Proof of Twin Prime Conjecture

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August 2019

Author’s Biography

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this proof of twin prime conjecture is completely K.H.K. Geerasee Wijesuriya's proof.

Geerasee is now 30 years old and she studied before at Faculty of Science, University of Colombo Sri Lanka. And she graduated with BSc (Hons) in Physics and Mathematics from the University of Colombo, Sri Lanka in 2014. And in March 2018, she completed her first Doctorate Degree in Physics with first class recognition. Now she is following her second PhD in Astrophysics with Belarusian National Technical University.

Geerasee has been invited by several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, She has received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Mathematics department, Faculty of Engineering, University of Moratuwa, Sri Lanka. Furthermore she has achieved several other scientific achievements already.

List of abbreviations

Faculty of Science, University of Colombo, Sri Lanka , Belarusian National Technical University Belarus

Acknowledgement

I would be thankful to my parents who gave me the strength to go forward with mathematics and Physics knowledge and achieve my scientific goals.

Keywords: prime; contradiction; greater than ; natural number

Abstract

A twin prime numbers are two prime numbers which have the difference of 2 exactly. In other words, twin primes is a pair of prime that has a prime gap of two. Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair. Up to date there is no any valid proof/disproof for twin prime conjecture. Through this research paper, my attempt is to provide a valid proof for twin prime conjecture.

Literature Review

The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that p + 2 is also prime. In 1849, de Polignac made the more general conjecture that for every natural number k, there are infinitely many primes p such that p + 2k is also prime. The case k = 1 of de Polignac's conjecture is the twin prime conjecture.

A stronger form of the twin prime conjecture, the Hardy–Littlewood conjecture (see below), postulates a distribution law for twin primes akin to the prime number theorem. On April 17, 2013, Yitang Zhang announced a proof that for some integer N that is less than 70 million, there are infinitely many pairs of primes that differ by N. Zhang's paper was accepted by Annals of Mathematics in early May 2013. Terence Tao subsequently proposed a Polymath Project collaborative effort to optimize Zhang's bound. As of April 14, 2014, one year after Zhang's announcement, the bound has been reduced to 246. Further, assuming the Elliott–Halberstam conjecture and its generalized form, the Polymath project wiki states that the bound has been reduced to 12 and 6, respectively. These improved bounds were discovered using a different approach that was simpler than Zhang's and was discovered independently by James Maynard and Terence Tao.

Assumption

Let's assume that there are finitely many twin prime numbers.

Therefore we proceed by considering that there are finitely many twin prime numbers. Then let the highest twin prime numbers are P_{n-1} and (P_{n-1} +2). Then for all prime numbers P_n greater than P_{n-1} , (P_n - 2) is not a prime number.

Methodology

With this mathematical proof, I use the contradiction method to prove the twin prime conjecture.

Let P_n is an arbitrary prime number greater than P_{n-1} (because there are infinite number of prime numbers). Then according to our consideration, (P_n - 2) is not a prime number. Since $P_n > 2$ and since P_n is a prime number and since P_n is an odd number, for all prime numbers P_i :

P_i (< P_n / 2): $P_n / P_i = r_1$

Thus $P_n = P_i * r_1$..................(01)

Where r_1 is a rational number (which is not a natural number)

But according to our consideration, (P_n - 2) is not a prime number. Also since P_n is a prime number greater than 2, (P_n - 2) is an odd number.

Thus for some prime number P_1 (< [(P_n - 2) / 2]) ; (P_n - 2) / $P_1 = x_1$. Where we choose P_1 such that x_1 is a natural number. But since previously chose P_i is any arbitrary prime number less than (Pn / 2); now we consider $P_1 = P_i$

Then (P_n - 2) = $P_1 * x_1$ ....(02) and $P_n = P_1 * r_1$..................(01)

Let P_N is a prime number (greater than P_n). Then according to our assumption, (P_N + 2) is not a prime number. Here P_N is a prime number such that (P_N + 2) is dividing by prime number P_2. …………………(1.1)

Thus (P_N + 2) = $P_2 * x_2$ for some x_2 natural number. Because there are infinitely many prime numbers.

Since P_N is a prime number, for some r_2 (rational number which is not a natural number):

$P_N / r_2 = P_2$.

Thus $(P_N + 2) = P_2 * x_2$ ………………(03) and $P_N = r_2 * P_2$ ……………….(04)

x_1 and x_2 are natural numbers and P_1 and P_2 are prime numbers.

