Refutation of the Gödel-McKinsey-Tarski translation of intuitionistic logic (IPC)

Abstract: We evaluate the Gödel–McKinsey–Tarski [GMT] translation of IPC "by reduction from intuitionistic logic (IPC) using a series of translations". The equation is *not* tautologous. This refutes GMT, the method approach using a series of translations, and IPC itself which form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Jeřábek, E. (2019). Rules with parameters in modal logic II. arxiv.org/pdf/1905.13157.pdf

Abstract: We analyze the computational complexity of admissibility and unifiability with ... parameters (constants) in transitive modal logics satisfying certain extension properties

3 Derivability: … what is the complexity of tautologicity or derivability in these logics.

We now turn to the lower bound. … We will use another method, namely by reduction from intuitionistic logic (IPC) using a series of translations. This route is more useful for our purposes, because the resulting statement is relatively more general in the context of transitive modal logics (it applies to all transitive logics with the disjunction property, and it also applies to various extensions of K4.2, which will be relevant in the sequel).

Definition 3.5 Let T denote the *Gödel–McKinsey–Tarski translation* of **IP**C (formulated using connectives {→,∧,∨,⊥}) in **S4**: T(ϕ)=□ϕ if ϕ is an atom, T commutes with ∧,∨, and ⊥, and T(ϕ→ψ)=□(T(ϕ)→T(ψ)). (3.5.1)

LET p, q: ϕ, ψ.

((p=p)&(p>q))=#(((p=p)&p)>((p=p)&q)) ;
NTTN NTTN NTTN NTTN (3.5.2)

Remark 3.5.2: Eq. 3.5.2 as rendered is not tautologous as the Gödel–McKinsey–Tarski [GMT] translation of IPC. This refutes GMT, the method approach of "reduction from intuitionistic logic (IPC) using a series of translations", and IPC itself.