).

Of course, the model needs to be confronted with the basic axioms of quantum mechanics, and in particular these two:

1. The superposition of wavefunctions is done in the complex space and, hence, the assumption of a real-valued envelope for the wavefunction is, therefore, not acceptable.
2. The wavefunction for spin-1/2 particles cannot represent any real object because of its 720-degree symmetry in space. Real objects have the same spatial symmetry as space itself, which is 360 degrees. Hence, physical interpretations of the wavefunction are nonsensical.

Let us tackle these objections head-on.

Real or complex amplitudes?

The term amplitude is ambiguous: it may refer to the maximum amplitude of some real-valued wave or, alternatively, to a complex-valued *probability* amplitude. In the first case, we think of the a in the $a \cdot e^{i\theta}$ expression and, hence, it is a coefficient, a scaling factor (think of normalization) or – when building the wave packet – a *weight*. In the second, the term amplitude refers to the whole $a \cdot e^{i\theta}$ function. We are obviously talking about the coefficient here: we have no doubt we need complex-valued functions to describe real-life particles.¹⁷

The point is: in any geometric and/or physical interpretation of the wavefunction we think of a as some real-valued number. Any introductory course on quantum physics will point out that this is nonsensical because, in quantum mechanics, we do linear operations using complex-valued coefficients. For example, when using the framework of *state vectors*, we write something like $|X\rangle = \alpha|A\rangle + \beta|B\rangle$, and α and β would be complex numbers. We also know that, if ψ_1 and ψ_2 are solutions to the Schrödinger equation, then $\alpha\psi_1 + \beta\psi_2$ will be a solution too—and, once again, α and β can be complex numbers. Of course, we can always multiply with $1/\alpha$ and then we get $|A\rangle + \frac{\beta}{\alpha}|B\rangle$ or $\psi_1 + \frac{\beta}{\alpha}\psi_2$ to get *one* complex parameter only: the β/α ratio, which – when thinking about the *degrees of freedom* of the system we are trying to describe – is equivalent to two *real-valued* parameters.¹⁸

¹⁴ See: Jean Louis Van Belle, *The anomalous magnetic moment: classical calculations*, 5 June 2019

(<http://vixra.org/abs/1906.0007>)

¹⁵ See: Jean Louis Van Belle, *The Emperor Has No Clothes: A Realist Interpretation of Quantum Mechanics*

(<http://vixra.org/pdf/1901.0105vG.pdf>)

¹⁶ As we are looking at the QED sector of the Standard Model, the electromagnetic force is the only candidate: the force grabs onto a *charge* here. There's nothing else to grab onto.

¹⁷ We must qualify this statement. We can, of course, write any wavefunction as a set of *two* real-valued functions—one for the real and one for the imaginary part. For example, we can write Schrödinger's equation as a set of two equations. But that is, obviously, *not* the point that we are trying to make here.

¹⁸ See: Prof. Dr. Barton Zwiebach, *Quantum Mechanics*, MITx 8.01.1x, Chapter 1, Section 4. A complex number $x + iy$ effectively consists of two parts (x and y) and can therefore reflect the (two) degrees of freedom of the physics of the situation. The

However, the *mathematical* objection remains the same: one should not think of a real-valued *envelope* for the wavefunction because we can always multiply a state by some *complex* number (α) and we'll get the same state: $\psi \cong \alpha\psi$. Here again, the obstinate refusal to think of the wavefunction as representing something *real*, obscures the obvious answer, which consists of two pieces:

1. A complex number $z = a + i \cdot b$ can always be re-written in terms of a real-valued magnitude r and a real-valued phase Δ : $a + i \cdot b = r \cdot e^{i\Delta}$.
2. A physical state will always be described in terms of *base states*. Hence, the multiplication by r and by $e^{i\Delta}$ of a state implies all base states should be multiplied by r and by $e^{i\Delta}$.

In a physical interpretation of the wavefunction¹⁹, multiplication by r amounts to a re-definition of the distance unit, while a multiplication by $e^{i\Delta}$ is just a (common) phase shift—i.e. a re-definition of the zero point in space, in time, or in both.²⁰

In short, the reasoning that a superposition of wavefunctions is done in the complex space and, hence, that the assumption of a real-valued envelope for the wavefunction is, therefore, not acceptable is fundamentally flawed: the former is true (wavefunctions should be superposed using complex coefficients), but the conclusion is *not*: the wavefunction always has a real-valued envelope.

It is rather weird that few – if any – physicists seem to have thought about this because, in practice, we always end up with wavefunctions with real-valued coefficients. Let us give two notable examples here—the solutions to the Schrödinger equation in a potential (the model of the hydrogen atom), and the standard representation of the wavefunction as a Fourier sum:

1. The correct description of the electron orbitals of the hydrogen atom is one of the main feats of quantum mechanics, and these descriptions are wavefunctions with real-valued coefficients. Of course, the wavefunction for an electron orbital will routinely include a factor like $-\frac{1}{\sqrt{2}} \cdot \sin\theta \cdot e^{i\phi}$ so, yes, there is a complex number there²¹, but note how the complex factor appears: it is just a phase shift. The *envelope* for the oscillation is some real number.