Since P_N is a prime number , $(P_N – 2)$ is also not a prime number (Since $P_N – 2 > P_{n-1}$)

Then for some prime P_3 , $(P_N – 2) / P_3 = x_3$

$(P_N – 2) = P_3 * x_3$ ……………………..(05)

By (04) and (05): $P_3 * x_3 = P_2 * r_2 - 2$ …………………..(06)

But according to the below induction method proof, there exists primes P_n and P_N such that $(P_N – 2)$ and $(P_n – 2)$ both are divisible by 3 (where $P_1 = 3$). * To see the induction method proof, please refer the 'Proof ' below.**

Then $(P_N – 2) = (P_n – 2) + 3.l$ for some l natural number.

Thus $P_N = P_n + 3.l$ ……………………(*)

By (*): $P_1. r_1 + 3.l = r_2 * P_2$. Thus by (06): $P_3 * x_3 = P_1. r_1 + 3.(l -1) + 1$ ……………(6.1)

But $P_1. r_1$ (= P_n) is an odd number. Thus $[P_1. r_1 + 1]$ is an even number. Thus $[P_1. r_1 + 1]$ is not divisible by 3 (=P_1). Therefore, when we write $[P_1. r_1 + 1] = P_1 .r$; where r is not a natural number and r is a rational number. Thus by (6.1): $P_3 * x_3 = P_1. r + 3.(l -1)$.

Thus $P_3 * x_3 - 3.(l -1) = P_1. r$ ……………………(08)

Choose $l = m.P_1 + 1$; where m is a natural number.

*****Please refer the proof below to see the existence of 'l ' natural number such that**

$P_N = P_n + 3.l$.

i.e. there exists l (= $(m.P_1 + 1)$) natural number such that $P_N = P_n + 3.(m.P_1 + 1)$

Where we consider that $l = m.P_1 + 1$ …………..(9.0.0)

NOW BEFORE READING THE NEXT PART BELOW, PLEASE REFER ‘PROOF’.

Then:

For l = m.P_1 + 1 , by (08): (P_3 * x_3)- [3* m.P_1] = P_1 * r ……………………………….(09)

where m is a natural number. Also (m.P_1 + 1) is a natural number.

But for the prime number P_N and P_n (greater than P_{n-1}), there exists P_1 (=3) prime number such that (P_N - 2) / P_1 = x_3 …………………………(10) (According to the equation (13) in the “Proof” mentioned below)

Because we chose (P_N - 2) in that manner. Where P_N is a prime number.

Thus in (09): $P_3 \equiv P_1$.

Thus by (09): (P_1 * x_3)- [3.m.P_1] = P_1 * (x_3 - 3.m) = P_1 * r

Thus x_3 - [3* m] = r . But (3*m) is a natural number. Thus x_3 - [3* m] is a natural number. But r is not a natural number……………..(11)

Thus by (11), there is a contradiction.

Therefore the only possibility is: our assumption is false.

Therefore there are infinitely many Twin Prime Numbers.

Proof

We know the equation related to “prime gap” as written below.

$P_N = 2 + \sum_{j=1}^{N-1} gj$ ……………..(i)

*** refer the 2nd reference.

But for all $\epsilon_{N-1} > 0$, there exists ‘N - 2’ natural number such that for all N -1 > N - 2,

$g_{N-1} < P_{N-1} * \epsilon_{N-1}$

*** refer the 2nd reference below.

Then for some C_{N-1} positive number, $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1}$ for all $\epsilon_{N-1} > 0$

But $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1}$ for all $N - 1 > N - 2$

Choose $\epsilon_{N-1} = [(P_n + 3.m.P_1 + 1 - A) + C_{N-1}] / P_{N-1} > 0$. Then $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1} =$

$(P_n + 3.m.P_1 - A + 1)$. Here the chosen m natural number is responsible for $\epsilon_{N-1} > 0$.

Here $g_j = a_j$ …..(ii) for all $j < (N – 1)$. Where a_j is a natural number. Where we considered $\Sigma a_j = A$ for $j < N – 1$.

Thus by (i): $P_N = 2 + P_n + 3. m.P_1 - A + 1 + A = P_n + 3.m.P_1 + 3 = P_n + 3. (m.P_1 + 1)$

Therefore there exists a natural number l (= $m.P_1 + 1$) such that $P_N = P_n + 3.(m.P_1 + 1)$ ……..….(12)

Now let's prove that there exists infinite number of Prime numbers P_N such that $3| (P_N - 2)$, by using mathematical induction method as below.

Let's consider the statement Q(n) : [P(n) - 2] / 3 = x(n) ; where P(n) is the nth prime number which obeys P(n) + 3 = 3. x(n). And the meaning of x(n) is similar to that.

Q(1): [5 – 2] / 3 = 1 = x(1) = a natural number. Thus for n =1 , the result holds.

