Refutation of the predicatively unprovable termination of the Ackermannian Goodstein process

Abstract: We evaluate the Goodstein theorem which is *not* tautologous. This refutes such follow-on as the Ackermanian Goodstein process and conjecture of its predicatively unprovable termination. Therefore that segment is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Arai, T.; Fernández-Duqaue, D.; Wainer, S.; Weiermann, A. (2019).
Predicatively unprovable termination of the Ackermannian Goodstein process.
arxiv.org/pdf/1906.00020.pdf

1. Introduction: Among the greatest accomplishments of mathematical logic in the first half of the twentieth century was the identification of true arithmetical statements unprovable in Peano arithmetic (PA): the consistency of PA, due to Gödel ... However, such statements do not clarify whether incompleteness phenomena should be pervasive in other disciplines such as combinatorics or number theory. In contrast, Goodstein's principle is a purely number-theoretic statement simple enough to be understood by a high school student yet unprovable in PA.

Theorem 1.1 (Goodstein). For every $m \in$ N there is $i \in$ N such that $G_i m = 0$. (1.1.1)

Remark 1.1: This is repeated as theorem 2.9.

LET p, q, r, s: *i*, G, *m*, N.

((#s<r)>(%q<r))>(((p&q)&s)=(s@s)) ;
`TTTT TTTT TTT`**`F`** `TTT`**`F`** (1.1.2)

Eq. 1.1.2 as rendered is *not* tautologous, and so colors combinations of it as in an Ackermannian process with predicatively unprovable termination.