2. This is also the case for the description of the wave packet in terms of a Fourier sum. We *can* use complex-valued coefficients but, *in practice*, we use real-valued coefficients. Let us also be explicit here so we are all clear on this. The description of a wave packet in space (freezing time) is given by:

$$\psi(x, 0) = \int_{-\infty}^{+\infty} \Phi(k) e^{ikx} dk$$

example that is given is that of an elliptically polarized wave, whose shape is determined by the ratio of the axes of the ellipse (b/a) and its tilt (θ).

¹⁹ See: *In Search of Schrödinger's electron* (<http://vixra.org/abs/1809.0350>, accessed on 30 October 2018).

²⁰ The absolute character of the speed of light implies a re-scaling of the distance unit should also imply a re-scaling of the time unit. This merits a deeper reflection.

²¹ The formula gives us the angular dependence of the amplitude for the orbital angular momentum number $l = 1$.

The $\Phi(k)$ function gives us the *weight* factors for each of the waves that make up the packet²² and we will want to think of $\Phi(k)$ as a real-valued function, centered around some value $k_0 = \frac{p_0}{\hbar}$ and width Δk .²³

Of course, the argument above is heuristic only: it is *not* a formal proof that we can always find a suitable base to ensure real-valued coefficients. However, it should debunk the myth that the coefficients in front of wavefunctions are generally complex and that, therefore, we should not try to find a physical or geometric interpretation of the wavefunction.

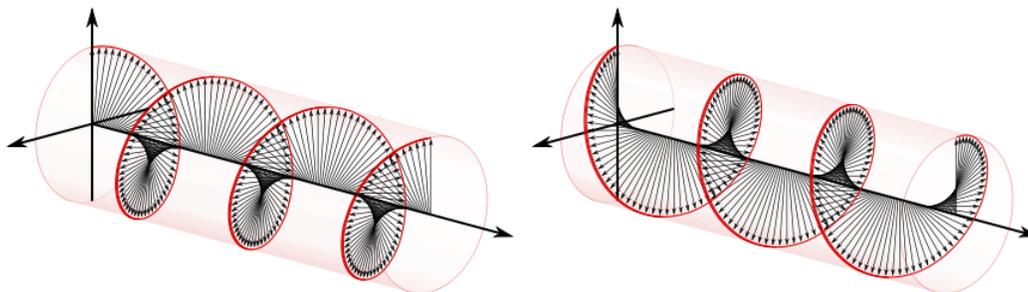
Let us now analyze the second casual objection to such interpretations which, in our view, is much more substantive: the theoretical 720-degree symmetry of the wavefunction for spin-1/2 particles. In the next section, we will show these weird symmetries come out of the same flawed thinking: spin-zero particles don't exist and, hence, one should include the idea of spin in the analysis from the outset.

Theoretical spin-zero particles versus real spin-1/2 particles

It is interesting that, using suitable conventions, we can rewrite Maxwell's equations using complex numbers. Indeed, if we think of the imaginary unit as a unit vector pointing in a direction that is perpendicular to the direction of propagation of the wave, we can write the magnetic field vector as $\mathbf{B} = -i\mathbf{E}/c$.

Note the minus sign in the $\mathbf{B} = -i\mathbf{E}/c$.²⁴ It is there because we need to combine several conventions here. Of course, there is the classical *physical* right-hand rule for \mathbf{E} and \mathbf{B} , but we also need to combine the right-hand rule for the coordinate system with the convention that multiplication with the imaginary unit amounts to a *counterclockwise* rotation by 90 degrees. Hence, the minus sign is necessary for the consistency of the description. It ensures that we can associate the $a \cdot e^{i\theta}$ and $a \cdot e^{-i\theta}$ functions with left- and right-handed polarization respectively.

Figure 2: Left- and right-handed polarization²⁵



²² One will usually see a $\frac{1}{\sqrt{2\pi}}$ factor in front of the integral, and it should be there, but we left it out for clarity.

²³ The <http://www.thefouriertransform.com/series/complexcoefficients.php> site gives examples of Fourier transforms of common functions using complex-valued coefficients, but shows that the same results can be obtained by using real-valued coefficients.

²⁴ Boldface letters represent geometric vectors – the electric and magnetic field vectors \mathbf{E} and \mathbf{B} in this case.

²⁵ Credit: <https://commons.wikimedia.org/wiki/User:Dave3457>.

It is, therefore, very peculiar that, in quantum mechanics, we do not have such consistency. For example, in the MIT's introductory course on quantum physics²⁶, it is shown that only $\psi = \exp(i\theta) = \exp[j(kx - \omega t)]$ or $\psi = \exp(-i\theta) = \exp[-j(kx - \omega t)] = \exp[j(\omega t - kx)]$ would be acceptable waveforms for a particle that is propagating in the x -direction – as opposed to, say, some real-valued sinusoid. We would then think some proof should follow of why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professor happily concludes that “*the choice is a matter of convention and, happily, most physicists use the same convention.*”

This is very surprising – and that's an understatement. Why? We *know*, from *experience*, that theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things. Think of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? *Occam's Razor* tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations.

What could be different? There is only one candidate here: spin.

This brings us to what is – without any doubt – the most challenging objection to a physical interpretation of the wavefunction: wavefunctions of spin-1/2 particles (which is what we are thinking of here) have a weird 720° symmetry.²⁷ Any *real* object that we can think of has a 360-degree symmetry in space. Why? Because space is three-dimensional.