Now assume for n = s, the result Q(s) holds. Then $[P_s - 2] / 3 = x(s)$ = natural number.

*** Here we must considered n = s [Here s ≡ (M - 1)] as:

for the positive real number $\epsilon_s = [(P_s - B + C_s - 2 + 6.k'] / P_s > 0$, the natural number (s - 1) is existing such that, for all s > (s - 1) , $g_s < P_s * \epsilon_s$. Where k' is a natural number. Here the chosen k' natural number is responsible for $\epsilon_{M-1} > 0$. Here $h_j = b_j$ for all $j < (M – 1)$. Here $\Sigma b_j = B$ for $j < (M – 1) = s$. *** refer the 2nd reference.

But s ≡ (M - 1)

But here we chose C_{M-1} such that $g_{M-1} = P_{M-1} * \epsilon_{M-1} – C_{M-1}$

But $g_{M-1} = P_{M-1} * \epsilon_{M-1} – C_{M-1} = (P_s - B – 2 + 6.k')$. Where k' is a natural number.

Then let's show for n = s +1, Q(s+1) holds. We denote $P(s+1) = P_M$

But we know $[P_s - 2] / 3 = x(s)$ ………………..…..(12.1)

Now let's use the 2nd reference to proceed further.

By 2nd reference, $P_M = 2 + \sum_{j=1}^{M-1} hj$ ……………….(ii)

But we know already that for $Є_{M-1} > 0$, there exists 'M - 2' natural number such that for all M -1 > M - 2, $G_{M-1} < P_{M-1} * Є_{M-1}$. Here $s \equiv (M - 1)$

(*** refer the 2nd reference below)

Then we already know that for some C_{M-1} positive number, $g_{M-1} = P_{M-1} * Є_{M-1} – C_{M-1}$.

But $g_{M-1} = P_{M-1} * Є_{M-1} – C_{M-1}$ for all $M – 1 > M – 2$

We know already that $Є_{M-1} = [(P_s - B + C_{M-1} – 2 + 6.k'] / P_{M-1} > 0$.

And $g_{M-1} = P_{M-1} * Є_{M-1} – C_{M-1} = (P_s - B – 2 + 6.k')$. Where k' is a natural number. We know already that the chosen k' natural number is responsible for $Є_{M-1} > 0$.

We know that $h_j = b_j$ for all $j < (M – 1)$. Where b_j is a natural number. Also we know that $\Sigma b_j = B$ for $j < M – 1$.

Thus by (ii): $P_M = 2 + P_s + 6.k' - B - 2 + B = P_s + 6.k'$ ……………..(12.2)

But $(P_s - 2)$ is divisible by 3 (= P_1) according to (12.1). Thus $(P_M - 2)$ is divisible by 3 (=P_1) according to (12.2), since 6.k' is divisible by 3.

Thus $(P_M – 2)$ is divisible by 3 (= P_1). i.e. $[P(s+1) – 2]$ is divisible by 3 (= P_1).

Thus for n = s + 1 , the result Q(n +1) holds. Thus by mathematical induction method:

There exists infinite number of prime numbers P_M such that $3 | (P_M - 2)$.

Thus there exists P_n and P_N primes (where we consider them as prime numbers greater than P_{n-1}) such that $(P_n – 2)$ and $(P_N - 2)$ both are divisible by 3 (=P_1). Thus $(P_N - 2)$ is divisible by P_1 (=3).

That means we have the capability to consider $P_3 \equiv P_1$ in the equations in the methodology…..........................(13)

Discussion

We assumed initially that there are finitely many twin primes. After proceeding with that, I ended up with a contradiction. But to get the contradiction, I used that P_n and P_N as primes numbers greater than P_{n-1} . Also to get the contradiction, I used the facts that (P_n - 2) and (P_N - 2) as non-primes. And also I have used that x_1 , x_3 as natural numbers (since P_n - 2 and P_N - 2 are not prime numbers). Therefore to get the contradiction, I have used the facts got from our assumption. Then the only possibility is our assumption is false.

Results

Therefore I have used our assumption to get a contradiction finally as showed in (11). Therefore it is possible to conclude that our assumption is false.

Thus there are infinitely many twin prime numbers.

Appendix

Prime number: A natural number which divides by 1 and itself only.

Twin Prime Numbers: Two prime numbers which have the difference exactly 2.

We denote 'i' th prime gap $g_i = P_{i+1} – P_i$

Then according to the 2nd reference; Prime number $P_N = 2 + \sum_{j=1}^{N-1} gj$

Also by 2nd reference: for all $\epsilon > 0$, there is a natural number 'n' such that for all N -1 > n;

$g_{N-1} < P_{N-1} . \epsilon$

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