We can try to solve this contradiction in two ways. The first way is to accept the 720° symmetry and try to interpret it by accepting the measurement apparatus and the object establish some *absolute* space. The metaphor here is Dirac's belt trick. We have written about this before and, hence, we will not repeat ourselves here.²⁸

The second way – much more radical – is to prove that the 720-degree symmetry would reduce to what we would expect for anything real in space – i.e. a 360-degree symmetry – when we would, effectively, use the two mentioned mathematical possibilities to distinguish between two particles that are identical but for their spin. The idea is that we would associate the $a \cdot e^{i\theta}$ and $a \cdot e^{-i\theta}$ functions with the quantum-mechanical equivalent of left- and right-handed polarization respectively. The wavefunction would then no longer describe a theoretical spin-zero particle, which should be fine – because we all know spin-zero particles don't exist: *real* particles (electrons and quarks) have spin-1/2.

In the next section, we will show this solves our problem.²⁹ Before we get going on this, we should note that the $a \cdot e^{i\theta}$ and $a \cdot e^{-i\theta}$ functions are each other's complex conjugate and we will, therefore, offer some reflections on the physical meaning of the complex conjugate.

²⁶ See, for example, the MIT's edX Course 8.04.1x, Lecture Notes, Chapter 4, Section 3.

²⁷ See, for example, Feynman's *Lectures*, Vol. III, Chapter 6.

²⁸ See: *Why it is hard to understand – and, therefore, explain – quantum math* (<http://vixra.org/pdf/1806.0183v1.pdf>, accessed on 21 October 2018).

²⁹ Our proof is heuristic, and we will develop it using standard reference material here, i.e. Feynman's *Lectures*, Vol. III, Chapter 6 (*Spin One-Half*).

The reality of the complex conjugate of a wavefunction

The idea of associating the complex conjugate of a wavefunction with a particle that is identical but for its (opposite) spin might be outlandish so, let us first explore a simpler idea. When we take the complex conjugate of $\psi = \exp(i\theta) = \exp[i(\mathbf{k}\cdot\mathbf{x}-\omega\cdot t)]$, we get $\psi^* = \exp(-i\theta) = \exp[i(-\mathbf{k}\cdot\mathbf{x}+\omega\cdot t)]$. Hence, \mathbf{x} becomes $-\mathbf{x}$ and t becomes $-t$. Hence, we may say that the complex conjugate of a wavefunction describes whose trajectory in space and in time is being reversed.

It is not merely time symmetry: we are talking *reversibility* here. It is like playing a movie backwards. We may relate this discussion to the *Hermiticity* of (many) quantum-mechanical operators. An operator A that is operating on some state $|\psi\rangle$ will be written as³⁰:

$$A|\psi\rangle$$

Now, we can then think of some (probability) amplitude that this operation produces some other state $|\varphi\rangle$, which we would write as:

$$\langle\varphi|A|\psi\rangle$$

We can now take the complex conjugate:

$$\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle$$

A^\dagger is, of course, the conjugate transpose of A – we write: $A^\dagger_{ij}=(A_{ji})^*$ – and we will call the operator (and the matrix) Hermitian if the conjugate transpose of this operator (or the matrix) gives us the same operator matrix, so that is if $A^\dagger = A$. Many quantum-mechanical operators are Hermitian. Because of the reversibility condition. Think of the meaning of the complex conjugate as presented above: a reversal of both the direction in time as well in space. Hence, what is the meaning of the complex conjugate of $\langle\varphi|A|\psi\rangle$?

The $\langle\varphi|A|\psi\rangle$ expression gives us the amplitude to go from some state $|\psi\rangle$ to some other state $\langle\varphi|$. Conversely, the $\langle\psi|A|\varphi\rangle = \langle\psi|A^\dagger|\varphi\rangle = \langle\varphi|A|\psi\rangle^*$ expression tells us we were in state $|\varphi\rangle$ but now we are in the state $\langle\psi|$, and the $\langle\psi|A|\varphi\rangle$ expression gives us the amplitude for that. Hence, the Hermiticity condition amounts to a reversibility condition.

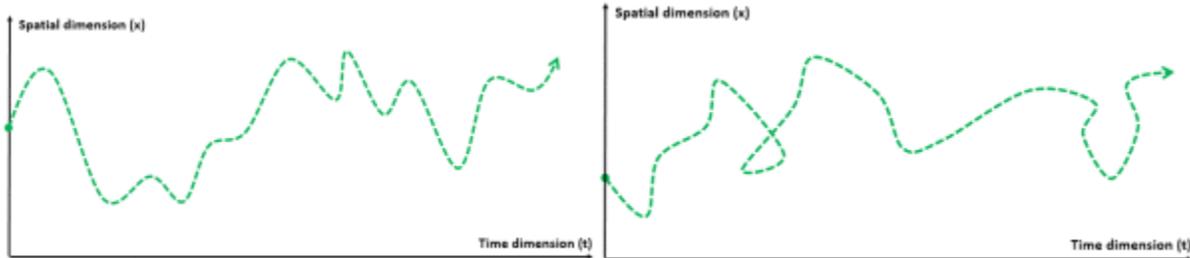
Here we need to highlight a subtle point. Time has one direction only: we cannot reverse time. We can only reverse the direction in space. We can do so by reversing the momentum of a particle. If we do so, our $\mathbf{k} = \mathbf{p}/\hbar$ becomes $-\mathbf{k} = -\mathbf{p}/\hbar$. However, the energy remains what it is and, hence, nothing happens to the $\omega\cdot t = (E/\hbar)\cdot t$ term. Hence, our wavefunction becomes $\exp[i(-\mathbf{k}\cdot\mathbf{x}-\omega\cdot t)]$, and we can calculate the wave velocity as negative: $v = -\omega/|\mathbf{k}| = -\omega/k$. The wave effectively travels in the opposite direction (i.e. the *negative* x -direction in one-dimensional space). Hence, we can think of opposite directions in space, but we can't reverse time. Why not?

We don't need to think of entropy here. Time has one direction only because – if it wouldn't – we would not be able to describe trajectories in spacetime by a well-behaved function. The diagrams below illustrate the point. The spacetime trajectory in the diagram on the right is not *kosher*, because our

³⁰ We should use the *hat* because the symbol without the hat is reserved for the *matrix* that does the operation and, therefore, A already assumes a representation, i.e. some chosen set of base states. However, let's skip the niceties here.

object travels back in time in not less than three sections of the graph. Spacetime trajectories need to be described by well-defined function: for every value of t , we should have one, and only one, value of x . The reverse is not true, of course: a particle can travel back to where it was. Hence, it is easy to see that our concept of time going in one direction, and in one direction only, implies that we should only allow well-behaved functions.

Figure 3: A well- and a not-well behaved trajectory in spacetime



It may be a self-evident point to make but it is an important one. Note that, once again, we have two *mathematical* possibilities to describe a theoretical spin-zero particle that would travel in the *negative x*-direction³¹: $\psi = \exp[i(-kx-\omega t)]$ or $\psi = \exp[i(kx+\omega t)]$.

Again, if we would *not* agree with the mainstream view that “the choice is a matter of convention” and that “happily, most physicists use the same convention”³² but, instead, dare to suggest that the two mathematical possibilities represent identical particles with opposite spin (i.e. *real* spin-1/2 particles as opposed to non-existing spin-zero particles), then we get the following table.

Figure 4: Occam’s Razor: mathematical possibilities versus physical realities

Spin and direction of travel	Spin up ($J = +\hbar/2$)	Spin down ($J = -\hbar/2$)
Positive x-direction	$\psi = \exp[i(kx-\omega t)]$	$\psi^* = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$
Negative x-direction	$\chi = \exp[-i(kx+\omega t)] = \exp[i(\omega t-kx)]$	$\chi^* = \exp[i(kx+\omega t)]$

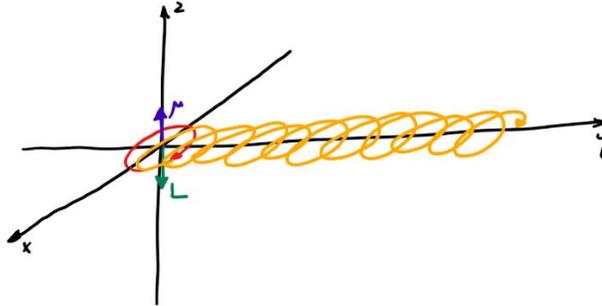
Of course, the above formulas only give us the *elementary* wavefunction. The wave *packet* will be a Fourier sum of such functions. Before we proceed, we should ask ourselves one more question: what is the physical meaning of $-\exp(i\theta)$?

Here we do need to think more carefully about the orientation of the plane of the oscillation. The illustrations of RHC and LHC waves above assume that plane is perpendicular to the direction of propagation, but there are other possibilities. In fact, a physical interpretation of the magnetic moment that we associate with the angular momentum or spin would require that plane to contain the direction of propagation, as illustrated below.

³¹ We are not just switching back and forth between one- and three-dimensional wavefunctions here: think of *choosing* the reference frame such that the x -axis coincides with the direction of propagation of the wave.

³² See, for example, the MIT’s edX Course 8.04.1x, Lecture Notes, Chapter 4, Section 3.

Figure 5: Is this the *Zitterbewegung* of an electron in a Stern-Gerlach apparatus?



If this sounds outlandish to the reader, then he or she may want to think of the remarkably simple result we get when calculating the angular momentum using the Compton wavelength for the radius a ³³:

$$L = I \cdot \omega = \frac{m \cdot a^2 c}{2} \frac{1}{a} = \frac{mc \hbar}{2 mc} = \frac{\hbar}{2}$$

A minus sign in front of our $\exp(i\theta)$ function reverses the direction of the oscillation. However, here we can use the $\cos\theta = \cos(-\theta)$ and $\sin\theta = -\sin(-\theta)$ formulas to relate $-\exp(i\theta)$ to the complex conjugate. We write:

$$-\psi = -\exp(i\theta) = -(\cos\theta + i \cdot \sin\theta) = \cos(-\theta) + i \cdot \sin(-\theta) = \exp(-i\theta) = \psi^*$$

This is a peculiar property that we will exploit in the next development. We should make one final note before we get into the meat of the matter. Where would this minus sign come from? We know we can always add an arbitrary phase change doesn't change the physical state: it is just like changing our zero point in time. Hence, $\exp(i\theta)$ and $\exp(i\alpha) \cdot \exp(i\theta) = \exp[i(\theta + \alpha)]$ should represent the same state. Our physical interpretation of the wavefunction does not challenge this at all. However, we should note the case of $\alpha = \pm\pi$, for which we can write:

$$\exp(\pm i\pi) \cdot \exp(i\theta) = \exp[i(\theta \pm \pi)] = -\exp(i\theta)$$

We will need this identity soon.

360° and 720° symmetries

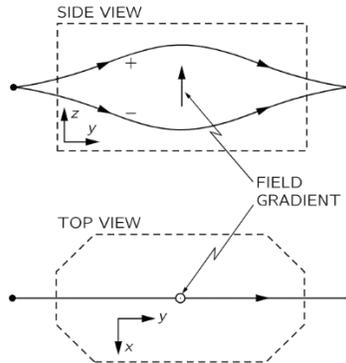
We are all familiar with the topic on hand: the angular dependence of amplitudes. To put it simply, it is about rotation matrices. The matter is best illustrated by sticking closely to Feynman's argument and so let us start with the two illustrations presenting the basic geometry of the situation on hand.

The first illustration shows the rather special Feynman-Stern-Gerlach apparatus (or the modified or *improved* Stern-Gerlach apparatus, as Feynman calls it): the apparatus splits a beam of electrons or

³³ The $\omega = c/a$ formula follows naturally from the same model (see: *In Search of Schrödinger's electron – and Einstein's atom*, <http://vixra.org/abs/1809.0350>, accessed on 21 October 2018). It is equally simple and intuitive as all the rest above, but we don't want to repeat ourselves repeatedly.

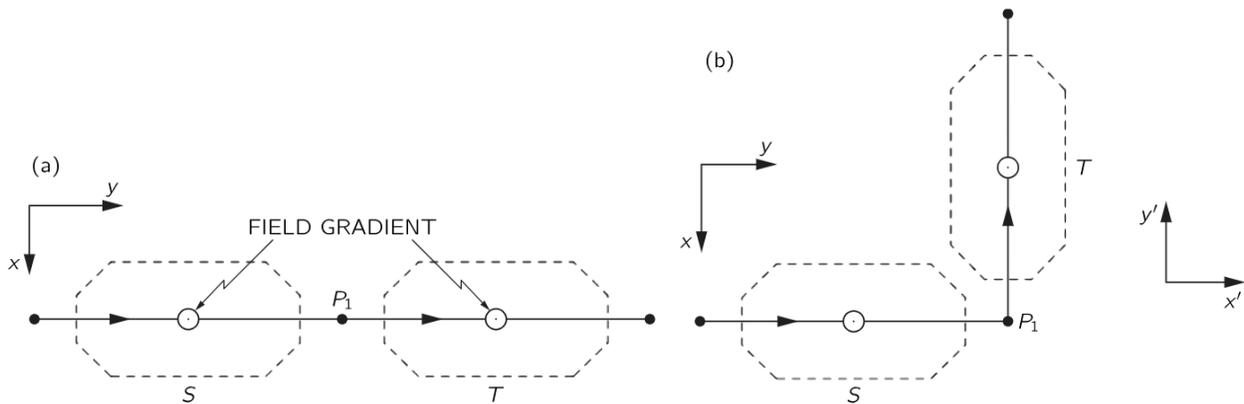
whatever spin-1/2 particles into two and then brings them together again. We can also block one of the two channels to select spin-up or spin-down particles. The y -direction is the direction of propagation and the z -direction is the direction along which we are measuring the magnetic momentum (or, what amounts to the same, the particle's angular momentum or *spin*). The field gradient is, obviously, the direction of the inhomogeneous magnetic field that causes our spin-1/2 particles to separate according to their magnetic moment (or spin), which is either *up* or *down*. Nothing in-between.

Figure 6: Feynman's modified (or improved) Stern-Gerlach apparatus



The objective is to find rotation matrices: we want to know how the wavefunction changes if we rotate it along the z -axis (the analysis for rotations along the other axes comes later). So that is what's shown below. On the left-hand side, our particles go through two apparatuses who are perfectly aligned (the rotation angle is zero). In the right-hand side, we have a rotation angle of 90 degrees ($\pi/2$).

Figure 7: Successive modified Stern-Gerlach apparatuses



The amplitudes for the *up* and *down* state – as our particle enters the second apparatus – may or may not be the same. We know we can no longer define them in terms of the base states that came with the first apparatus (S): being *up* or *down* with respect to S is not the same thing as being *up* or *down* with respect to T . We can write, more generally, something like this:

$$C'_j = \sum_i R_{ji}^{TS} C_i$$

Of course, we know the *probabilities* to be *up* or *down* are going to be the same, so we should probably *not* write something like $C'_{up} = C_{up}$ and $C'_{down} = C_{down}$ but writing something like $|C'_{up}| = |C_{up}|$ and $|C'_{down}| = |C_{down}|$ is plausible. So, the amplitudes differ by a phase factor only. Feynman writes:

$$C'_{up} = e^{i\lambda}C_{up} \text{ and } C'_{down} = e^{i\mu}C_{down}$$

Again, using the rule that we can always shift the phase of the amplitudes with some arbitrary number, we find that μ must be equal to $-\lambda$, so the equations become:

$$C'_{up} = e^{i\lambda}C_{up} \text{ and } C'_{down} = e^{-i\lambda}C_{down}$$

In the special case where the rotation angle is zero (so that's the left-hand diagram), we have that $\lambda = 0$. Same representation, same amplitudes. Simple. But, of course, we want to see what λ and $-\lambda$ are going to be when the rotation angle – which we'll denote by ϕ – is *not* equal to zero. Feynman starts by making a reasonable assumption: λ and ϕ are probably proportional, so let's try to see where we get by writing:

$$\lambda = m\phi$$

Of course, when we rotate the thing by 360 degrees ($\phi = 2\pi$), we are back where we were, so we write:

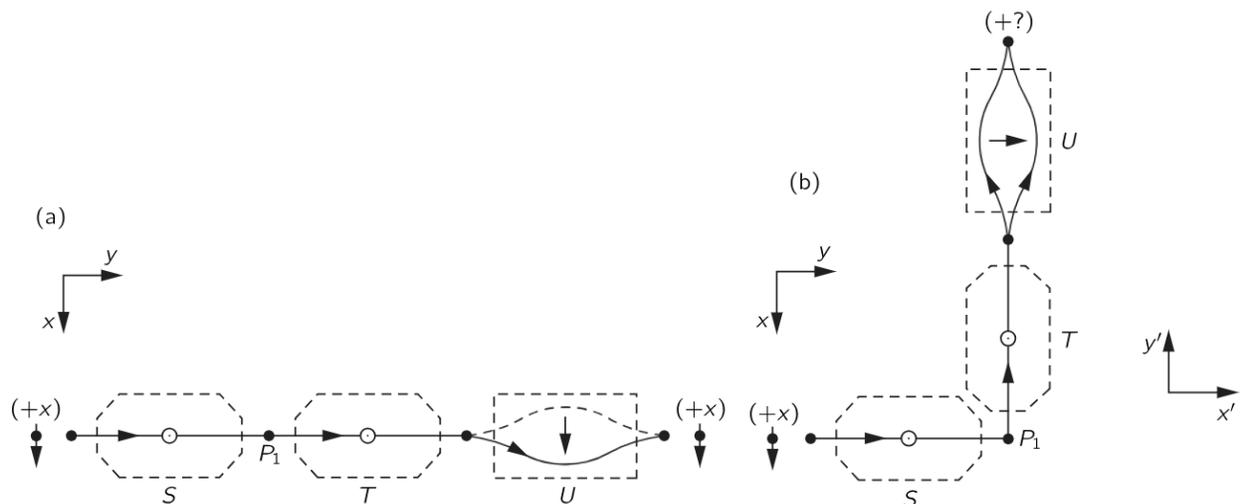
$$C'_{up} = e^{i\lambda}C_{up} = e^{im\phi}C_{up} = e^{im2\pi}C_{up} = C_{up}$$

$$C'_{down} = e^{-i\lambda}C_{down} = e^{-im\phi}C_{down} = e^{-im2\pi}C_{down} = C_{down}$$

For these two equalities to hold, m must be 1, right? So, we do have a 360-degree symmetry rather than this weird 720-degree symmetry, right?

Well... No. Not according to Feynman. He constructs a terribly complicated – and, in my view, potentially flawed – argument designed to sort of prove that the symmetry must be a 720-degree symmetry or, what amounts to the same, to prove that $m = \frac{1}{2}$. The argument is based on a thought experiment that imagines a third apparatus U , as shown below.

Figure 8: Feynman's series of modified Stern-Gerlach apparatuses



The argument goes as follows: we have some filter in front of the S apparatus that produces a pure $+x$ state. In other words, our particle (think of an electron) is up but, importantly, it's up along the x -direction. This orientation has nothing to do with the S and T representations, because these apparatuses measure spin along the z -direction. However, the U apparatus does measure spin along the x -direction and, hence, Feynman expects the particle to sail through but use one channel only, as depicted above. The result is that we still have a particle coming out with its spin up in the x -direction ($+x$).

What happens in the second set-up? We have the same electrons – with up spin along the x -direction – going through and coming out of apparatus S , but then they take a turn, so its wavefunction (that's what an amplitude is) must change. And then the particle goes through T and U , which analyze spin along the y -direction with respect to S . So far, so good. So, what can we say about the state of our electron when it comes out of U in the set-up on the right-hand side.

Well... Let us assume that the argument above is correct and that m is equal to 1. Let us now also consider a set-up for which the T and U apparatuses are rotated over a 180-degree angle (π). Hence, we sort of fold T onto S , so to speak. So, our rotation makes the particle go back in the direction where it came from – through the T and U apparatus. Now, if $m = 1$, then we get:

$$C'_{up} = e^{i\lambda}C_{up} = e^{i\pi}C_{up} = -C_{up}$$

$$C'_{down} = e^{-i\lambda}C_{down} = e^{-i\pi}C_{down} = -C_{down}$$

According to Feynman, this result cannot be possible. Let us quote him here:

“This result ($C'_{up} = -C_{up}$ and $C'_{down} = -C_{down}$) is just the original state all over again. Both amplitudes are just multiplied by -1 which gives back the original physical system. (It is again a case of a common phase change.) This means that if the angle between T and S in (b) is increased to 180° , the system (with respect to T) would be indistinguishable from the zero-degree situation, and the particles would again go through the (+) state of the U apparatus. At 180° , though, the (+) state of the U apparatus is the ($-x$) state of the original S apparatus. So a ($+x$) state would become a ($-x$) state. But we have done nothing to change the original state; the answer is wrong. We cannot have $m = 1$.”³⁴

This is where our physical interpretation – which, rather than making an arbitrary choice, maps all *mathematical* possibilities to all possible *physical* situations – differs from the mainstream interpretation. The $C'_{up} = -C_{up}$ and $C'_{down} = -C_{down}$ do represent two different realities – two different physical *states*, that is. Putting a minus sign in front of the wavefunction amounts to taking its complex conjugate. Hence, it effectively *does* reverse the spin direction.

Of course, the attentive reader will immediately cry wolf. We *do* have a common phase change here, don't we? Therefore, Feynman must be right and the $C'_{up} = -C_{up}$ and $C'_{down} = -C_{down}$ amplitudes *must* represent the same states. The answer is: no. There is no *common* phase change here. The phase change is $+\pi$ for the up state and $-\pi$ for the $down$ state.

Q.E.D. Quantum electrodynamics. Or: quod eram demonstrandum.

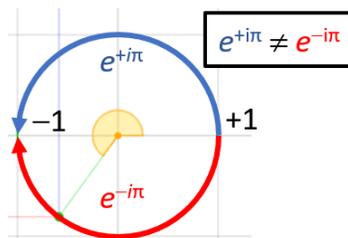
³⁴ Feynman *Lectures*, Vol. III, Chapter 6, Section 3.

Intermezzo: Occam's Razor Principle

Thomas Aquinas starts his *de Ente et Essentia* (on Being and Essence) quoting Aristotle: *quia parvus error in principio magnus est in fine*. A small error in the beginning can lead to great errors in the conclusions. This philosophical warning – combined with Occam's quest for mathematical parsimony – made us think about the mathematical framework of quantum mechanics: its rules explain reality, but no one understands them. Perhaps some small mistake has been made – early on – in the *interpretation* of the math. This has been a long quest – with little support along the way (see the acknowledgments above) – but we think we have found the small mistake – and we do believe it has led to some substantial misunderstandings – or, at the very least, serious *ambiguities* in the description.

We think that the power of Euler's function – as a mathematical description of what we believe to be a real particle – has not been fully exploited. We, therefore, have a redundancy in the description. The fallacy is illustrated below. When we combine -1 with an amplitude, we should not think of it as a scalar: we should think of -1 as a complex number itself. Hence, when we are multiplying a set of amplitudes – let's say *two* amplitudes, to focus our mind (think of a beam splitter or alternative paths here) – with -1 , we are *not* necessarily multiplying them with the same thing: -1 is *not* necessarily a common phase factor. The phase factor may be $+\pi$ or, alternatively, $-\pi$. To put it simply, when going from $+1$ to -1 , it matters how you get there – and vice versa.

Figure 9: $e^{+i\pi} \neq e^{-i\pi}$



Let us, to conclude this paper, elaborate two other implications. The first explains why taking the absolute square of some amplitude will give us a probability. The second is on a physical interpretation of the property of Hermiticity.

Interpreting state vectors and absolute squares

How should we interpret the product of the elementary function with its complex conjugate? In orthodox quantum mechanics, it is just this weird thing: some number that will be *proportional* to some *probability*. In our interpretation, this probability is proportional to energy densities – or, because of the energy-mass equivalence – to mass densities. Let us take the simplest of cases and think of the $\langle \psi |$ state as some very *generic* thing being represented by a *generic* complex function³⁵:

³⁵ Our critics will cry wolf and say we should be more general. They are right. However, let us make two remarks here. First, we should note that QED is a linear theory and, hence, we can effectively – and very easily – generalize anything we write to a Fourier superposition of waves. We use the \cong symbol to indicate an *equivalence*. It's not an identity. To mathematical purists – who will continue to cry wolf no matter what we write because they won't accept the $e^{-\pi} \neq e^{-\pi}$ expression either – we will admit it is more like a symbol showing *congruence*. Second, we do get some physical laws out of physics (both classical as well as quantum-mechanical) that are likely to justify the general $a \cdot e^{\theta}$ shape.

$$\langle \psi | \equiv a \cdot e^{i\theta}$$

The $\langle \psi | \langle \psi |^* = \langle \psi | | \psi \rangle$ product then just eliminates the *oscillation*. It freezes time, we might say:

$$\langle \psi | \langle \psi |^* = \langle \psi | | \psi \rangle = a \cdot e^{i\theta} \cdot a \cdot e^{-i\theta} = a^2 \cdot e^0 = a^2$$

Hence, we end up with one factor of the energy of an oscillation: its amplitude (a). Let us think about this for a brief moment. To focus our minds, let us think of a photon. The energy of any oscillation will always be proportional to (1) its amplitude (a) and (2) its frequency (f). Hence, if we write the proportionality coefficient as k , then the energy of our photon will be equal to:

$$E = k \cdot a^2 \cdot \omega^2$$

What should we use for the amplitude of the oscillation here? It turns out we get a nice result using the *wavelength*³⁶:

$$E = k a^2 \omega^2 = k \lambda^2 \frac{E^2}{h^2} = k \frac{h^2 c^2 E^2}{E^2 h^2} = k c^2 \Leftrightarrow k = m \text{ and } E = mc^2$$

However, we should immediately note that – in our interpretation(s) of the wavefunction – this assumes a circularly polarized wave. Its linear components – the sine and cosine, that is – will only pack *half* of that energy. Our electron model – *zbw* electron as well an orbital electron – is based on the same.

So, yes, now that we are here, let us quickly recap the formulas we found:

Table 2: Intrinsic spin versus orbital angular momentum

Spin-only electron (<i>Zitterbewegung</i>)	Orbital electron (Bohr orbitals)
$S = h$	$S_n = nh$ for $n = 1, 2, \dots$
$E = mc^2$	$E_n = -\frac{1}{2} \frac{\alpha^2}{n^2} mc^2 = -\frac{1}{n^2} E_R$
$r = r_C = \frac{\hbar}{mc}$	$r_n = n^2 r_B = \frac{n^2 r_C}{\alpha} = \frac{n^2 \hbar}{\alpha mc}$
$v = c$	$v_n = \frac{1}{n} \alpha c$
$\omega = \frac{v}{r} = c \cdot \frac{mc}{\hbar} = \frac{E}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3 \hbar} mc^2 = \frac{1}{n^2} \frac{\alpha^2 mc^2}{n \hbar}$
$L = I \cdot \omega = \frac{\hbar}{2}$	$L_n = I \cdot \omega_n = n \hbar$
$\mu = I \cdot \pi r_C^2 = \frac{q_e}{2m} \hbar$	$\mu_n = I \cdot \pi r_n^2 = \frac{q_e}{2m} n \hbar$
$g = \frac{2m \mu}{q_e L} = 2$	$g_n = \frac{2m \mu}{q_e L} = 1$

³⁶ We use the $E\lambda = hc \Leftrightarrow \lambda = hc/E$ identity. The reader might think we should use the amplitude of the electric and magnetic field. We could – the model is consistent – but it requires some extra calculations as we then need to think of the energy as some force over a distance. We refer to our papers for more details.

We also developed a photon model, based on which we think we can explain Mach-Zehnder interference in an equally *realist* fashion.³⁷

What's Hermiticity?

To conclude this paper, we offer a discussion on the physical meaning of the *Hermiticity* of (many) quantum-mechanical operators. If A is an operator³⁸, then it could operate on some state $|\psi\rangle$. We write this operation as:

$$A|\psi\rangle$$

Now, we can then think of some (probability) amplitude that this operation produces some other state $|\varphi\rangle$, which we would write as:

$$\langle\varphi|A|\psi\rangle$$

We can now take the complex conjugate:

$$\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle$$

A^\dagger is, of course, the conjugate transpose of A : $A^\dagger_{ij}=(A_{ji})^*$, and we will call the operator (and the matrix) Hermitian if the conjugate transpose of this operator (or the matrix) gives us the same operator matrix, so that is if $A^\dagger = A$. Many operators are Hermitian. Why? Well... What is the meaning of $\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle = \langle\psi|A|\varphi\rangle$? Well... In the $\langle\varphi|A|\psi\rangle$ we go from some state $|\psi\rangle$ to some other state $\langle\varphi|$. Conversely, the $\langle\psi|A|\varphi\rangle$ expression tells us we were in state $|\varphi\rangle$ but now we are in the state $\langle\psi|$.

So, is there some meaning to the complex conjugate of an amplitude like $\langle\varphi|A|\psi\rangle$? We say: yes, there is! Read up on time reversal and CPT symmetry! Based on the above – and your reading-up on CPT symmetry – we would think it is fair to say we should interpret the Hermiticity condition as a *physical* reversibility condition.

We are not talking mere time symmetry here: reversing a physical process is like playing a movie backwards and, hence, we are actually talking CPT symmetry here.

Conclusion

This paper offers a philosophical-epistemological basis for the realist interpretation of quantum mechanics on the basis of the *Zitterbewegung* model of an electron. We hope it becomes the *manifesto* for a scientific revolution—for the kind of ‘radical conceptual renewal’ that John Stewart Bell and so many other pioneering minds were hoping to see.

Jean Louis Van Belle, 7 June 2019

³⁷ See Chapter XV of our manuscript (<http://vixra.org/pdf/1901.0105vG.pdf>).

³⁸ We should use the *hat* because the symbol without the hat is reserved for the *matrix* that does the operation and, therefore, already assumes a representation, i.e. some chosen set of base states. However, let us skip the niceties here.

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to:

1. Feynman's *Lectures on Physics* (<http://www.feynmanlectures.caltech.edu>). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

One should also mention the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, *The Strange Theory of Light and Matter*, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Francesco Celani:

3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.
4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, <https://arxiv.org/pdf/0802.2728.pdf>.
5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

We would like to mention the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (<https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html>). In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, <https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf>

We should, perhaps, also mention the following critical appraisal of the quantum-mechanical framework:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

It is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: “Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research.” (p. 1-10)

Finally, I would like to thank Prof. Dr. Alex Burinskii for taking me seriously. He is in a different realm – and he has made it clear that my writings are extremely simplistic and probably serve pedagogic purposes only. However, his confirmation that I am not making any *fundamental* mistakes while trying to understand the fundamentals, have kept me going on this.